

AOE 4144: Applied CFD

A series of 12 lectures by Prof. Raj (course co-instructor)

Reflections on the Effectiveness of Applied Computational Aerodynamics for Aircraft Design

Pradeep Raj, Ph.D.

Collegiate Professor Emeritus

Kevin T. Crofton Department of Aerospace and Ocean Engineering

Virginia Tech, Blacksburg, Virginia, USA

<http://www.aoe.vt.edu/people/emeritus/raj.html>

Program Management Director, Lockheed Martin (Retired)

Deputy Director, Technology Development & Integration

The Skunk Works®, Palmdale, California, USA



LOCKHEED MARTIN



Lecture 5

*Topic 5: Evolution of Applied Computational Aerodynamics
(Part 1 of 5)*

List of Topics

Preface

1. Introduction
2. Genesis of Fluid Dynamics (*Antiquity to 1750*)
3. Fluid Dynamics as a Mathematical Science (*1750–1900*)
4. Emergence of Computational Fluid Dynamics (*1900–1950*)
5. Evolution of Applied Computational Aerodynamics (*1950–2000*)

5.1 *Infancy through Adolescence (1950–1980)*

Level I: Linear Potential Methods (LPMs)

Level II: Nonlinear Potential Methods (NPMs)

5.2 *Pursuit of Effectiveness (1980–2000)*

Level III: Euler Methods

Level IV: Reynolds-Averaged Navier-Stokes (RANS) Methods

6. ACA Effectiveness: Status and Prospects (*2000–20xx*)

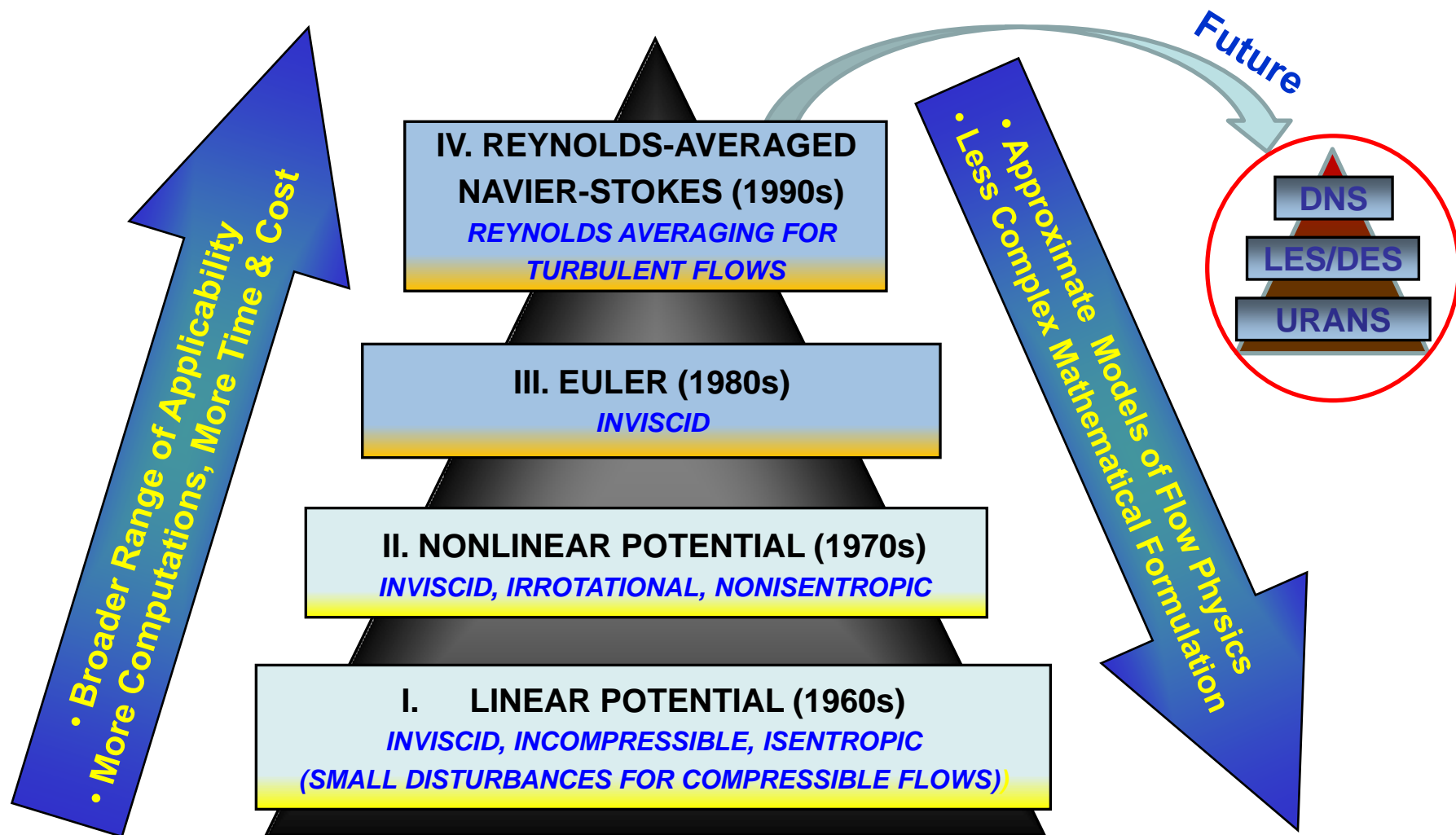
6.1 *Assessment of Effectiveness (2000–2025)*

6.2 *Prospects for Fully Effective ACA (Beyond 2025)*

7. Closing Remarks

Appendix A. An Approach for ACA Effectiveness Assessment

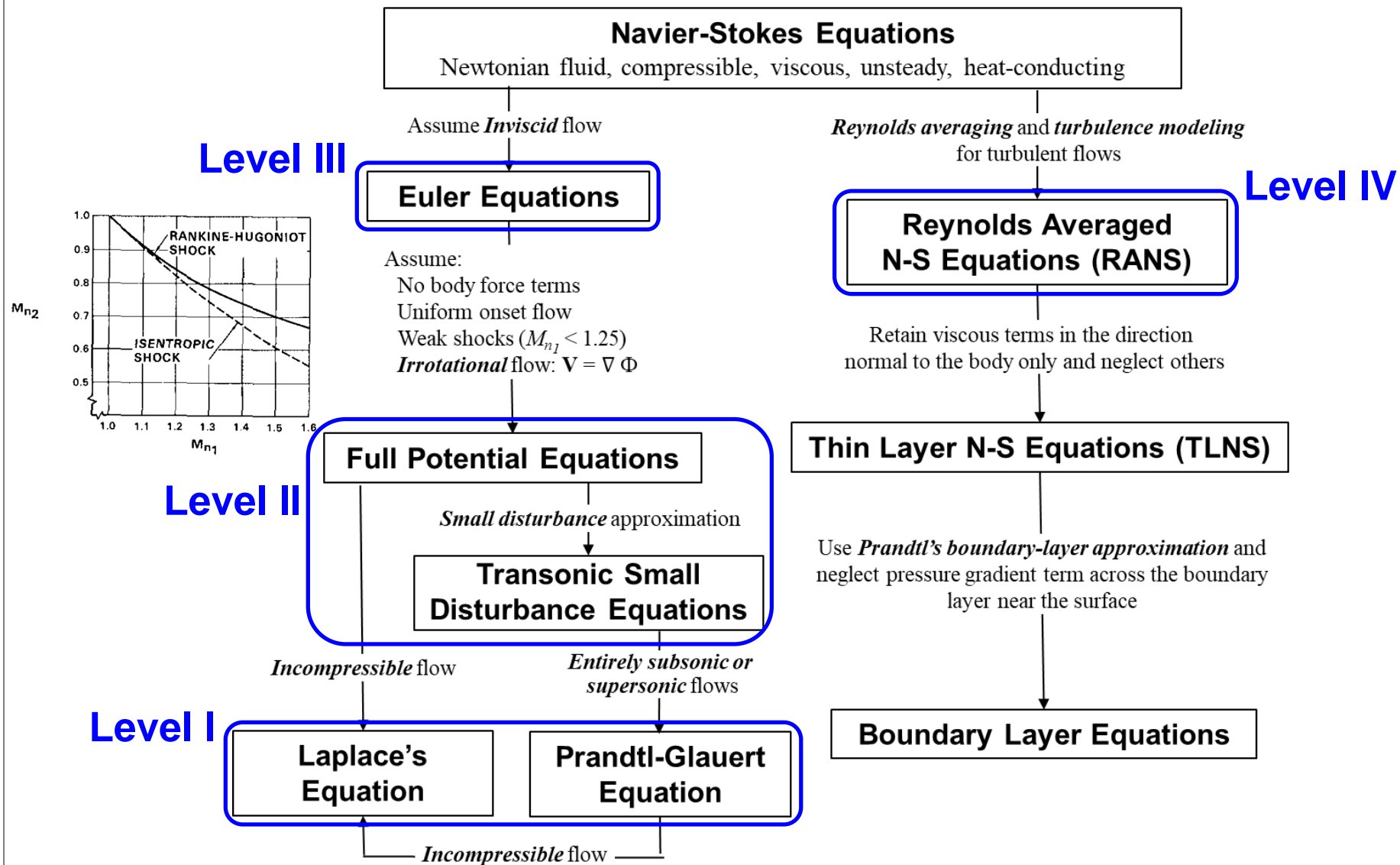
Directly Related to the Evolution & Maturity of Four Levels of CFD Technology



Paced by Impressive Advances Since The 1950s

Note: Time frames in parenthesis indicate *widespread* adoption by industry

Approximations of N-S Equations Mapped to Four Levels of CFD



Adapted from Fig. 2-10, Configuration Aerodynamics
by W.H. Mason

On “Fidelity” and “Credibility” of CFD Methods for Aerodynamic Simulations

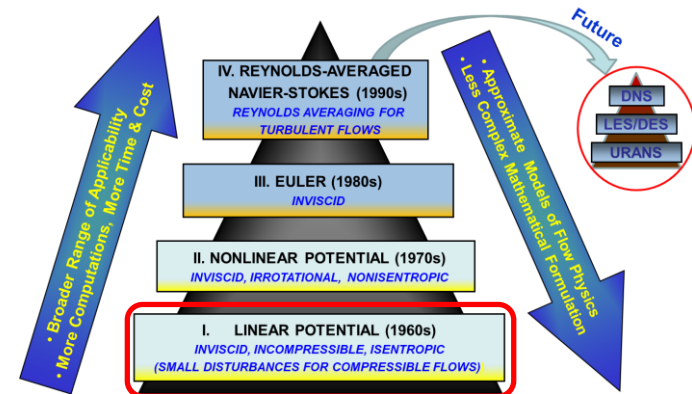
- Many CFD developers characterize *lower level* (potential [flow]) methods as “***low fidelity***” and “***higher level***” (Euler and RANS) methods as “***high fidelity***.” In this context, ***fidelity implies exactness*** with which governing equations approximate flow physics, not necessarily ***trustworthiness of computational solutions in replicating reality***. ***For ACA, credibility implying trustworthiness of solutions is of paramount importance.***
- Experience has shown that ***higher level RANS methods do not necessarily produce credible data especially for complex flows*** that are dominated by vortices and boundary-layer separation. Therefore, one could argue that RANS methods are not “*high fidelity*” for replicating complex flows. When considering *fidelity*, more is not always better. ***Using the “highest fidelity” CFD in all instances can lead to misuse of valuable resources.***
- Since each CFD method is (should be?) carefully designed to solve a selected set of equations as *accurately* as possible, a potential flow method may not be inherently *low fidelity*—as long as the method is *accurately* solving the governing potential flow equations, and *producing credible results for the target application*.
- ***For ACA, it’s the credibility of aerodynamic data that is of utmost importance. Customers must have enough trust in data to use it in making decisions without incurring undue risk. This requires that data produced by a CFD method closely replicate reality. Validation is the most common approach for assessing credibility—albeit not without its own set of challenges to be highlighted later.***

What Matters Most to the Customer is Results, Not Tools!

Level I

Linear Potential Methods

1950s – present



Flow Model

- Inviscid, Irrotational, Isentropic (*Small Disturbances for Compressible Flow*)

$$\mathbf{U} = \mathbf{U}_{\infty} + \nabla \phi$$

$$(\phi_{tt} + 2U_{\infty} \phi_{xt}) / a_{\infty}^2 = (1 - M_{\infty}^2) \phi_{xx} + \phi_{yy} + \phi_{zz}$$

- ✓ Linear second-order PDEs with appropriate boundary conditions
- ✓ Laplace's equation for steady, incompressible flow
- ✓ Prandtl-Glauert equation for steady, compressible flow
- ✓ *Wakes not captured as part of the solution—must be explicitly modeled*

Applicability

- Attached flows that are entirely subsonic or supersonic; *not transonic*
- Flows not dominated by shocks, vortices, or boundary-layer separation

Linear Potential Methods (LPMs)

Overview

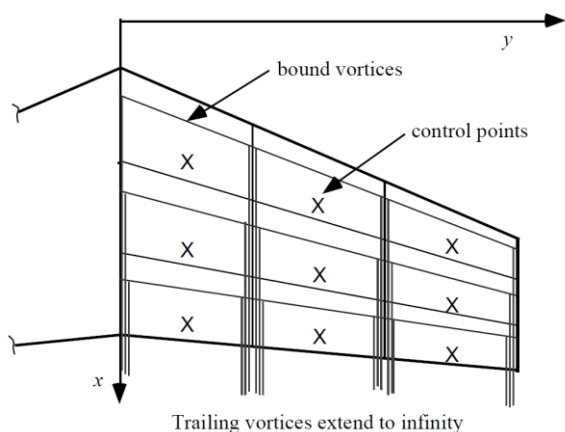
L5

Basic Formulation

- Discretize geometry into small elements
- Distribute singularities (source, doublets, vortex filaments) on each element
- Impose no-normal-flow boundary condition (BC) at control points (one per element), and Kutta condition at sharp trailing edge
- Solve resulting system of linear algebraic equations to determine singularity strengths
- Use Bernoulli's equation to compute airloads

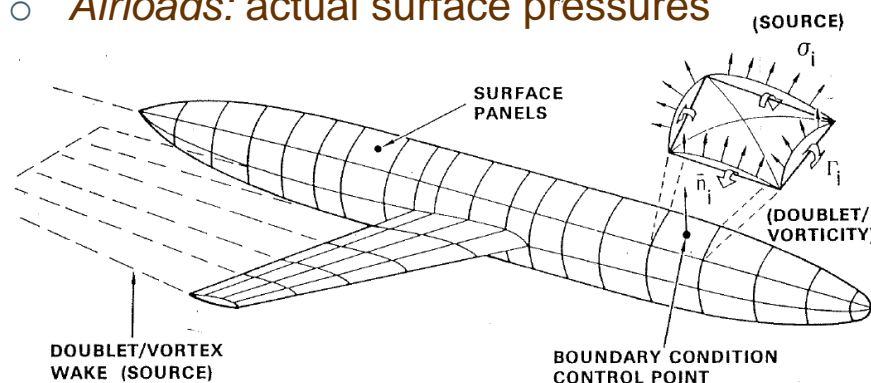
Vortex Lattice Methods (VLMs)

- Geometry:* mean surface
- Singularity type:* horseshoe vortices
- BCs:* control points on mean surface
- Airloads:* net pressure



Panel Methods

- Geometry:* actual surface
- Singularity type:* sources, doublets or both
- Singularity distribution:* constant, linear or higher order
- BCs:* control points on actual surface
- Airloads:* actual surface pressures



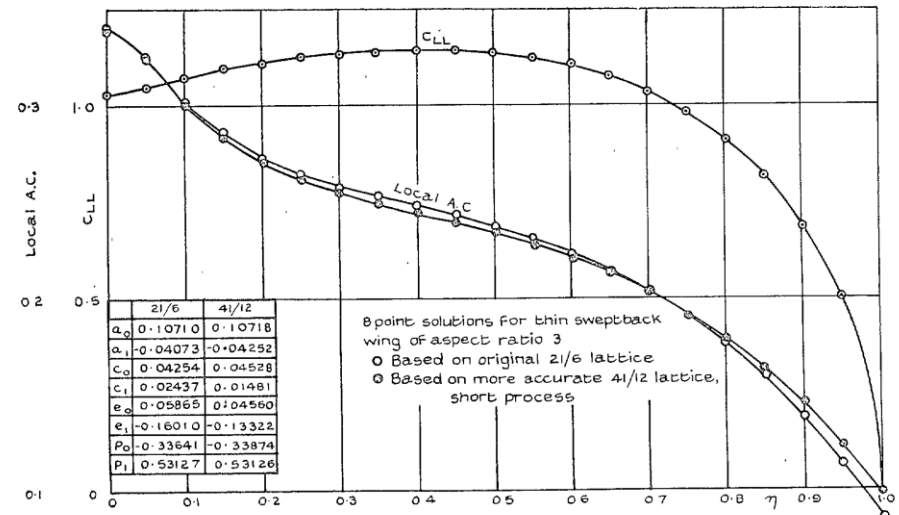
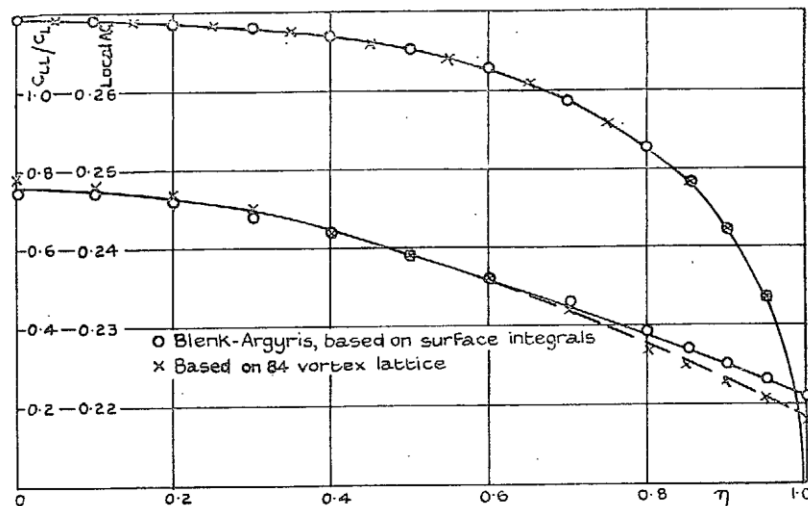
LPMs (VLMs & Panel Methods): Today's Workhorse!

Birth of Vortex Lattice Methods

1940s

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- **V.M. Falkner (1949): “The Scope and Accuracy of Vortex Lattice Theory” Report & Memoranda 2740, Aeronautical Research Council, United Kingdom**
 - Research motivated by the need to calculate loading distribution on a wing of arbitrary plan form including wing twist, discontinuities due to flaps, compressibility, etc.; initiated in early '40s
 - Paper outlines principles of using vortex lattice to solve potential flow problems in lifting plane theory; highlights key developments from Falkner's R&M 2591 (1947) and R&M 1910 (1943)



- **Variations were tried extensively throughout industry during the 1950s**

Advances in Electronic Computers and Numerical Methods Made Practical VLM Applications Possible in the 1960s

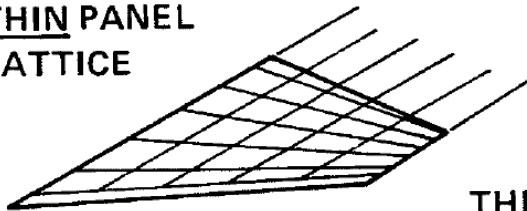
Vortex Lattice Methods (VLMs)

Rapid Development (1960s & 70s)

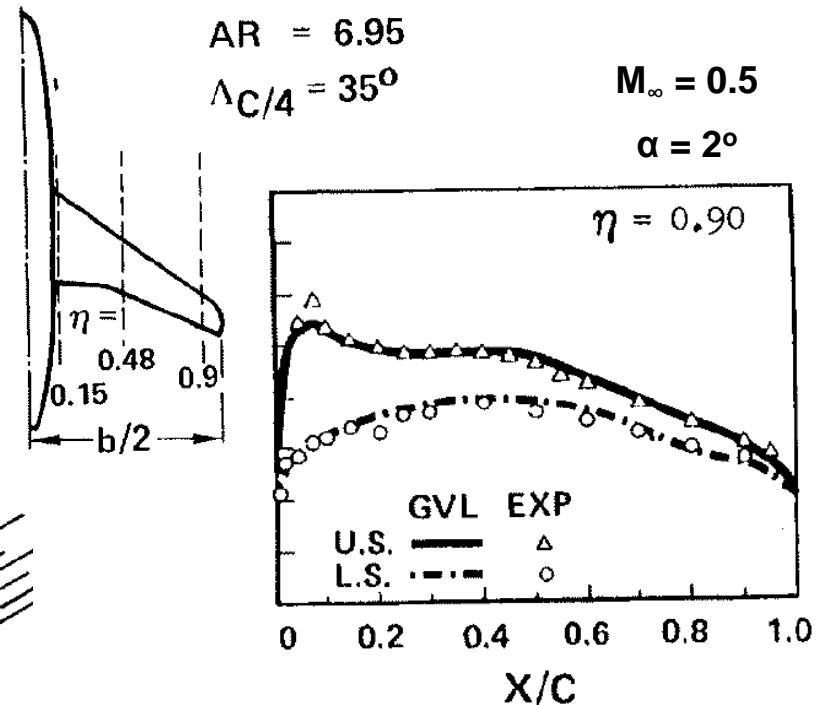
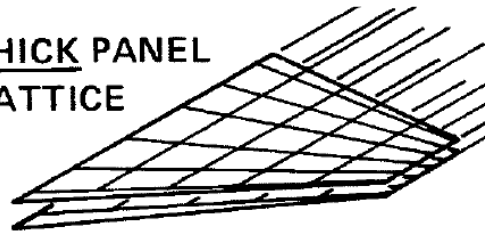
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- **Rubbert (1964)**
 - Non-planar Vortex Lattice Methods; arbitrary wings—*Boeing Co. Document D6-9244*
- **Margason and Lamar (1971)**
 - Vortex-lattice Fortran program for estimating subsonic aerodynamic characteristics of complex planforms—*NASA TN D-6142*
- **Vortex-Lattice Utilization workshop (1976)**
 - Compilation of many papers—*NASA SP-405*
- **Miranda, Elliott and Baker (1977)**
 - A generalized vortex-lattice (GVL) method for subsonic and supersonic flow applications, the VORLAX code—*NASA CR 2865*

**THIN PANEL
LATTICE**



**THICK PANEL
LATTICE**



Falkner's Method Extended and Adapted to Electronic Computers

Birth of Panel Methods

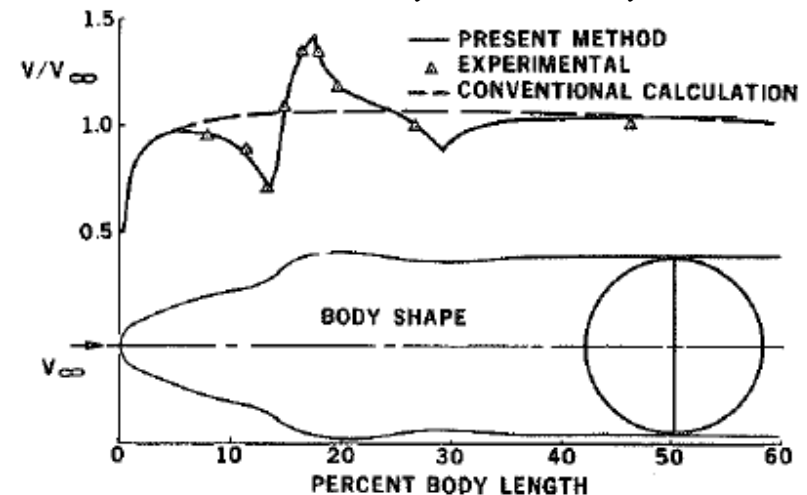
1950s

- **A.M.O. Smith and J. Pierce, Douglas Aircraft Co., Long Beach, CA**
 - Non-circulatory plane [2-D] and axially symmetric flows
 - 1953--Serious work began to solve **Neumann problem**
 - **Continuous source distribution on surface panels**
 - 1954--Programming on **IBM/701** *in machine language!*
 - Test cases selected based on availability of theoretical [analytical] solutions
 - From 24-point body of revolution solutions in 1954 to **150-points** by the end of 1955!
 - DAC financed all work through **1958**
 - ONR contract: extend the method to 3-D non-lifting flows
- **DAC Report E.S. 26988, April 1958**

A.M.O. Smith



Chief Aerodynamics Engineer, Research
2 July 1911 – 1 May 1997

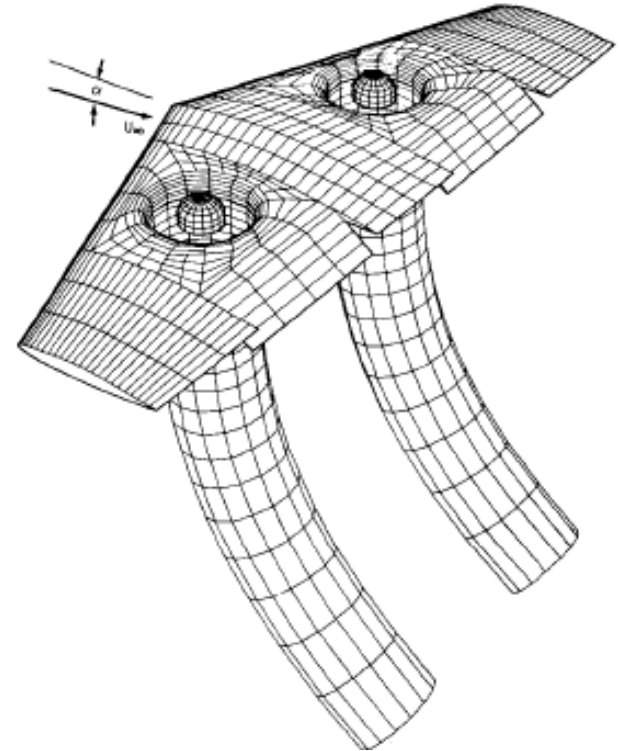


And the Rest is History!

Panel Methods

Rapid Development (1960s & 70s)

- **Hess (1962)**
 - Arbitrary bodies of revolution with axes perpendicular to the free stream direction—*Journal of the Aerospace Sciences*
- **Hess and Smith (1967)**
 - Extensive description of panel methods—*Progress in Aeronautical Sciences, Vol. 8* (138 pages!)
- **Rubbert and Saaris (1968)**
 - Incompressible flow; arbitrary configurations; source and doublet distributions—Fan-in-wing simulation, *SAE Paper 680304*
- **Hess (1970)**
 - Arbitrary 3-D lifting bodies—*McDonnell Douglas Rept. MDC J0971-01* (Also in *Comp. Methods in Applied Mechanics and Engineering*, 1974)
- **Woodward (1973)**
 - Subsonic or *supersonic* flow; wing-body-tail configurations; source and vortex distributions—*NASA CR-2228*



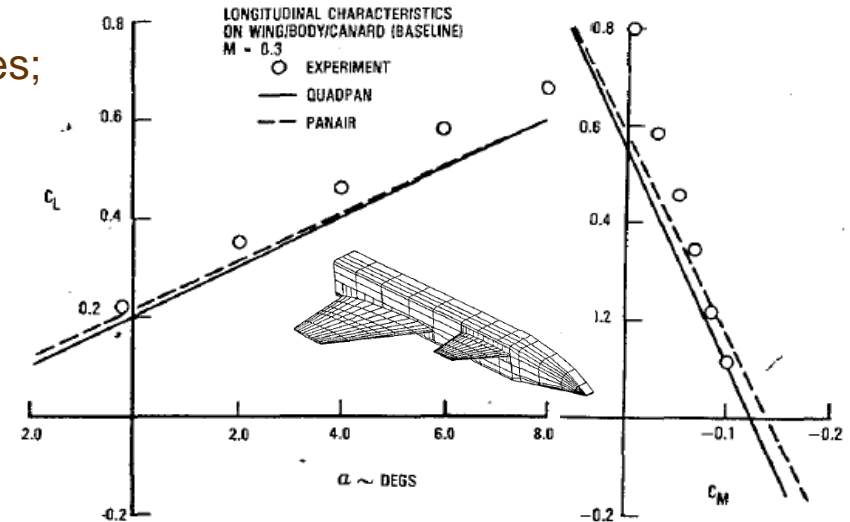
Panels for a fan-in-wing configuration

Powerful Capability to Simulate Flow About Realistic Geometries to Support Aircraft Design Needs

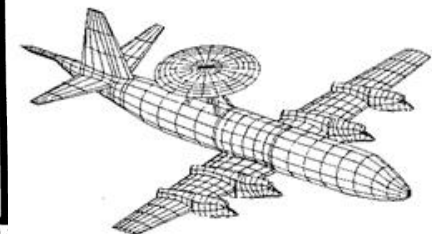
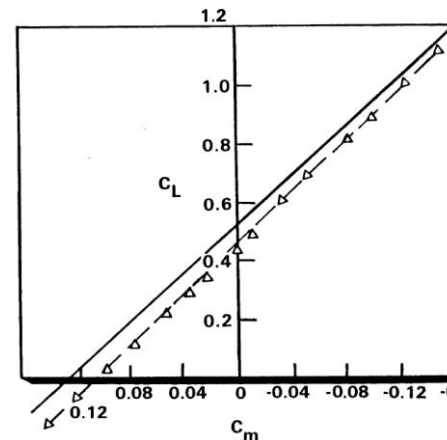
Technology Comes of Age (1980s)

- **PANAIR (Boeing):** Magnus, Ehlers and Epton—NASA CR 3251, April 1980
 - Subsonic or supersonic flow; arbitrary bodies; higher order singularity distribution
- **MCAIR (McDonnell):** Bristow and Hawk—NASA CR 3528, March 1982
 - Subsonic flow; arbitrary bodies; constant source, quadratic doublet singularities
- **VSAERO (AMI):** Maskew—NASA CR 166476, Dec 1982
 - Subsonic flow; arbitrary bodies; piecewise constant doublet and source singularities
- **QUADPAN (Lockheed):** Youngren, Bouchard, Coopersmith, and Miranda—AIAA 83-1827, July 1983
 - QUADriletral PANel code: subsonic flow; arbitrary bodies; low-order constant source and doublet singularities

Wing-Body-Canard Analysis

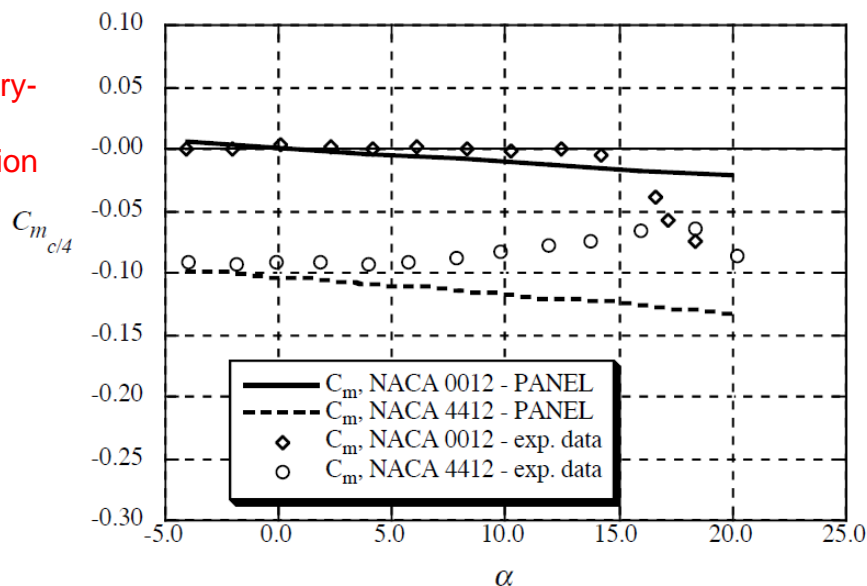
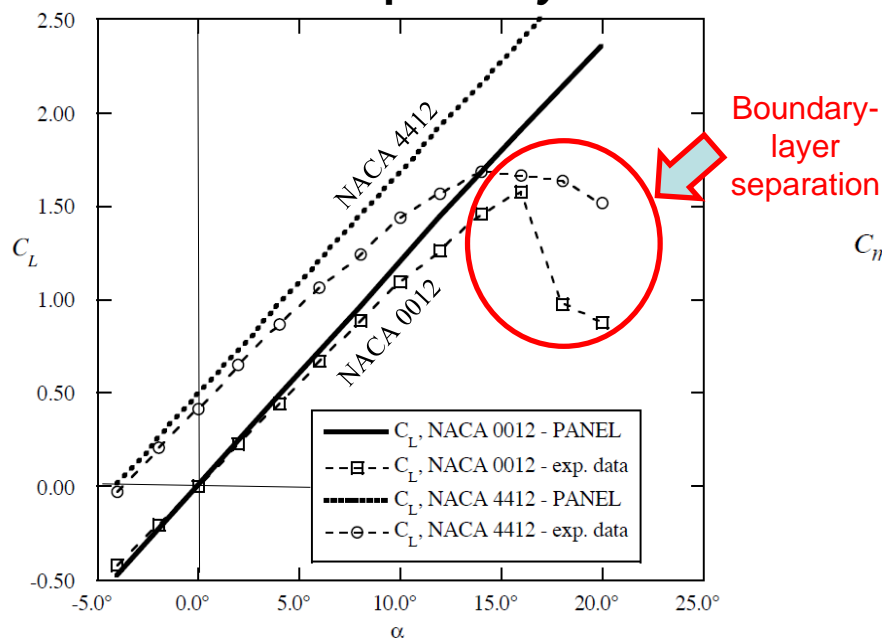


P-3 AEW&C Development

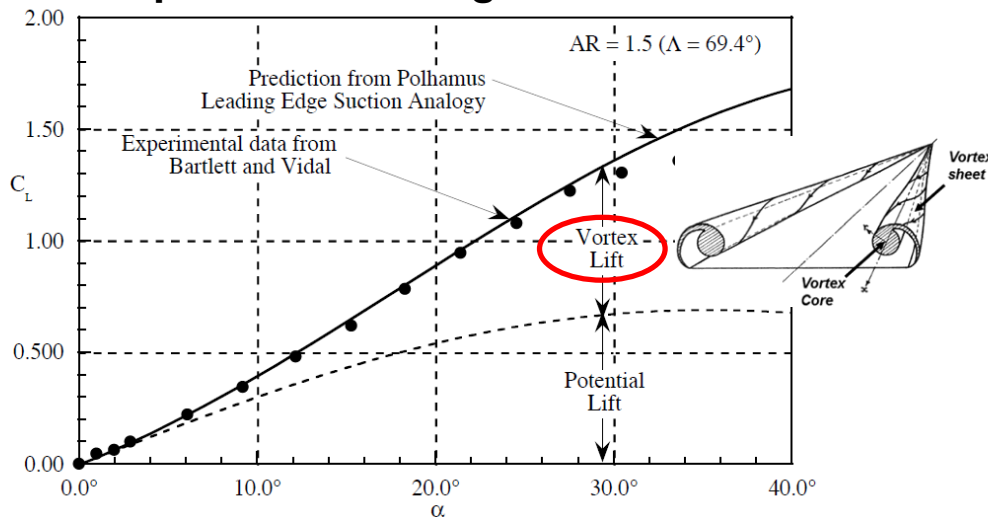


Applicable to Simulating Entirely Subsonic or Supersonic Attached Flows on Full Aircraft Configurations

Example 1: Symmetric and Cambered Airfoils Flow Simulation



Example 2: Delta Wing Flow Simulation



LPMs applicable to simulate

- *attached* flows that are entirely subsonic or supersonic; not transonic
- flows not dominated by shocks, vortices, or boundary-layer separation

Assessment Based on Comparing LPM Results With Experimental Data!

“Higher, Faster, Farther” Jet Transports

US SST (Supersonic Transport) Aircraft (1960s)

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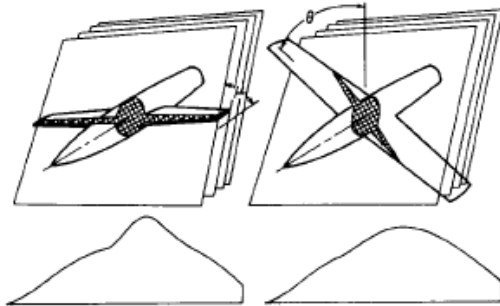
- **June 5, 1963:** FAA launched the SST program to *improve upon the Anglo-French Concorde* with quite aggressive targets
 - 250 passengers
 - $M_{\text{cruise}} = 2.7 - 3.0$
 - 4,000 miles Range
- **January 15, 1964:** Proposals submitted
 - Boeing and Lockheed entries downselected for further development
 - Boeing developed swing-wing B 2707, and Lockheed L-2000
- **January 1, 1967:** Boeing won the competition
- **May 20, 1971:** Development work stopped; US Congress canceled funding
 - Rising costs and lack of a clear market were likely factors



Design Needs of SST Stimulated Research in Many Areas

Leverage Computers to Meet SST Aerodynamic Design Needs (1960s)

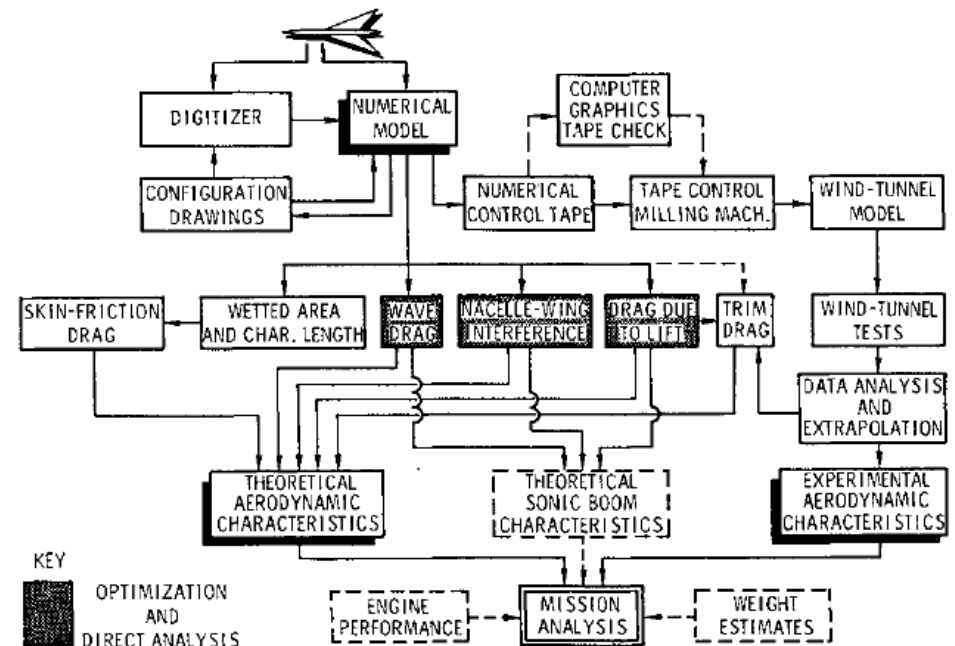
- **Wave Drag Analysis—Harris (1964)**
 - Analysis and correlation of aircraft wave drag—NASA TM X-947
- **Supersonic Aircraft Design Integration – Baals et al (1968)**
 - Aerodynamic design integration of



$$D(\theta) = -\frac{\rho V^2}{4\pi} \int_0^l \int_0^l A''(x_1) A''(x_2) \text{LOG} |x_1 - x_2| dx_1 dx_2$$

$$D = \frac{1}{2\pi} \int_0^{2\pi} D(\theta) d\theta$$

- **Supersonic Wing Camber Design**
 - ***Carlson and Middleton (1964)***
 - Numerical method for designing camber surfaces of supersonic wings with arbitrary planform corresponding to specified load distributions—*NASA TN D-2341*



“Computer-Aided Aerodynamics” Demonstrated Its Usefulness

“Long-Haul, High-Capacity” Jet Transports L5

Transonic Aircraft (1960s)

- Jet transport designs in the 1960s pushed cruise speed into transonic regime to maximize Range Factor, $M_{cruise} (L/D)$
 - C-5A (1968): $M_{cruise} = 0.77$
 - B747 (1969): $M_{cruise} = 0.84 - 0.88$
 - L-1011 (1970): $M_{cruise} = 0.86$
- Drag rises with speed due to added wave drag + shock-induced separation drag
 - *The higher the drag rise Mach number, the better!*
 - Sweep helps...but design tradeoffs limit it to about 35° in practice



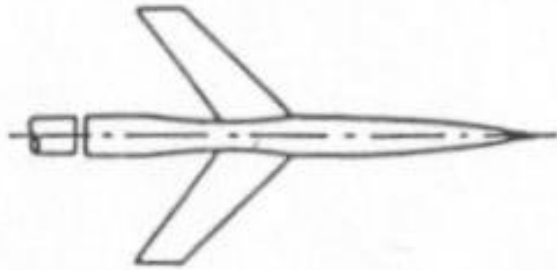
LPMs of Little Use for Accurate Transonic Flow Simulation

Transonic Aircraft Design

EFD: Primary Means of Flow Simulation

- **Whitcomb (1954 Collier Trophy)**

- “Area Rule”

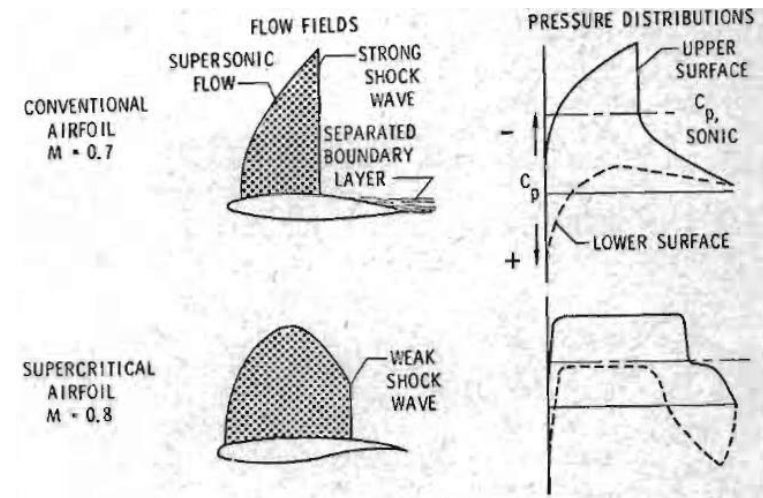


- **Pearcy (1962)**

- “Peaky” airfoils: 0.02 to 0.03 increase in drag rise Mach number over NACA 6-series

- **Whitcomb (1967)**

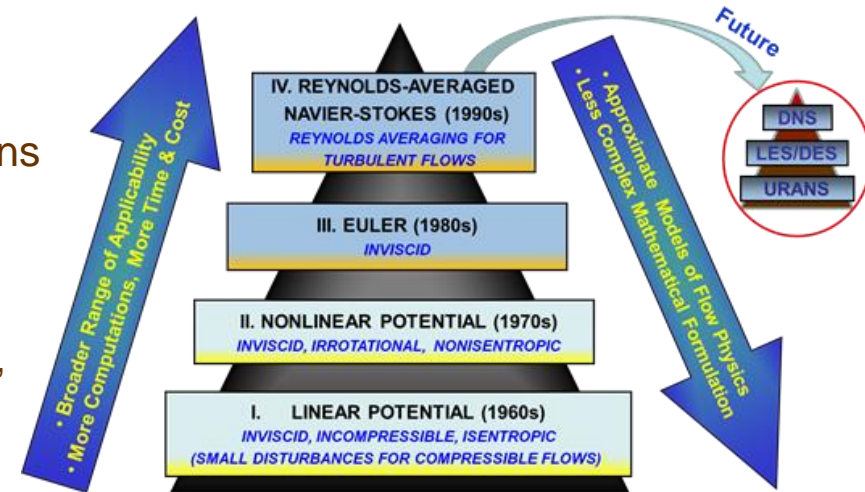
- Supercritical “roof top” airfoils



ACA Capability Urgently Needed to Support Design Needs!

Lecture 5: Key Takeaways

- **ACA evolution paced by impressive advances in CFD since the 1950s**
 - Capabilities directly related to four levels of CFD methods, each based on approximations of Navier-Stokes equations
 - *Level I:* linear potential methods for inviscid, irrotational, isentropic flows
 - *Level II:* nonlinear potential methods for inviscid, irrotational, isentropic flows
 - *Level III:* Euler methods for inviscid flows
 - *Level IV:* RANS methods for viscous flows



- **Linear Potential Methods (LPMs)**
 - Vortex Lattice Method (VLM) and Surface Panel Method: 1950s
 - Technology comes of age in 1980s—*Today's workhorse for early stages of design*
 - Range of applicability limited to purely subsonic or supersonic attached flows
- **“Computer-aided Aerodynamics” Demonstrated Its Usefulness for Meeting Supersonic Aircraft Design Needs: 1960s**
 - Harris Wave Drag analysis
 - Aerodynamic design integration of supersonic aircraft
- **Meeting Transonic Aircraft Design Needs: 1960s**
 - **LPMs woefully inadequate**

Topic 5

5. Evolution of Applied Computational Aerodynamics (1950-2000)

5.1 Infancy through Adolescence (1950–1980)

- 5.1.1 Falkner, V.M., “The Scope and Accuracy of Vortex Lattice Theory,” R & M No. 2740, A.R.C. Technical Report, 1949.
- 5.1.2 Rubbert, P.E., “Theoretical Characteristics of Arbitrary Wings by a Nonplanar Vortex Lattice Method,” Boeing Report D6-9244, The Boeing Company, 1964.
- 5.1.3 Margason, R.J. and Lamar, J.E., “Vortex-Lattice FORTRAN Program for Estimating Subsonic Aerodynamic Characteristics of Complex Planforms,” NASA TN D-6142, 1971.
- 5.1.4 “Vortex Lattice Utilization,” NASA SP-405, May 1976.
- 5.1.5 Miranda, L.R., Elliott, R.D., and Baker, W.M., “A Generalized Vortex Lattice Method for Subsonic and Supersonic Flow Applications,” NASA CR-2865, 1977.
- 5.1.6 Smith, A.M.O., “The Panel Method: Its Original Development,” Chapter 1, Applied Computational Aerodynamics, Progress in Astronautics and Aeronautics, Vol. 125, AIAA, Washington D.C., 1990, Henne, P.A. (Editor).
- 5.1.7 https://en.wikipedia.org/wiki/Apollo_M._O._Smith
- 5.1.8 Hess, J.L., “Calculation of potential flow about bodies of revolution having axes perpendicular to the free-stream direction,” Journal of the Aerospace Sciences, Vol. 29, No. 6 (1962), pp. 726-742. <https://doi.org/10.2514/8.9591>
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- 5.1.12 Woodward, F.A., “An Improved Method for the Aerodynamic Analysis of Wing-Body-Tail Configurations in Subsonic and Supersonic Flow,” NASA CR-2228, 1973.
- 5.1.13 Magnus, A.E., Ehlers, F.E., and Epton, M.A., “PANAIR - A Computer Program for Predicting Subsonic or Supersonic Linear Potential Flow About Arbitrary Configurations Using a Higher-Order Panel Method,” NASA CR-3251, April 1980.
- 5.1.14 Johnson, F.T., “A General Panel Method for the Analysis and Design of Arbitrary Configurations in Incompressible Flows,” NASA CR-3079, 1980.
- 5.1.15 Bristow, D.R. and Hawk, J.D., “Subsonic Panel Method for the Efficient Analysis of Multiple Geometry Perturbations,” NASA CR-3528, March 1982.

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- 5.1.17 Maskew, B., "Program VSAERO: A Computer Program for Calculating the Non-linear Aerodynamic Characteristics of Arbitrary Configurations, User's Manual" NASA CR-166476, December 1982.
- 5.1.18 Hess, J.L. and Friedman, D.M., "An Improved Higher-Order Panel Method for Three-Dimensional Lifting Flows," NADC Report 79277-60, U.S. Naval Air Development Center, December 1981.
- 5.1.19 Maskew, B., "Prediction of Subsonic Aerodynamic Characteristics: A Case for Low-Order Panel Methods," *Journal of Aircraft*, Vol. 19, February 1982, pp. 157-163.
- 5.1.20 Coopersmith, R.M., Youngren, H.H., and Bouchard, E.E., "Quadrilateral Element Panel Method (QUADPAN)", User's Manual (Version 3), Lockheed-California Company, LR 29671, June 1983.
- 5.1.21 Coopersmith, R.M., Youngren, H.H., and Bouchard, E.E., "Quadrilateral Element Panel Method (QUADPAN)", Theoretical Report (Version 3), Lockheed-California Company, LR 30500, July 1983.
- 5.1.22 Youngren, H.H., Bouchard, E.E., Coopersmith, R.M., and Miranda, L.R., "Comparison of Panel Method Formulations and Its Influence on the Development of QUADPAN, an Advanced Low Order Method," AIAA Paper 83-1827, July 1983.
- 5.1.23 Fornasier, L., "HISSS—A Higher-Order Subsonic/Supersonic Singularity Method for Calculating Linearized Potential Flow," AIAA Paper 84-1646, June 1984.
- 5.1.24 Margason, R.J., Kjelgaard, S.O., Sellers, W.L., Morris, C.E., Walkey, K.B., and Shields, E.W., "Subsonic Panel Methods—A Comparison of Several Production Codes," AIAA Paper 85-0280, January 1985.
- 5.1.25 Johnston, C.E., Youngren, H.H., and Sikora, J.S., "Engineering Applications of an Advanced Low-Order Panel Method," SAE Paper 851793, October 1985.
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- 5.1.27 Tinoco, E.N., Ball, D.N., and Rice, F.A., II, "PANAIR Analysis of a Transport High-Lift Configuration," *Journal of Aircraft*, Vol. 24, March 1987, pp. 181-187.
- 5.1.28 Fornasier, L. and Heiss, S., "Application of HISSS Panel Code to a Fighter Type Aircraft Configuration at Subsonic and Supersonic Speeds," AIAA Paper 87-2619, August 1987.
- 5.1.29 Lednicer, D., "A VSAERO Analysis of Several Canard Configured Aircraft," SAE Paper 881485, SP-757, October 1988.
- 5.1.30 Harris, R.V., "An Analysis and Correlation of Aircraft Wave Drag," NASA TM X-947, March 1964.
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- 5.1.33 Pearcy, H.H., "The Aerodynamic Design of Section Shapes for Swept Wings," Advances in Aeronautical Sciences, Vol. 3, Pergamon Press, 1962.
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<https://doi.org/10.2514/3.6131>
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- 5.1.39 Caughey, D.A. and Jameson, A., "Progress in Finite-Volume Calculations for Wing-Fuselage Combinations," AIAA Journal, Vol. 18, No. 11 (1980), pp 1281-1288. <https://arc.aiaa.org/doi/pdf/10.2514/3.50883>
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