Studies of plasma sheaths with secondary electron emission using continuum kinetic model

Cagas P.,[1] Hakim A.,[2] and Srinivasan B.[1]

[1]Virginia Tech, Blacksburg, VA, USA, [2]Princeton Plasma Physics Laboratory, Princeton, NJ, USA

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Introduction

A continuum kinetic plasma model is used to study magnetized plasma sheaths by directly evolving the ion and electron distribution functions along with Maxwell's equations. Appropriate boundary conditions need to be included to account for secondary electron emissions at the walls. Secondary electron emission (SEE) from a solid surface can drastically influence the plasma behavior – some recent works suggest that SEE can even reverse the gradient of the electrostatic potential in the plasma sheath. Therefore, a self-consistent SEE model based on real material parameters needs to be included in numerical models. Currently, SEE is commonly implemented using Monte-Carlo algorithms. However, this work presents a novel approach where the full velocity distribution function of SEE is directly constructed using the incident electron population and phenomenological material fits. This distribution function then can be used as the boundary condition in the continuum kinetic simulation. Parts of this work have been published [Cagas et al., 2017].

Numerical Model

This work uses the discontinuous Galerkin (DG) [Cockburn and Shu, 2001] discretization of the full non-relativistic Vlasov-Maxwell system implemented in the `Gkeyll` framework [Juno et al., 2017]. This approach involves directly discretizing the Boltzmann equation for each species s

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\cdot\frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s}\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\frac{\partial f_s}{\partial \mathbf{v}} = C_s, \tag{1}$$

where f is the particle distribution defined in phase-space. The species are then coupled together using the full Maxwell equations.

- Unlike the fluid methods, which are derived by taking the moments of Eq. (1), the continuum kinetic method does not make any assumptions about the shape of the particle distribution function.
- Since the full continuous distribution function is discretized, the results are not affected by statistical noise.
- Phase-space has high dimensionality (up to 6D) and needs to be discretized directly.

Plasma Sheaths

In the simplest hydrogen plasma, ion (protons) are 1836× heavier in comparison to electrons. This results in higher electron mobility and higher thermal flux even if the species have the same temperature. Therefore, when plasma comes into contact with a solid surface acting as a sink, electrons quickly leave the domain and the quasineutrality no longer holds. This non-quasineutral region near the wall is called sheath. Inside sheath, electric field self-consistently arises and equalizes the fluxes – accelerates ions and retards electrons. Even though it usually spans only couple Debye lengths, it can have global effects on plasma.

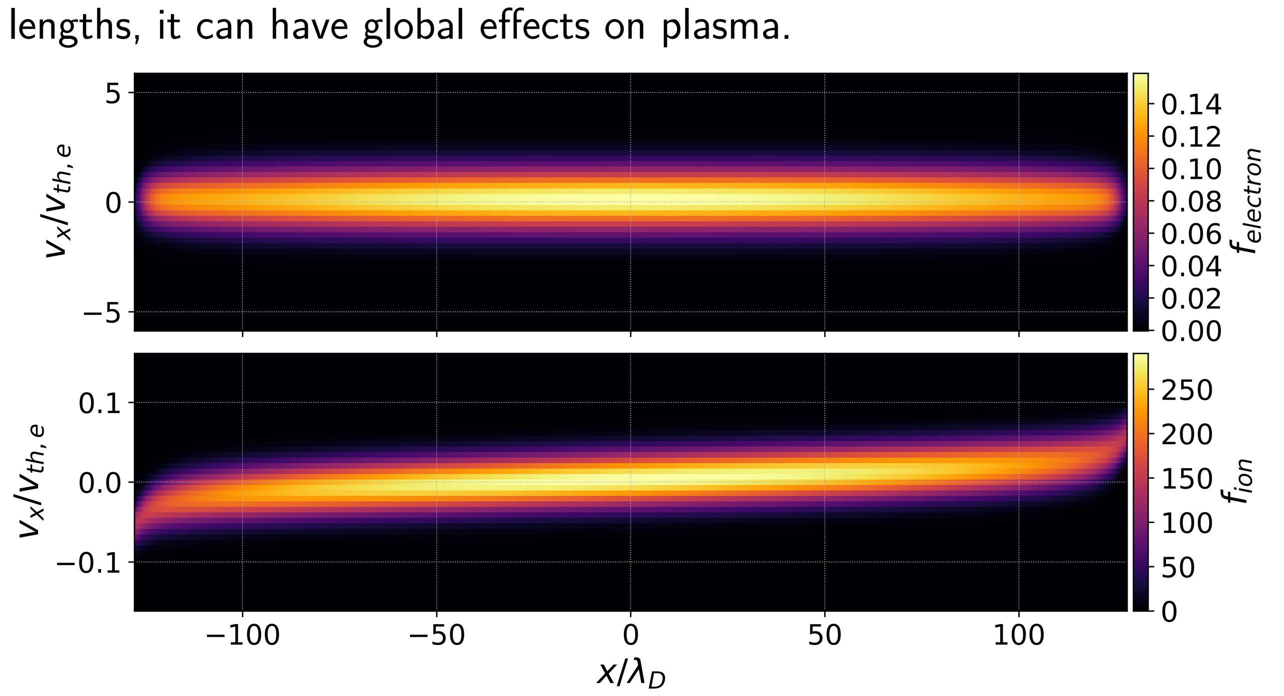


Figure 1: Electron (top) and ion (bottom) distribution function from plasma sheath simulations with one configuration space dimensions.

Numerical Simulations

To compensate for particles leaving the simulation domain, ionization and elastic collisions are added to the right-hand-side of the Boltzmann equation (Eq. 1).

Inelastic collisions (particle impact ionization) act as a source

$$C_s^{ionization} = f_n(\mathbf{v})\int_{\mathcal{V}}\sigma(|\mathbf{v}-\mathbf{v}'|)|\mathbf{v}-\mathbf{v}'|f_e(\mathbf{v}')d\mathbf{v}' \tag{2}$$

BGK operator is implemented in order to account for elastic collisions

$$C_s^{BGK} = \nu_{ss}\left(f_{M,s}-f_s\right), \quad \nu_{ss} = \frac{e^4}{2\pi\varepsilon_0^2 m_s^2}\frac{n_s}{v_{th,s}^3}\ln(\Lambda) \tag{3}$$

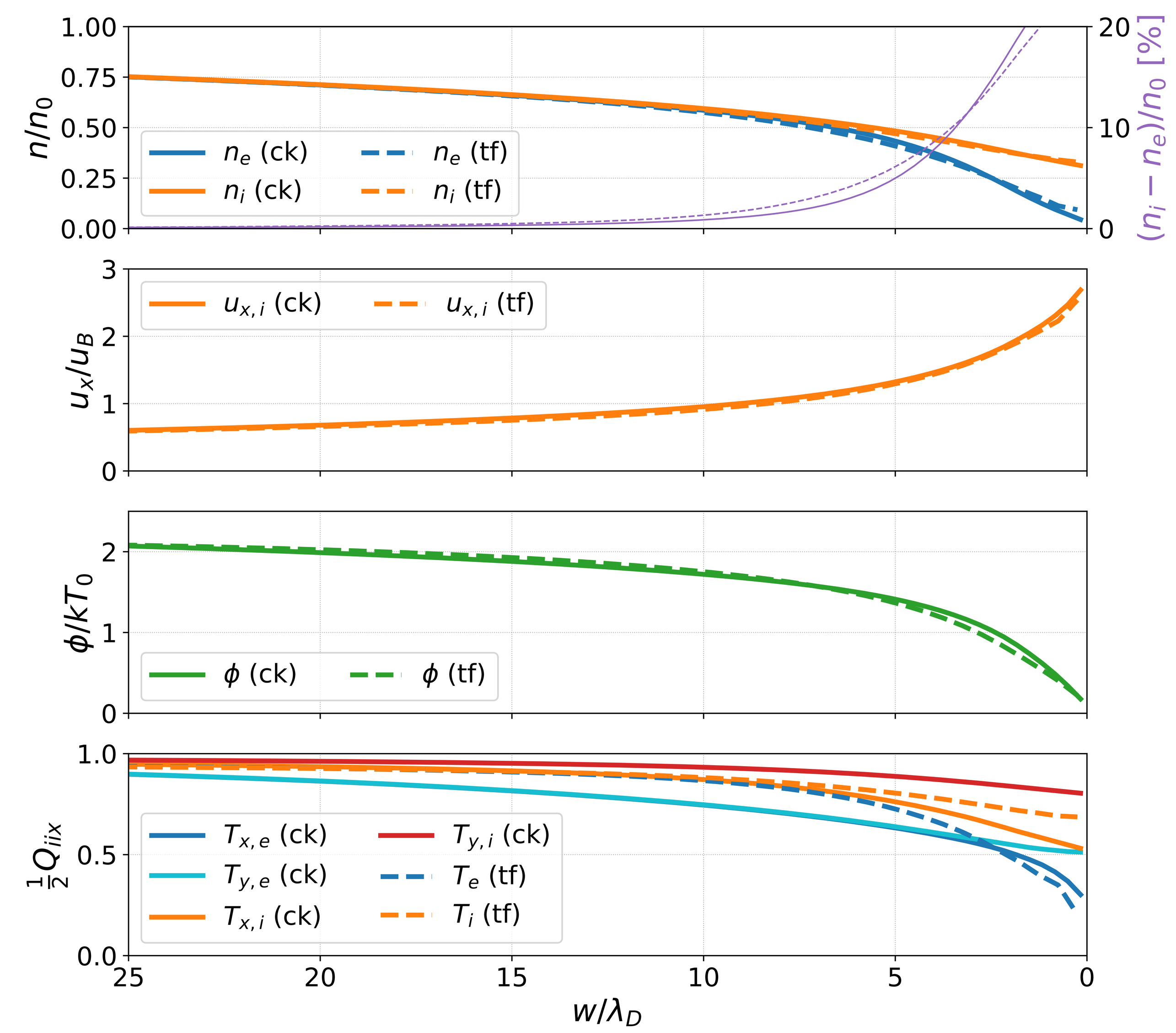


Figure 2: Plasma profiles in the region near the wall.

The difference between 1X2V continuum kinetic simulations and 5-moment two-fluid simulations lies in the heat flux tensor

$$\mathcal{Q}_{ijk} = m\int v_i v_j v_k f \, d^3\mathbf{v}. \tag{4}$$

Its contraction leads to

$$\frac{1}{2}\mathcal{Q}_{iix} = \underbrace{q_x + u_x\Pi_{xx}}_{non-ideal} + \underbrace{\frac{5}{2}pu_x + \frac{1}{2}mnu_x^3}_{ideal}, \tag{5}$$

where Π_{xx} is the parallel component of the stress tensor, p is pressure, and

$$q_x = \frac{1}{2}m\int_{-\infty}^{\infty}\int_0^{\infty}\left(w_x^2+v_\perp^2\right)w_x f(v_x,v_\perp)2\pi v_\perp dv_\perp dv_x \tag{6}$$

is the heat flux vector in the plasma frame, and $w_x = v_x - u_x$

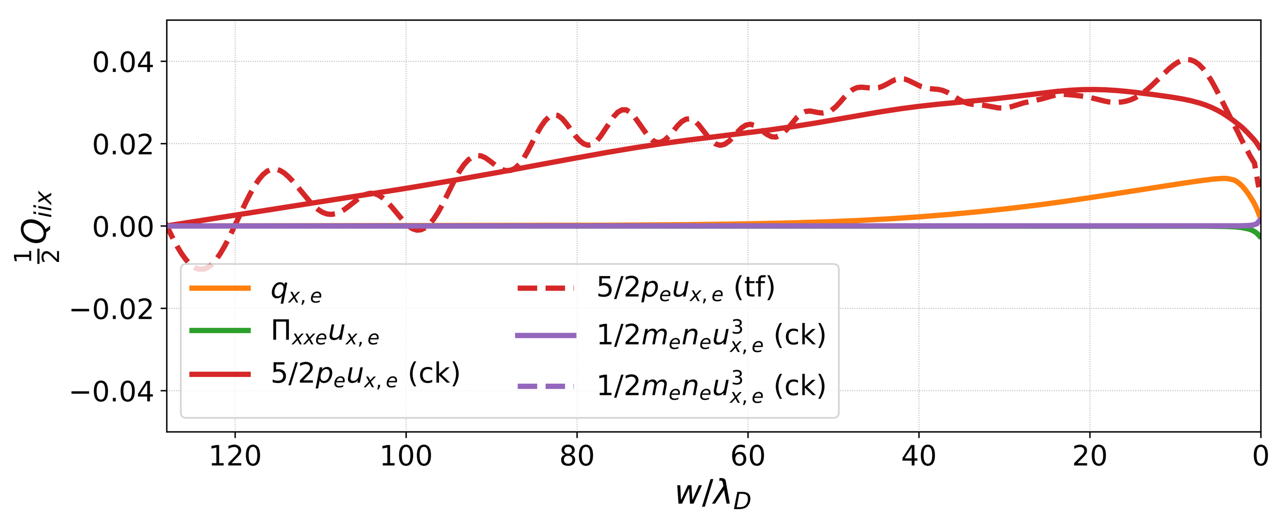


Figure 3: Components of the heat flux tensor. Results of the two-fluid plasma model (tf) are also plotted in addition to the continuum kinetic results (ck).

Secondary Electron Emission

We use the probabilistic model derived by Furman and Pivi [2002]. The model describes the three types of the secondary electrons

Backscattered Electrons which are elastically reflected from the wall. They are fitted with narrow Gaussian.

Rediffused Electrons which penetrate the material, scatter on atoms inside and then leave the material again. They are fitted with a polynomial.

True-secondary Electrons originally from the material which are excited by incoming flux and emitted out. In the model, they are described with a product of polynomial and exponential function. Unlike with the previous two types, single incoming electron can cause an emission of many true-secondary electrons.

All together give

$$\frac{d\delta(E,\theta)}{dE} = f_{1,e}(E,\theta) + f_{1,r}(E,\theta) + \sum_{n=1}^{\infty} f_{n,ts}(E,\theta), \tag{7}$$

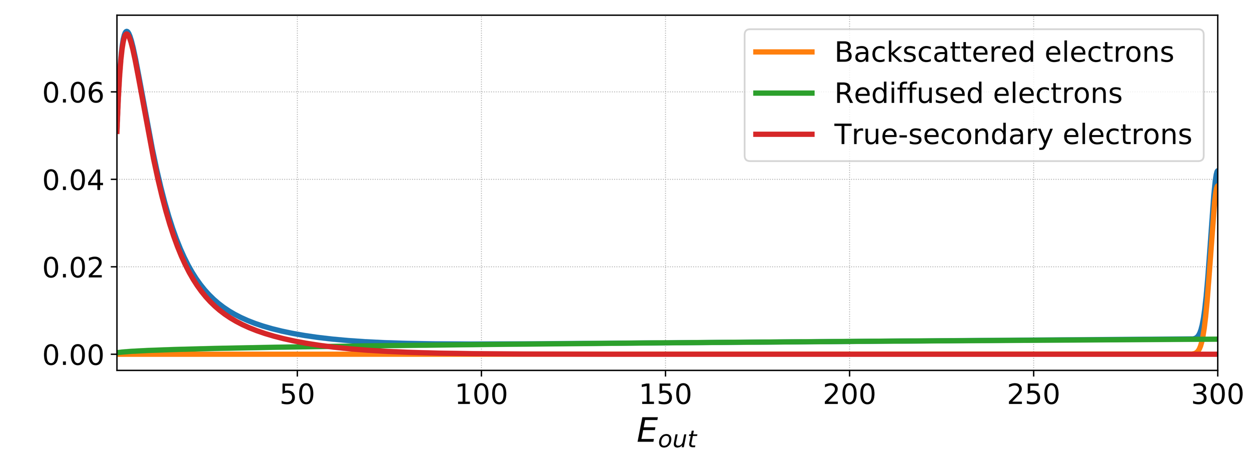


Figure 4: Energetic distribution of SEE based on the phenomenological model by Furman and Pivi [2002]. Calculated for 300 eV electron beam with normal incidence.

In the original work, the function $d\delta/dE$ is used to initialize Monte-Carlo SEE from a single particle. For continuum kinetic method, it can be used as a part of a reflection function to initialize the full distribution function of the emitted particles.

$$\begin{aligned} f_{SEE}(v_{x,o},v_{y,o}) = \int_{-\infty}^{\infty}\int_0^{\infty}\Bigg(&\frac{d\delta}{dE}(v_{x,i},v_{y,i},v_{x,o},v_{y,o})\cos\left[\theta(v_{x,o},v_{y,o})\right] \\ &f_{wall}(v_{x,i},v_{y,i})\cos\left[\theta(v_{x,i},v_{y,i})\right]m_e\Bigg)dv_{x,i}dv_{y,i} \end{aligned} \tag{8}$$

$$= \int_{-\infty}^{\infty}\int_0^{\infty}\left(F(v_{x,i},v_{y,i},v_{x,o},v_{y,o})f_{wall}(v_{x,i},v_{y,i})m_e\right)dv_{x,i}dv_{y,i} \tag{9}$$

In the continuum kinetic method, the full distribution function at the wall is known at all times. The reflection function can be then used to directly obtain the boundary conditions at the wall.

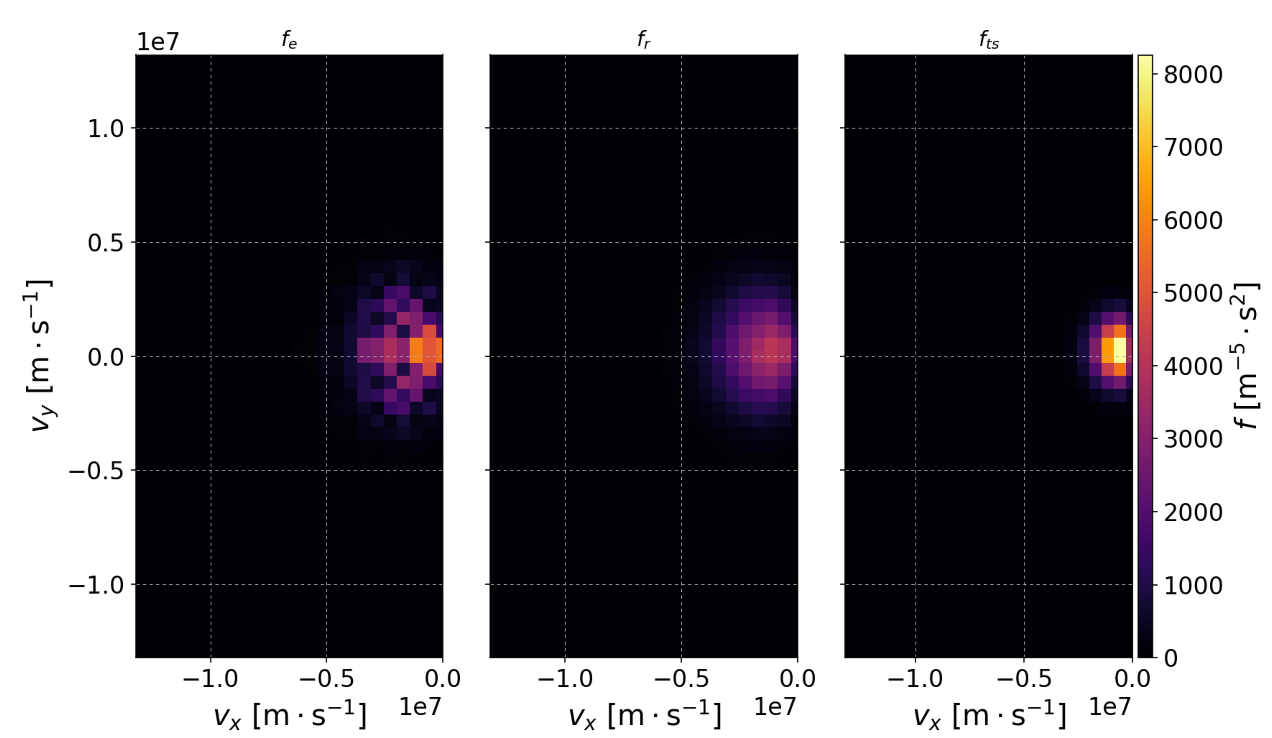
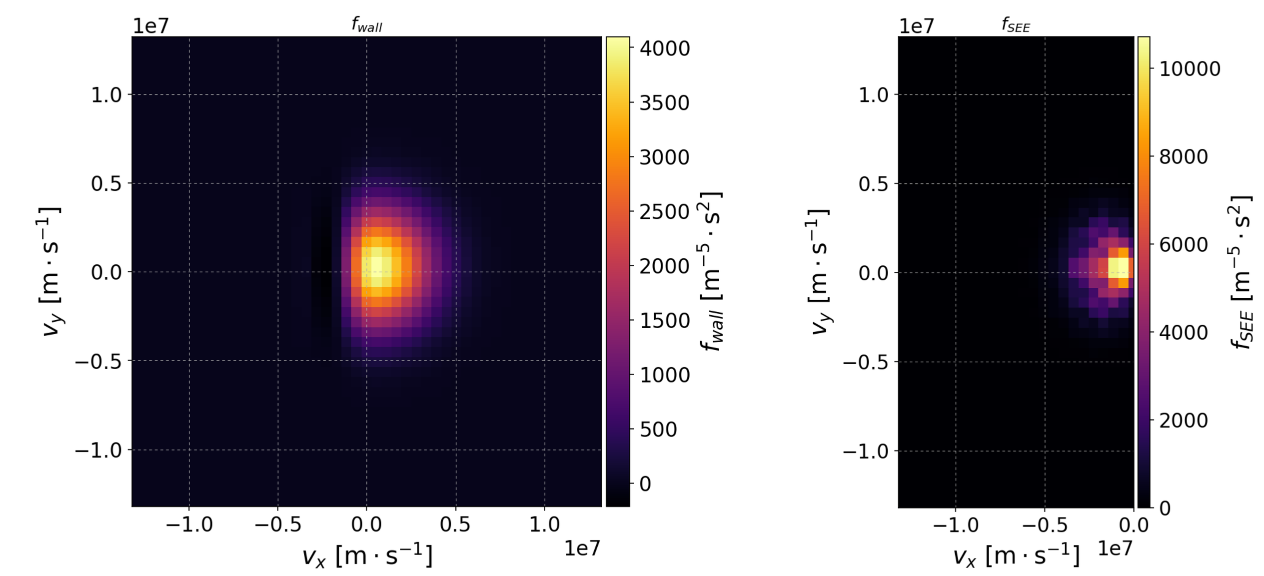


Figure 5: Three populations of SEE calculated from the known distribution function at the wall.

Direct SEE Boundary Condition

For continuum kinetic DG model, SEE needs to be implemented as non-linear boundary condition.



In order to treat SEE properly, integral over the whole velocity space needs to be performed for each cell at wall. This can became computationally expensive. SEE can, however, significantly alter the sheath behavior and is vital for many applications.

Summary

- The kinetic and two-fluid results of sheath are in good agreement for momentum and density of both species over the sheath region, however, the heat flux vector, which is not captured in the five-moment two-fluid model is significant for temperatures in the sheath region.
- Since the distribution of SEE is a function of incoming energy and angle, precise temperature (both parallel and perpendicular) in the sheath region is required.
- Phenomenological model of SEE by Furman and Pivi [2002] can be used as a "reflection function" to directly obtain the continuum kinetic boundary condition at the wall.

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