Applied Computational Aerodynamics (ACA):
Reflections on My Long and Exciting Journey in
the Never-Ending Pursuit of Effectiveness

Pradeep Raj, Ph.D.
Professor, Kevin T. Crofton Department of Aerospace and Ocean Engineering
Virginia Tech, Blacksburg, VA
https://www.aoe.vt.edu/people/faculty/raj.html

Program Management Director, Lockheed Martin (Retired)
The Skunk Works®, Palmdale, CA

8 May 2019
DISCLAIMERS

The author has gathered and compiled the material included herein from publicly available sources and personal archives. Although a good-faith attempt has been made to cite all sources of material, the author regrets any inadvertent errors or omissions.

This presentation reflects the views, thoughts, and opinions solely of the author, and not necessarily those of the author’s employers or other groups or individuals.

Outline

• Introductory Remarks

• ACA Evolution and Pursuit of Effectiveness
  o 1900 – 1950: Shifting Landscape of Fluid Mechanics
  o 1950 – 1980: Infancy through Adolescence
    - Level I: Linear Potential Methods
    - Level II: Nonlinear Potential Methods
  o 1980 – 2000: Quest for Effectiveness
    - Level III: Euler Methods
    - Level IV: Reynolds-Averaged Navier-Stokes (RANS) Methods
  o 2000 – 2050: Challenges and Prospects for Effective ACA

• Closing Remarks
Why Look Back?

Study the past, if you would define the future. — Confucius

Most of what follows has been said before. Then why say it again?
Everything has been said before, but *since nobody listens* we have to keep going back and beginning all over again.

- André Gide

French author
Nobel Prize in Literature (1947)

22 November 1869 – 19 February 1951
More Than 100 Years Ago

The Approximate Arithmetical Solution by Finite Differences of Physical Problems involving Differential Equations, with an Application to the Stresses in a Masonry Dam.

By L. F. Richardson, King’s College, Cambridge.

Read January 13, 1910

IX. The Approximate Arithmetical Solution by Finite Differences of Physical Problems involving Differential Equations, with an Application to the Stresses in a Masonry Dam.

By L. F. Richardson, King’s College, Cambridge.

Communicated by Dr. R. T. Glashuise, F.R.S.

Received (in revised form) November 2, 1909.—Read January 13, 1910.

§ 1. INTRODUCTION.—§ 1.0. The object of this paper is to develop methods whereby the differential equations of physics may be applied more freely than hitherto in the approximate form of difference equations to problems concerning irregular bodies.

Though very different in method, it is in purpose a continuation of a former paper by the author, on a “Freehand Graphic Way of Determining Stream Lines and Equipotentials” (“Phil. Mag.”, February, 1908; also “Proc. Physical Soc.”, London, vol. xxi.). And all that was there said, as to the need for new methods, may be taken to apply here also. In brief, analytical methods are the foundation of the whole subject, and in practice they are the most accurate when they will work, but in the integration of partial equations, with reference to irregular-shaped boundaries, their field of application is very limited.

Both for engineering and for many of the less exact sciences, such as biology, there is a demand for rapid methods, easy to be understood and applicable to unusual equations and irregular bodies. If they can be accurate, so much the better; but 1 per cent. would suffice for many purposes. It is hoped that the methods put forward in this paper will help to supply this demand.

The equations considered in any detail are only a few of the commoner ones occurring in physical mathematics, namely:—Laplace’s equation \( \nabla^2 \phi = 0 \); the oscillation equations \( (\nabla + k) \phi = 0 \) and \( (\nabla - k) \phi = 0 \); and the equation \( \nabla^2 = 0 \). But the methods employed are not limited to these equations.

The Number of Independent Variables.—In the examples treated in the paper this never exceeds two. The extension to three variables is, however, perfectly obvious. One has only to let the third variable be represented by the number of the page of a book of tracing paper. The operators are extended quite simply, and the same

Lewis Fry Richardson
FRS, British Mathematician, Physicist, Meteorologist, Psychologist

11 October 1881 – 30 September 1953

Source: Refs. 2 & 3
“The object of this paper is to develop methods whereby the differential equations of physics may be applied more freely than hitherto in the approximate form of difference equations to problems concerning irregular bodies.”

“...analytical methods are the foundation of the whole subject, and in practice they are the most accurate when they will work, but in the integration of partial equations, with reference to irregular-shaped boundaries, their field of application is very limited.”

“So far I have paid piece rates for the operation of about $n/18$ pence per coordinate point, $n$ being the number of digits. The chief trouble to the computers has been the intermixture of plus and minus signs. As to the rate of working, one of the quickest boys averaged 2,000 operations per week, for numbers of three digits, those done wrong being discounted.”

**Extension to Fluid Mechanics**

FLOW ABOUT IRREGULARLY SHAPED BODIES
1. Use difference form of differential equations of fluid flow physics.
2. Cannot apply analytical methods to irregularly shaped bodies.
3. Employ ‘computers’ to perform the arithmetic operations.

_The What, the Why and the How of CFD (the rest is DETAIL!)_
Richardson’s Insightful Observation: 1910

“Both for engineering and for many of the less exact sciences, such as biology, there is a demand for rapid methods, easy to be understood and applicable to unusual equations and irregular bodies. If they can be accurate, so much the better; but 1 per cent, would suffice for many purposes.”

COMPUTATIONAL METHODS SHOULD BE

• Rapid
• Easy to understand
• Applicable to unusual equations
• Applicable to irregularly shaped bodies
• Accurate (1% would suffice!)

For applications to fluid flow problems in engineering

Key Tenets of Effectiveness!

Source: Ref. 3
Outline

• Introductory Remarks

• ACA Evolution and Pursuit of Effectiveness
  o 1900 – 1950: Shifting Landscape of Fluid Mechanics
  o 1950 – 1980: Infancy through Adolescence
    ▪ Level I: Linear Potential Methods
    ▪ Level II: Nonlinear Potential Methods
  o 1980 – 2000: Quest for Effectiveness
    ▪ Level III: Euler Methods
    ▪ Level IV: Reynolds-Averaged Navier-Stokes (RANS) Methods
  o 2000 – 2050: Challenges and Prospects for Effective ACA

• Closing Remarks
At the Dawn of the 20th Century…

- 17 December 1903 to be precise, the first manned, controlled, powered flight by the Wright brothers.

- Dramatic evolution of civil and military aviation followed.

...12 Seconds Changed Human History Forever!

Source: Internet
Analytical Fluid Mechanics: 
(1900 – 1950)

• Early 1900s, fluid mechanics considered as *mathematical science* from which all observed characteristics of fluid motion could be determined.
   
   “All the theory of the motion of fluids has just been reduced to *solution of analytical formulas*.”
   
   – LEONARD EULER (1755)

• But…available analytical models were inadequate for meeting the *emerging engineering design needs of airplanes* (*irregular shaped bodies*)

• **Aerodynamics became the most exciting research frontier of fluid mechanics** leading to development of noteworthy new models

  - Airfoil theory (Kutta-Joukowsky)
  - Subsonic wing theory (Lanchester & Prandtl)
  - Statistical theory of turbulence (G.I. Taylor)
  - Boundary layer theory (Prandtl)
  - Supersonic wing theory (Busemann)

Source: Refs. 4 & 5; Wikipedia
“…no exact analytical model describing physically interesting flows that depend significantly on Re [Reynolds number] is known.”

– GARRETT BIRKHOFF (1981)

Garrett Birkhoff
American Mathematician
19 January 1911 – 22 November 1996

Analytical Models Remain Inadequate for Simulating Realistic Flows…Even Today!
Experimental Aerodynamics
(1900 – 1950)

Rapid advancements to support development of new airplane designs

- Bigger tunnels; high-speed tunnels; low-turbulence tunnels; special purpose tunnels; …

“data for 78 classical airfoil shapes: see TR 460, 1935”

“aircraft development work”

“solve the mysteries of flight beyond Mach 1”

- Techniques and instruments for accurate measurements (e.g., hot-wire anemometry) and visualization (e.g., Schlieren, interferometry)

Source: NASA websites; Refs. 6 & 7
Numerical Aerodynamics
(1900 – 1950)

• Noteworthy foundational research in numerical methods
  o Richardson (1910) – point iterative scheme for Laplace’s equation
  o Liebmann (1918) – improved version of Richardson’s method with faster convergence
  o Courant, Friedrichs, and Lewy (1928) – uniqueness and existence of numerical solutions of PDEs (origins of the CFL condition well known to all CFDers)
  o Southwell (1940) – improved relaxation scheme tailored for hand calculations
  o Frankel (1950) – first version of successive over-relaxation scheme for Laplace’s equation

• The bottleneck: Humans as computers – not practical for large, complex problems!

• John von Neumann’s vision (1945 - 1946)
  “really efficient high-speed [digital] computing devices” may “break the present stalemate created by the failure of the purely analytical approach to nonlinear problems”… in fluid mechanics and in “many other fields”

Novel Techniques for Numerically Solving PDEs of Aerodynamics!
Digital Computers (1900 – 1950)

- **Alan Turing (1936)** – a universal machine capable of computing anything that is computable

- **Atanasoff (1937)** – first computer without gears, cams, belts and shafts

- **Atanasoff and Berry (1941)** – a computer that can solve 29 equations simultaneously, and store information on its main memory

- **Mauchly and Eckert (1943-44)** – Electronic Numerical Integrator and Calculator (**ENIAC**) using 18,000 vacuum tubes
  - Speed: 500 floating point operations per second
  - Size: 1,800 square feet

- **Mauchly and Presper (1946)** – Universal Automatic Computer (**UNIVAC**), the first commercial computer for business and government

**The Key to Converting von Neumann’s Vision into Reality!**

Source: Ref. 9
By 1950, all fundamental elements were in place for the emergence of an exciting new field of [what we now call] Computational Fluid Dynamics (CFD).

In the second half of the 20th century, phenomenal advances in CFD methods and computing capabilities led to the evolution of Applied Computational Aerodynamics (ACA).

ACA Evolution was Fueled by the Promise of CFD Serving as a Powerful Complement to Analytical and Experimental Techniques for Simulating Aerodynamics of Irregularly Shaped Bodies!
I. LINEAR POTENTIAL (1960s)

II. NONLINEAR POTENTIAL (1970s)
- INVISCID, IRROTATIONAL, ISENTROPIC
  (SMALL DISTURBANCES FOR COMPRESSIBLE)
- + NONLINEAR
- + NONLINEAR

III. EULER (1980s)
- + ROTATIONAL & NONISENTROPIC

IV. REYNOLDS-AVERAGED NAVIER-STOKES (1990s)
- + VISCOUS

Dramatic Advances in CFD Methods – Key Driver for ACA

ACA Evolution
Paced by 4 Levels of CFD Methods

[Diagram with levels: DNS, LES/DES, URANS]
ACA Evolution

Enabled by an Order of Magnitude Increase in Speed and Memory Every FIVE Years!

**Speed**

<table>
<thead>
<tr>
<th>Year</th>
<th>MegaFLOPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>IBM 7094</td>
</tr>
<tr>
<td>1970</td>
<td>CDC</td>
</tr>
<tr>
<td>1980</td>
<td>Cray 1</td>
</tr>
<tr>
<td>1990</td>
<td>Cray X-MP/48</td>
</tr>
<tr>
<td>2000</td>
<td>Cray Y-MP/C90</td>
</tr>
<tr>
<td>1970</td>
<td>Cray Y-MP/832</td>
</tr>
</tbody>
</table>

**Memory**

<table>
<thead>
<tr>
<th>Year</th>
<th>MegaWords</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>CDC</td>
</tr>
<tr>
<td>1970</td>
<td>Cray 1</td>
</tr>
<tr>
<td>1980</td>
<td>Cray X-MP/48</td>
</tr>
<tr>
<td>1990</td>
<td>Cray Y-MP/C90</td>
</tr>
<tr>
<td>1970</td>
<td>Cray Y-MP/832</td>
</tr>
</tbody>
</table>

**Phenomenal Advances in Computing – Key Driver for ACA**
Applied Computational Aerodynamics: Much More than CFD Methods + Computing

It’s C³I

CFD methods
(aerodynamic analysis and design software)

Customer
(Engineering need, resources, and constraints)

Computing
(High-speed, low-cost)

Impact
(Meeting customer needs and expectations)

ACA

ACA Extracts Value from CFD for Customers
ACA and CFD are **not** Synonymous

**CFD produces data.**

Computational Fluid Dynamics (CFD) is a tool (à la wind tunnels) to simulate flow of fluids about bodies of arbitrary shape.

Both use a 3-step process

1. Build a model
2. Blow air on it
3. Collect data

(Data include forces, moments, and flow quantities—on and off the surface)

---

**ACA produces solutions.**

Applied Computational Aerodynamics (ACA) is all about using CFD to meet engineering needs.

**CFD is to ACA as Airplane is to Air Transportation!**
Outline

• Introductory Remarks

• ACA Evolution and Pursuit of Effectiveness
  o 1900 – 1950: Shifting Landscape of Fluid Mechanics
  o 1950 – 1980: Infancy through Adolescence
    ▪ Level I: Linear Potential Methods
    ▪ Level II: Nonlinear Potential Methods
  o 1980 – 2000: Quest for Effectiveness
    ▪ Level III: Euler Methods
    ▪ Level IV: Reynolds-Averaged Navier-Stokes (RANS) Methods
  o 2000 – 2050: Challenges and Prospects for Effective ACA

• Closing Remarks
Flow Model

- Inviscid, Irrotational, Isentropic (Small Disturbances for Compressible Flow)

\[ U = U_\infty + \nabla \phi \]

\[ (\phi_{tt} + 2U_\infty \phi_{xt})/a_\infty^2 = (1 - M_\infty^2) \phi_{xx} + \phi_{yy} + \phi_{zz} \]

- Linear second-order PDEs with appropriate boundary conditions
- Laplace’s equation for steady, incompressible flow
- Prandtl-Glauert equation for steady, compressible flow
- Wakes must be explicitly modeled—not captured as part of the solution

Range of Applicability

- Flow fields that are entirely subsonic or supersonic; not transonic
- Flows not dominated by shocks, vortices, or boundary-layer separation

Refs. 11 to 40
Back in the 1950s

- A.M.O. Smith and J. Pierce, Douglas Aircraft Co., Long Beach, CA
  - Non-circulatory plane and axially symmetric flows
  - 1953—Serious work began to solve Neumann problem
    - Continuous source distribution on surface panels
  - 1954—Programming on IBM/701 in machine language!
  - Test cases selected based on availability of theoretical solutions
  - From 24-point body of revolution solutions in 1954 to 150-points by the end of 1955!
  - Douglas financed all work through 1958
  - ONR contract: extend the method to 3-D non-lifting flows
- DAC Report E.S. 26988, April 1958

And the Rest is History!

Source: Refs. 11 & 12
Linear Potential Methods (LPMs)

• **Basic Formulation**
  o Discretize geometry into small elements
  o Distribute singularities (source, doublets, vortex filaments) on each element
  o Impose no-normal-flow boundary condition (B.C.) at control points (one per element)
  o Solve system of linear algebraic equations to determine vortex strengths
  o Bernoulli’s equation to compute airloads (pressure distribution)

• **Vortex Lattice Methods (VLMs)**
  o **Geometry:** mean surface
  o **Singularity type:** horseshoe vortices
  o **B.C.:** control points on mean surface
  o **Airloads:** net pressure

• **Panel Methods**
  o **Geometry:** actual surface
  o **Singularity type:** sources, doublets or both
  o **Singularity distribution:** constant, linear or higher order
  o **B.C.:** control points on actual surface
  o **Airloads:** actual surface pressures

**VLMs & Panel Methods: Today’s Workhorse!**
Vortex Lattice Methods (VLMs): Rapid Development (1960s & 70s)

- **Falkner (1949)**
  - Scope and accuracy of Vortex Lattice theory—R. & M. No. 2740, British A. R. C.

- **Rubbert (1964)**
  - Non-planar Vortex Lattice Methods; arbitrary wings—*Boeing Co. Document D6-9244*

- **Margason and Lamar (1971)**
  - Vortex-lattice Fortran program for estimating subsonic aerodynamic characteristics of complex planforms—*NASA TN D-6142*

- **Vortex-Lattice Utilization workshop (1976)**
  - Compilation of many papers—*NASA SP-405*

- **Miranda, Elliott and Baker (1977)**
  - A generalized vortex-lattice method for subsonic and supersonic flow applications, the VORLAX code—*NASA CR 2865*

---

**Easier to Use than Panel Methods: Simple Model of Geometry**

Source: Refs. 13 to 17
Panel Methods: 
Rapid Development (1960s & 70s)

• Hess (1962)
  - Arbitrary bodies of revolution with axes perpendicular to the free stream direction— *Journal of the Aerospace Sciences*

• Hess and Smith (1967)
  - Extensive description of panel methods—*Progress in Aeronautical Sciences, Vol. 8* (138 pages!)

• Rubbert and Saaris (1968)
  - Incompressible flow; arbitrary configurations; source and doublet distributions—*SAE Paper 680304*

• Hess (1970)
  - Arbitrary 3-D lifting bodies—*McDonnell Douglas Rept. MDC J0971-01* (Also in *Comp. Methods in Applied Mechanics and Engineering, 1974*)

• Woodward (1973)
  - Subsonic or supersonic flow; wing-body-tail configurations; source and vortex distributions—*NASA CR-2228*
Panel Methods:
Technology Comes of Age in the 1980s

- **PANAIR** *(Boeing)*: Magnus, Ehlers and Epton—*NASA CR 3251, April 1980*
  - Subsonic or supersonic flow; arbitrary bodies; higher order singularity distribution

- **MCAIR** *(McDonnell)*: Bristow and Hawk—*NASA CR 3528, March 1982*
  - Subsonic flow; arbitrary bodies; constant source, quadratic doublet singularities

- **VSAERO** *(AMI)*: Maskew—*NASA CR 166476, Dec 1982*
  - Subsonic flow; arbitrary bodies; piecewise constant doublet and source singularities

- **QUADPAN** *(Lockheed)*: Youngren, Bouchard, Coopersmith, and Miranda—*AIAA 83-1827, July 1983*
  - QUADriletral PANel code: subsonic flow; arbitrary bodies; low-order constant sources and doublet singularities

**Powerful Capability to Support Aircraft Design Needs**

Source: Refs. 24 to 40
**Key Enabler of (a) LPM Applications to Complex Configurations, and (b) Research in RANS Methods and Vortex Methods**
1960s - Supersonic Aircraft Design

“Computers Enabled Application of Aerodynamic Theory”

- Wave Drag Analysis—Harris (1964)
  - Analysis and correlation of aircraft wave drag—NASA TM X-947

- Supersonic wing camber design—Carlson and Middleton (1964)
  - Numerical method for designing camber surfaces of supersonic wings with arbitrary planform corresponding to specified load distributions—NASA TN D-2341

- Supersonic aircraft design integration—Baals et al (1968)
  - Aerodynamic design integration of supersonic aircraft—AIAA Paper 68-1018; also in Journal of Aircraft, 7(5), 1970

“Computer-Aided Aerodynamics” Demonstrated Its Usefulness

Source: Refs. 42 - 44
1960s - Transonic Aircraft Design

No ACA Capability to Meet the Need!

- Jet transport designs in the 1960s pushing cruise Mach number into transonic regime to maximize Range Factor, $M_{\text{cruise}} (L/D)$
  - L-1011-1: $M_{\text{cruise}} = 0.86$
  - B747: $M_{\text{cruise}} = 0.84 - 0.88$
  - C-5A: $M_{\text{cruise}} = 0.77$
  - Drag rises with speed due to added wave drag + shock-induced separation drag
  - *The higher the drag rise Mach number the better!*
  - Sweep helps...but design tradeoffs limit it to about $35^\circ$ in practice

- **Pearcy (1962)**
  - “Peaky” airfoils: 0.02 to 0.03 increase in drag rise Mach number over NACA 6-series

- **Whitcomb (1967)**
  - Supercritical “roof top” airfoils

---

**Linear Potential Methods Woefully Inadequate**

Source: Refs. 45 & 46
Level II
Nonlinear Potential Methods
1970s

Flow Model

• Inviscid, Irrotational, Isentropic

\[ \mathbf{U} = (u, v, w) = \nabla \Phi \]

\[ \Phi_{tt} + 2 \mathbf{U} \cdot \mathbf{U}_t = a^2 \nabla^2 \Phi - \mathbf{U} \cdot \nabla(U^2/2) \]

✓ Nonlinear second-order PDEs with appropriate boundary conditions
✓ Transonic Small Disturbance (TSD) or Full Potential formulations
  o Mass conserved across discontinuities
  o Momentum deficiency provides an estimate of wave drag
  o *Wakes must be explicitly modeled—not captured as part of the solution*

Range of Applicability

• Transonic flow with weak shocks
• Flows with no distributed vorticity or boundary-layer separation

Refs. 47 – 51, 62 & 63
Transonic Small Disturbance (TSD) Methods

- Murman and Cole (1970)
  - Mixed finite difference scheme for perturbation potential equation of plane steady transonic flow—**shocks appear naturally!**
  - Landmark paper—*AIAA J*, 9 (1), 1971

- Bailey and Ballhaus (1975)
  - Good comparisons of computed and measured pressures for transonic flows on wing and wing-fuselage configurations—*NASA SP-347*

- Boppe (1978)
  - Transonic flow about realistic aircraft configurations—*AIAA Paper 78-104*, 1978
  - Finite-difference scheme applied to an improved TSD equation
    - Unique grid embedding scheme to improve solution accuracy
  - Approx. **45 minutes on IBM 370** (15 mins. on CYBER 175)

*New Transonic Aerodynamic Analysis and Design Capability!*

Source: Refs. 47-49
Transonic Full Potential Methods

- **Jameson and Caughey (1976)**
  - FLO-22: transonic flow past 3-D swept wings
    - Full Potential equation transformed into sheared parabolic coordinates
    - Solved using coordinate invariant *rotated difference scheme*
  - Theory, results, and computer program in *ERDA Research and Development Report, COO-3077-140, 1977*

- **Caughey and Jameson (1980)**
  - FLO-28 & FLO30: transonic flow past wing-body combinations
  - FPE solved using *finite-volume method* on boundary conforming grids—*AIAA J, 18(11), 1980*

---

**Antony Jameson**
FRS, Hon Fellow AIAA
‘Father of FLO & SYN Series of CFD Codes’
Hawker Siddeley, Grumman
NYU, Princeton, Stanford, Texas A&M
Born: 20 Nov 1934

Source: Refs. 50 & 51
On the Other Side of the World…
...a lad was growing up* oblivious of CFD

1950s (Foundational Years)

1960s (Formative Years)

1963 High School (10th grade): Government Higher Secondary School, Muzaffarnagar, U.P., India (1st division; distinction in English, Math, Science and Sanskrit; ranked 15th in statewide exam)

1965 Intermediate School (12th grade): S.D. Intermediate College, Muzaffarnagar, U.P., India (1st division; distinction in Physics, Chemistry and Math; ranked 7th in statewide exams; too young for IIT)

1967 Bachelor of Science: S.D. College, Muzaffarnagar, U.P., India; College affiliation--Agra University, later Meerut University (1st division; distinction in Physics, Chemistry, and Math; graduated at the top of the class; Chancellor’s Medal)

1970 Bachelor of Engineering (with Distinction), Electrical Technology, Indian Institute of Science, Bangalore, India (graduated at the top of the class; Hay medal)

*has grown old now (born 15 Dec 1949), but debatable if he ever grew up!
1970s (Early Adulthood Years)

1970 - 1972

- **Master of Engineering** (with Distinction), **Aeronautical Engineering**, Indian Institute of Science, Bangalore, India
- Advisor: **Dr. Suresh M. Deshpande**
- Project: *Numerical Determination of Periodic Solutions for Gravity Gradient Stabilized Satellites*
  - First exposure to FORTRAN computer programming
    - Integrate two coupled 1\textsuperscript{st} order ODEs
    - Use **IBM 360/44** for processing

1972 - 1976

- **Ph.D., Aerospace Engineering**, Georgia Institute of Technology, Atlanta, Georgia, USA
- Advisor: **Dr. Robin B. Gray**
- Dissertation: *A Method of Computing the Potential Flow on Thick Wing Tips*
  - Developed **LPM** using **surface vorticity distribution**
    - Strengths determined using iterative procedure; avoided inverting large ill-conditioned matrices
    - **CDC Cyber 70/74** NOS 1.1-419/420
  - 2-D results in *AIAA Journal of Aircraft*, 15 (10), 1978
  - 3-D results in *Journal of Aircraft*, 16 (3), 1979

Source: Refs. 52 & 53; images from internet
Entrée into the “CFD World”!

1976 - 1978
• Research Assistant Professor, Aerospace Engineering, Iowa State University, Ames, Iowa
• NASA-Ames sponsored project: *Alleviation of wake-vortex hazard through merging of co-rotational vortices*
• Principal Investigator: **Dr. James D. Iversen**
• Conducted computational investigations to complement experimental research of Steve Brandt
  ✓ *Wonderful memories of working with, and learning from, Dr. Joseph L. Steger—a CFD pioneer, a professional, and a gentleman—at NASA-Ames Research Center*
  ✓ *Experienced the challenge of simulating vortical flows using zero-, one-, and two-equation turbulence models*

1978 - 1979
• Assistant Professor, University of Missouri-Rolla
• Taught Undergraduate courses: Fluid Mechanics, Thermodynamics, and Heat Transfer

1979
• Sr. Aerodynamics Engineer, Computational Aerodynamics Group, Lockheed-California Co., Burbank, California
• Group Engineer: **Mr. Luis R. Miranda**

Source: Refs. 54 - 57
First Day on the Job: May 1979

- Dr. A. Richard Seebass (University of Arizona, Tucson) visits Lockheed in Burbank!
  - Raj assigned to work with Dick Seebass on **shock-free supercritical wing design** procedure using **fictitious gas concept**
    [for future L-1011-500 wing design]
  - Results using **FLO-22** in *AIAA Paper 81-0383*; also in *AIAA Journal of Aircraft, 19(4), 1982*

---

**A. Richard Seebass**
Renowned Aerodynamicist and Educator
(1936-2000)

\[ M_\infty = 0.8 \]
\[ C_L = 0.63 \]
**Inviscid Drag reduced by \(~35\%)**

---

**Overnight Immersion into Transonic Aerodynamics!**

Source: Ref. 58
Oh, The Strange Seventies!

- **“The Lockheed Debacle”**
  - 1969-71: C-5 Galaxy cost overruns and serious wing design issues
  - 1971: Saved from bankruptcy by U.S. Congress approval of $250 million ‘Loan Guarantee’
  - 1974: Stock Price drops to a Low of 33/8 (High of 737/8 in 1967!)
  - 1976: Foreign Bribery Scandals cost $1.3 Billion order to Japan

- **Rolls-Royce Bankruptcy**
  - 1971: Could not proceed with RB-211 engine for Lockheed’s L-1011 Tristar
    - Cost of each engine increased by 30% over fixed-price contract value
    - Additional $360 million required to put the new engine into production

- **“The Great Boeing Bust”**
  - **Business**
    - 1969: Introduced now iconic B747
    - 1970-71: Not a single new order from any U.S. airline for 17 months
    - 1971: SST program cancelled by U.S. Government
  - **Workforce**
    - 32,500 employees by late 1971—down from about 80,000 in 1969
    - “Optimists brought lunch to work, pessimists left the car running in the parking lot”

- **New Exciting Endeavors!**
  - 1970: Pan Am 747 NY–London service
  - 1970: First operational C-5A Galaxy
  - 1975: New starts: GD F-16 and MDC F/A-18
  - 1976: Concorde entered service
ACA in the 1970s
The Adolescent Years with “Irrational Exuberance”

Computers vs. Wind Tunnels for Aerodynamic Flow Simulations
DEAN R. CHAPMAN, HANS MARK, and MELVIN W. PIRTLE
NASA Ames Research Center

“AIAA Astronautics & Aeronautics
APRIL 1975 VOLUME 13, NO. 4

“To displace wind tunnels as the principal source of flow simulations for aircraft design….the required computer capability would be available in the mid-1980s.”

“…within a decade computers should begin to supplant wind tunnels in the aerodynamic design and testing process…”

“Experience is what you get when you didn't get what you wanted.” – Prof. Randy Pausch, The Last Lecture

Source: Ref. 59
Symbiosis: Why CFD and wind tunnels need each other

By JOE STUMPE

_AIAA Aerospace America_

JUNE 2018

As powerful as computational fluid dynamics and supercomputing are, they have not come close to relegating wind tunnels to history. In fact, in the U.S., a new tunnel is going up at MIT, and NASA is deliberating whether it should close a historic tunnel at NASA’s Langley Research Center in Virginia four years from now as planned.
Outline

• Introductory Remarks

• ACA Evolution and Pursuit of Effectiveness
  o 1900 – 1950: Shifting Landscape of Fluid Mechanics
  o 1950 – 1980: Infancy through Adolescence
    ▪ Level I: Linear Potential Methods
    ▪ Level II: Nonlinear Potential Methods
  o 1980 – 2000: Quest for Effectiveness
    ▪ Level III: Euler Methods
    ▪ Level IV: Reynolds-Averaged Navier-Stokes (RANS) Methods
  o 2000 – 2050: Challenges and Prospects for Effective ACA

• Closing Remarks
ACA Effectiveness Codified: 1980-81

- $E = Q \times A$ unveiled in AIAA 82-0018, Jan 1982; also published in Journal of Aircraft, 21(6), 1984

Effectiveness = Quality $\times$ Acceptance

- “E” (Effectiveness): measure of merit of computational methods in practical aircraft design environments
  - “Q” (Quality) Factor: accuracy and realism of simulation
  - “A” (Acceptance) Factor: usability, applicability and affordability of computational methods

“If increasing the accuracy of a computational procedure will detract from its ease and economy of use, the implied tradeoff between quality and acceptance should be considered carefully to determine if its effectiveness will actually be enhanced by the increase in accuracy.”

Miranda’s “Law” of ACA Effectiveness: Guiding Principle of Author’s Efforts Ever Since

Source: Ref. 61
Lockheed ACA Development
Early 1980s

- **QUADPAN (QUADrilateral PANel)**
  - **Linear Potential Method** (developed by “The Quad Squad”)
    - **Low-order Formulation**: As accurate as high-order for subsonic flows at greatly reduced cost
    - **Source/doublet Singularities with Dirichlet BC**: Essential for robustness
    - **Pressure Formula Consistent with Linear Theory**: Accurate force calculations
    - **Modified Kutta Condition**: For trailing edges with large included angles

- **FLO-22.5: More Effective Nonlinear Full Potential Method** *(Raj & Reaser)*
  - **Better Geometry Modeling**: Planform-conforming grid for tapered wings
  - **Faster Turnaround**: Multi-grid acceleration
  - **Simulation Realism**: Fuselage effects and viscous effects
  - **Wing Design**: Garabedian-McFadden supercritical wing design technique
  - **Documentation**: LR 29759; AIAA 83-0262; also Journal of Aircraft, 21(2), 1984

**Key Driver: Effectiveness**

Source: Refs. 33, 61 - 63
1981: A Pivotal Year

- **December 7, 1981**
  - Lockheed discontinues L-1011 (*lost $2.5B in 13 years!*)
    - Concentrate instead on defense opportunities expected under Reagan military buildup

- **June 1981**
  - US Air Force Request for Information (RFI) for **Advanced Tactical Fighter (ATF)**—*a new air superiority fighter* (to replace F-15)
  - **Strong shocks and free-vortex flows** dominate fighter aerodynamics

- **Linear and Nonlinear Potential Methods** *ineffective* for flows with strong shocks and free vortices

---

*ATF - Impetus for Exploring Euler Methods*
Level III
Euler Methods
1980s

Flow Model

- **Inviscid, Irrotational, Isentropic**

\[
Q_t + F_X + G_Y + H_Z = 0
\]

\[
Q = (\rho, \rho u, \rho v, \rho w, \rho E)
\]

- System of nonlinear 1\text{st} order PDEs with appropriate boundary conditions

Range of Applicability

- All Mach numbers and attitude angles
- Flow may have shocks and free vortices but no boundary-layer separation

Refs. 64 to 85
Four Major Developments of
the Eighties

VIDEO CASSETTE RECORDER
COMPACT DISK PLAYER
EULER SOLVER
ГЛА́СНОСТЬ

1980s: ‘Golden Era’ of Euler Methods

Bram van Leer
Professor Emeritus
University of Michigan
Major contributions to CFD, Fluid Dynamics and Numerical Analysis

Image Source: Internet
A Small Sample of Euler Solvers of 1980s

- **Rizzi and Eriksson (1981)**
  - Grid generation: Transfinite interpolation for 3-D boundary-conforming grids on wings or wing-bodies; O-O and C-O topologies most efficient
  - Euler solver: Explicit pseudo time-marching scheme; nonreflecting boundary conditions; damping filter to improve convergence—*AIAA Paper 81-0999*
  - Shocks and wakes automatically “captured”; no explicit imposition of Kutta condition as long as the trailing edge was sharp

- **Jameson, Schmidt, and Turkel (1981)**
  - FLO-57: flow past 3-D wings; *finite volume formulation decouples solver and grid*
  - Cell-centered spatial discretization; a blend of second- and fourth-differences added for numerical dissipation with pressure gradient sensor; accelerated convergence to steady state using multi-stage pseudo-time stepping procedure—*AIAA Paper 81-1259*

- **Jameson and Baker (1984)**
  - Multigrid solution for aircraft configurations—*AIAA Paper 84-0093*

- **Benek, Buning and Steger (1985)**
  - A 3-D Chimera grid embedding scheme—*AIAA Paper 85-1523*

- **Lohner, Morgan, Peraire and Zienkiewicz (1985)**
  - Finite-element methods for high speed flows—*AIAA Paper 85-1531*

- **Mavriplis (1988)**
  - Accurate multigrid solutions on unstructured and adaptive meshes—*NASA CR 181679*
Lockheed Focus in 1980s: Full Aircraft Euler Analysis to Meet ATF Needs

1981
- Jameson creates FLO-57 code based on JST scheme (AIAA 81-1259)
- Finite volume formulation decouples solver and grid
- **Shocks and wakes automatically “captured”**
  - No explicit imposition of Kutta condition as long as trailing edge is sharp

1982
- Lockheed initiates **FLO-57GWB** development (*Pl: Raj*)
  - Extend Jameson’s FLO-57 (with JST scheme) to generalized wing-body configurations
  - **FLO-57 code courtesy of R.M. Hicks of NASA Ames**

1984
- Lockheed wins USAF Wright Research & Development Center contract to develop **Three-dimensional Euler Aerodynamic Method (TEAM)**
- **Antony Jameson** visits Lockheed! *A fascinating individual with singular intellect!*

1987
- USAF amends contract scope and extends period of performance **Three-dimensional Euler/Navier-Stokes Aerodynamic Method (TEAM)**

1989
- USAF Contract successfully completed; work documented in three USAF reports
Customer Requirements

• Aerodynamic analysis of fighter, transport, and flight research configurations with multiple lifting surfaces and flow-through or powered nacelles

• Symmetric or asymmetric flights at subsonic through hypersonic speeds for wide range of attitude angles

• Forces, moments, surface pressures, off-body pressures, velocities, etc.

• Validate code using 10 test cases

Lockheed Team

• Raj (Principal Investigator) with Brennan, Keen, Long, Mani, Olling, Sikora, and Singer under Miranda’s leadership and supervision

USAF Monitors

• Jobe, Sirbaugh, Jochum, Witzeman, Sedlock, Kinsey

Strategy for Effectiveness

• Modular Computational System: (i) Pre-processor; (ii) Grid Generator; (iii) Euler Solver; and (iv) Post-processor—easier to incorporate technology advances

• Patched Zonal Hexahedral Grids: arbitrary topology, grid generator of user’s choice—facilitates analysis of complex configurations

• Spatial Discretization: FLO-57 finite-volume formulation, cell-centered scheme with
  o JST adaptive dissipation—balanced accuracy and robustness
  o Characteristics-based—increased robustness for hypersonic flows

• Time Discretization: multistage pseudo-time stepping to steady state—faster turnaround

Towards Full Aircraft Euler Analysis

Source: Refs. 74 - 76
TEAM (Euler) Validation: Transonic Flow

NLR 7301 Airfoil*

\[ M_\infty = 0.721 \quad \alpha = -0.194^\circ \]

161 x 321 O Grid
(Far-field boundary 80 chords away)

Surface pressure distribution

Surface total pressure loss distribution

*Exact transonic shock-free hodograph solution

Source: Ref. 74
Team (Euler) Validation: 
Transonic Flow

Canard-Wing-Body Configuration

168 x 84 x 34 H-H grid

Source: Ref. 74
TEAM (Euler) Validation: Hypersonic Flow

Cone-derived Mach 6 Waverider

\( M_{\infty} = 6 \)

\( \alpha = -4^\circ, 0^\circ, 4^\circ \)

45 x 30 x 39 O-H Grid

Source: Ref. 74
Free-vortex Flows: 
Encounters of the Euler Kind*

- **Eriksson and Rizzi (1981)**
  - Euler equation solutions on delta wing at 0.9 and 1.5 Mach numbers and $\alpha = 15^\circ$; free vortices captured automatically—*IV GAMM Conference*
  - Similar solutions by others including Hitzel & Schmidt (1984), and Murman & Rizzi (1986)

- **Raj and Sikora (1984)**
  - Free-vortex flows simulated using FLO-57GWB: sharp-edged cropped delta wing, $M = 0.6$

- **Raj, Sikora and Keen (1986)**
  - Free-vortex flows simulated on a strake-wing-body using TEAM

  “…the generation of vortices about sharp-edged wings due to the total pressure losses is quite insensitive to the actual magnitude of numerical dissipation, *as long as there is some.*”

**Euler Codes More Effective than the-then RANS Codes**

*with due apologies to Steven Spielberg who wrote and directed *Close Encounters of the Third Kind*—a 1977 American classic*
TEAM (Euler) Capability: Free-Vortex Flow Simulation

Raj, Keen and Singer (1988)
75°/62° Double-Delta Wing Body

$M_\infty = 0.3$, $\alpha = 20^\circ$

TEAM Computations
49 x 145 x 33 C-H Grid

Measurements

Fig. 12.42 (p 457)

Recognition by Aircraft Designer—Doesn’t Get Better Than That!
TEAM (Euler) Application: ATF (YF-22)

1988: Full-aircraft Airloads Prediction (Reaser and Singer)

- Several transonic and supersonic Mach numbers
- Symmetric and asymmetric flight conditions
- Flow-through as well as powered nacelles

\[ \alpha = 8^\circ \]

Transonic flow

\[ \alpha = 16^\circ \]

mid-span

mid-span

43-zone H-H grid

1.5 million grid points

- TEAM results generated before wind-tunnel pressure model test
  - Code used in predictive mode*; no grid adjustments made for ‘better’ correlations!

*not necessarily by choice!!!

Source: Ref. 85
ATF (YF-22) TEAM (Euler) Analysis: Effectiveness Assessment

- **Tedious and time consuming grid generation**
  - Two engineers spent **few hundred man-hours** over **several weeks** to build a 43-zone H-H hexahedral grid with approximately **1.5 million nodes** for half the configuration.

- **Limited value of extensive ‘validation’**: A more demanding application uncovered several ‘bugs’ and other deficiencies!

- **Inability to predict total drag**: Project personnel disappointed.

  ![Graph of Lift vs. Angle of Attack](image)

  **Lift reasonably well predicted for transonic flight conditions**

- **Detailed surface pressures very useful**: For structural analysis as well as thermodynamic analysis.

- **Structural Design group’s desire**: Provide force, moment, and surface pressure increments due to control surface deflections!

  ![F-22 Aircraft Dimensions](image)

  **Effort Greatly Contributed to ‘Customer Confidence’**
Quest for Increasing TEAM Effectiveness

• **Total Drag:** *add viscous effects*
  - Coupling with integral boundary-layer codes: not well suited for fighter analyses
  - Extend TEAM by adding N-S viscous terms: in-house efforts initiated in 1986

• **Grid Generation:** *make it faster and less labor-intensive*
  - **Hexahedral grids**—Steinbrenner, Chawner, and Fouts: *A structured approach to interactive multiple block grid generation*—AGARD FDP Specialists’ Meeting, Loen, Norway, 1989
  - **Overlapping grids**—Benek, Buning and Steger: *A 3-D Chimera grid embedding scheme*—AIAA Paper 85-1523
  - **Cartesian grids**—Clarke, Salas and Hassan: *Euler calculations on multi-element airfoils*—AIAA Journal, 24 (3), 1986
  - **Unstructured tetrahedral grids**—Jameson, Baker, and Weatherill: *Calculation of inviscid transonic flow over a complete aircraft*—AIAA 86-0103

• **1990:** Lockheed acquired AIRPLANE
  **Unstructured Tetrahedral Grid Euler Code**
  - Vendor: Intelligent Aerodynamics, Inc., Princeton, NJ

• **The Path Forward**
  - Insufficient time and personnel to develop skills and experience with alternative methods to burn down the risk enough by 1991 for potential applications to F-22 EMD
  - **Strategy:** make maximum use of multi-zone structured grid—*once it’s built!*
Innovative Solution of Customer Problem!

- **Problem:** Estimate incremental aerodynamic forces, moments, and surface pressures due to control surface deflections for multiple settings and flight conditions to support structural analyses.

- **Solution:** Used *surface transpiration concept* to “simulate” the effect of control surface deflection by appropriately changing the no-normal-flow surface boundary condition; **NO NEED TO CHANGE THE INITIAL GRID!**

- The concept—originally proposed by Lighthill—had great success in simulating the effect of boundary layer on inviscid flow modeled using potential or Euler methods.

Method and results in *Raj and Harris, AIAA Paper 93-3506*
The Exciting Eighties!

**Personal**

1980
- Granted US Permanent Resident status
- And…

1981
- 1st son

1985
- 2nd son

1985
- Naturalized US Citizen

**Professional**

- **AIAA & SAE**
  - AIAA Euler Solvers Workshop: Monterey (1987)

- **ICAS Congress**


- **After-hours teaching**
  - UCLA Continuing Education (*Introduction to Aerodynamics*)
  - Lockheed Tech Institute (*Computational Fluid Dynamics*)

- **Lockheed consolidation** (1987)
  - Three companies into one: Lockheed Aeronautical Systems Company headquartered in Burbank, California
  - Loss of CFD and ACA talent and expertise in Georgia

  - Member, Corporate Task Force on Advanced Computing Methods
The Nasty Nineties!

- **US Aerospace industry in depression**
  - Lost 495,000 people (37% of workforce) in just five years (1990-1994)
  - Overall sales down 9% in 1994 after single-year 10% drop in 1993
  - Dramatic reductions in Research & Development by industry

- **Major consolidations, mergers, and reorganizations**
  - Dec 1992: *Lockheed acquires General Dynamics military aircraft division*
  - March 1995: *Lockheed and Martin Marietta formally merge*
  - Dec 1996: *Boeing and McDonnell Douglas announce merger*

---

* Major consolidations, mergers, and reorganizations

- Grumman
- Westinghouse*
- Northrop
- Texas Instruments*
- Raytheon*
- E-Systems
- GM Hughes*
- General Dynamics*
- Loral
- GE Aerospace
- Martin Marietta
- Lockheed
- McDonnell Douglas
- Rockwell*
- Boeing

(size of the bar represents sales volume)

15 down to 4 in 7 years!
New Opportunities: Early 1990s

- May 1990: Lockheed Reorganization—*one company into two!*
  - Lockheed Advanced Development Company (LADC), Palmdale, California
  - Lockheed Aeronautical Systems Company (LASC), Marietta, Georgia

- 23 April 1991: *YF-22 is the winner!*
  - Secretary of the USAF Donald Rice announced Lockheed’s YF-22 as the winner
  - LASC to work the F-22 Engineering and Manufacturing Development (EMD) contract in Georgia
  - *Raj relocates to Georgia in August 1991*

- 13 December 1991: *LASC selects two engineers to the first inaugural year of Technical Fellows program*
  - Chellman (Structures)
  - Raj (CFD)
  - *Increased emphasis on mentoring and technical leadership*
  - *Key Challenge: Rebuild ACA team in GA*
“A Dose of Reality”

• Engineer’s Week Celebration, San Fernando Valley (23 February 1991)
  o Conversation over drinks about CFD and YF-22
  o Caren asks: How many more “design cycles” on YF-22 could we do because of CFD?

• As Tech Fellow, Raj embarks on a mission to understand and address design related issues
  ▪ 1993–1997: AIAA Multi-disciplinary Design Optimization (MDO) TC member
  ▪ 1994: Multi-disciplinary Aerodynamic Design Environment (MADE) Proposal to DARPA by Jameson (IAI–Lead), Gregg (Boeing), Raj (Lockheed); not funded
  ▪ 1997: CFD at a Crossroads: An Industry Perspective (Invited) at the Thirty Years of CFD and Transonic Flow Symposium to honor Prof. Earll Murman on his 55th Birthday, Everett, WA; also in Frontiers of Computational Fluid Dynamics, Caughey & Hafez (eds.),1998, pp. 429-445
  ▪ 2007: Computational Uncertainty: Achilles’ Heel of Simulation Based Aircraft Design (Invited), NATO/RTO Air Vehicle Technology (AVT) Symposium, Athens, Greece

Source: Ref. 90 - 92
F-22 EMD TEAM (Euler) Analysis: 1991

- Full-aircraft airloads prediction ($40M estimated cost avoidance!)
  - 42-zone grid, 1.25 million nodes (for half the configuration)
    - built in 6 weeks from CATIA design loft by two engineers (Kinard and Harris)
  - 370 airloads cases; 3 months; 1600 CPU hours* on Cray-Y/MP 2/16
    - 6 Mach numbers (0.6 to max speed)
    - Angles of attack: -4° to +24°; Side-slip angles: 0° to 5°
    - Leading and trailing-edge flap deflections
    - Horizontal tail deflections and rudder deflections

MACH = 0.9; AOA = 8°

*equivalent to 24 hours a day, 5 days a week, for 13 weeks! Probably an industry record at that time.

Source: Ref. 93
Impact of Errors in Drag Prediction: C-141 Cruise Drag

- Predicted drag (early 1960s) based on wind-tunnel tests within *One Count of flight data*...
  
  ...*but good agreement due to Compensating Errors!*
  
  - Minimum Profile Drag: *Underpredicted*
  - Compressibility Drag: *Overpredicted*

- DoD Aeronautical Test Facilities Assessment Team (1997)
  
  - *Question*: Can we do better with improved wind-tunnel test techniques combined with CFD?
  
  - *Answer*: Cruise drag would be *Underpredicted* by 3.5%
    
    - Considering only Reynolds Number Scaling
      
      - Minimum Profile Drag Underprediction—about Eight counts
      - Compressibility Drag Overprediction—eliminated

*Erroneous Predictions would Increase Fuel Cost by $688M (FY96 dollars) for Entire Fleet over Service Life*
Impact of Errors in Drag Prediction:

C-5 Cruise Drag (mid 1960s)

- Total drag overpredicted (mid 1960s) by 2.5% based on wind-tunnel tests
  - Minimum Profile Drag: Underpredicted by one scale-up method and correctly predicted by another
  - Compressibility Drag: Overpredicted

- DoD Aeronautical Test Facilities Assessment Team (1997)
  - Question: Can we do better with improved wind-tunnel test techniques combined with CFD?
  - Answer: Cruise drag would be Underpredicted by 1.5%
    - Considering only Reynolds Number Scaling
      - Minimum Profile Drag Underprediction—1% to 3%
      - Compressibility Drag Overprediction—eliminated

Erroneous Predictions Would Increase Fuel Cost by $153M (FY96 dollars) for Entire Fleet over Service Life!

Source: Ref. 94
Impact of Errors in Drag Prediction: 
**F-22 Cruise Drag (1990s)**

- Drag predicted using wind-tunnel test matched well with flight test data for Mach 0.9 and 1.5

- Differences may be due to a combination of interpolated pieces
  - Thrust effects, auxiliary inlet and vents, control surface scheduling

- Subsonic and transonic drag rise poorly predicted
  - *Impacts accelerations, decelerations, cruise and loiter performance*

Source: Ref. 95
Impact of Errors in Drag Prediction:
HSCT Conceptual Design MDO Study (mid 1990s)

- High Speed Civil Transport
  - Cruise Mach Number: 2.4
  - Range: 5,500 nm
  - Payload: 250 passengers

- TOGW = 772,907 lbs.
- Fuel Weight Fraction = 0.52
- Empty Weight Fraction = 0.39
- Aspect Ratio = 2
- L/D\text{max} = 9.16

\textbf{Just Two-count Cruise Drag Overprediction Increases Take-Off Gross Weight by More Than 7%!}
Computing Advances: Key Enabler

Two orders of magnitude improvement in speed and memory in the 1980s

---

**Speed**

![Graph showing speed improvement over time with Cray Y-MP/C90, Cray Y-MP/832, Cray X-MP/48, and CDC.]

**TEAM Computational Time**
Cray Y-MP: 15 to 30 seconds per time step for a million cell grid

---

**Memory**

![Graph showing memory improvement over time with Cray Y-MP/C90, Cray Y-MP/832, Cray X-MP/48, and CDC.]

**TEAM Computational Memory**
Cray Y-MP: Approx. 40 times the maximum number of cells

---

By 1990, Euler Solutions on Million-cell Grid in 6 to 8 Hours...But Weeks of Grid-Generation Time Hampers Effectiveness!
Tackling the Grid Generation Challenge: 1990s

- **Objective:** 24 hour aero analysis turnaround without increased cost by 2000!
- **Approach:** Automate grid generation to reduce time and labor hours
- **1993-1996:** NASA-LaRC (Jim Luckring) Sponsored Studies - assess capabilities and limitations of rapidly evolving unstructured-grid *Euler methods* for preliminary design applications
  - *Kinard and Harris, Evaluation of two unstructured CFD methods—AIAA Paper 94-1877*
    - AIRPLANE code (Meshplane/FLOPLANE)
    - TetrUSS code (Vgrid/USM3D)
    - Three test cases: 74° delta wing; Wing C; and Arrow wing-body
    - Needs for improvement identified
  
<table>
<thead>
<tr>
<th></th>
<th>Memory (words/cell)</th>
<th>CPU time μs/cell/cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLOPLANE</td>
<td>34</td>
<td>11</td>
</tr>
<tr>
<td>USM3D</td>
<td>45</td>
<td>18</td>
</tr>
</tbody>
</table>
  
  - *Kinard, Finley and Karman, Prediction of compressibility effects using unstructured Euler analysis on vortex dominated flow fields—AIAA Paper 96-2499*
    - SPLITFLOW code (Cartesian grids)
    - TetrUSS code (Vgrid/USM3D)
    - Compressibility increments predicted well for forces, but not for moments
    - More details in NASA CR 4710 and 4711

---

*All Unstructured Grid Methods More Effective than TEAM*
Level IV

RANS Methods

1990s

Flow Model

\[ Q_t + F_x + G_y + H_z = Re^{-1} ( R_x + S_y + T_z ) \]

\[ Q = (\rho, \rho u, \rho v, \rho w, \rho E) \]

- Laminar flows—Reverts to N-S equations; no assumption (other than continuum)
- Turbulent flows—solve Reynolds-Averaged Navier-Stokes (RANS) equations
  - Turbulence models of nonlinear Reynolds stress terms needed for closure

Range of Applicability

- All Mach numbers and all flow configurations

Source: Refs. 105 to 115
Motivation for RANS: 
More Realism

RAE 2822 (AGARD Case 10)
\[ M_\infty = 0.75, \ \alpha_{\text{corrected}} = 2.81^\circ \]
\[ Re = 6.2 \times 10^6 \]
129 x 257 C Grid

Simulation of shock/boundary-layer interaction improves realism.

- Olling, Raj, and Miranda (1986)
  - Initiated TRANSAM* (Three-dimensional Reynolds-Averaged Navier-Stokes Aerodynamic Method) development by adding viscous terms to the TEAM Euler solver to serve as a testbed for turbulence models
    - Incorporate turbulence models: zero, one- and two-equation; all with fixed transition location

- Goble, Raj and Kinard (1993)
  - Many improvements along with Baldwin-Lomax and Chien k-\varepsilon turbulence models

- Raj, Olling and Singer (1988)
  - TEAM renamed (Three-dimensional Euler/Navier-Stokes Aerodynamic Method) with ability to perform Euler or RANS analyses
  - Applied to many test cases: airfoils to wings to full aircraft results in ICAS-90-6.4.4 and iPAC 911990

Labor-intensive Structured Grid Generation: Major Stumbling Block!

Source: Refs. 85, 105-107
NASA TetrUSS Replaces TEAM at Lockheed: Late 1990s

**TetrUSS Software: A Modular System**
- GridTool—GUI for surface definition
- VGRID/ VGRIDns—advancing front method for tetrahedral grids
- USM3D/ USM3Dns—cell-centered finite-volume upwind flow solver
- VPLOT3D—interactive, menu-driven extraction and display of flow data

**Rapid Advancement of Capabilities in the 1990s**
- Pirzadeh: *Structured Background Grids for Generation of Unstructured Grids by Advancing Front Method*, AIAA J, 31(2), 1993

*Choice Driven by Careful Cost-Benefit Assessment*

\[ M_\infty = 0.95 \quad \alpha = 4^\circ \quad Re_{mac} = 2 \times 10^6 \]
2000: Another Pivotal Year

24-hour turnaround of full aircraft RANS analysis using TetrUSS
(Thanks to the ACA team in Georgia!)

Mission Accomplished

- **P-3C Airloads (Goble and Hooker)**
  - Supported US Navy’s Service Life Assessment Program (SLAP)
  - Full aircraft grids with more than 7 million cells
  - Nearly 300 aerodynamic loads cases over entire flight envelope using Cray T3E and SGI Origin 2000
  - Details in AIAA 2001-1003

- **KC-130J Refueling Pod (Hooker)**
  - Design and integration of refueling pods
  - Full aircraft viscous grid with 7 million cells
  - *Six full aircraft viscous solutions per day* with dedicated use of two 64-node PC clusters: each node made up of dual 850 MHz Intel Pentium III processors with 768 MB RAM
  - Details in AIAA 2002-2805

- **Lockheed Martin Aeronautics Company**
  - January 2000: Three separate companies combined into *one Company with three sites* (California, Georgia, Texas) to improve chances of winning Joint Strike Fighter
  - *Raj selected as Senior Manager, Vehicle Science and Systems, Technology Development & Integration, Advanced Development Programs (the Skunk Works®)*

RANS: full steam ahead!
Outline

• Introductory Remarks

• ACA Evolution and Pursuit of Effectiveness
  o 1900 – 1950: Shifting Landscape of Fluid Mechanics
  o 1950 – 1980: Infancy through Adolescence
    ▪ Level I: Linear Potential Methods
    ▪ Level II: Nonlinear Potential Methods
  o 1980 – 2000: Quest for Effectiveness
    ▪ Level III: Euler Methods
    ▪ Level IV: Reynolds-Averaged Navier-Stokes (RANS) Methods
  o 2000 – 2050: Challenges and Prospects for Effective ACA

• Closing Remarks
Impressive RANS Capability Demonstrations: Early 2000s

- **F-22 Tail Buffet (2005)**
  - TetrUSS code

- **C-5M Re-engining (2006)**
  - TetrUSS code

- **F-35 Performance (2007)**
  - Top View
  - Side View
  - Bottom View
  - Falcon code

- **Low-boom Supersonic Airliner (2012)**
  - CFD++ code

Reasonably Quick and Inexpensive Simulation of Wide Variety of Flows about Full Aircraft. But...
Effectiveness of RANS Methods: Less than Satisfactory

The Overarching Challenge

RANS methods produce less than credible* data especially when applied to simulate complex flows characterized by separation and free vortices!

*How faithfully predictions mimic the real world?

• Most problematic when designing novel configurations
  o If RANS simulations predict flow separation or free vortices, are the data credible enough to invest additional time and effort for configuration redesign?
  o If expensive and time consuming wind-tunnel testing must be done for further validation—doesn’t it defeat the purpose of using RANS simulations in the first place for a cost-effective design process?

Assessing and Overcoming This Challenge Has Been a Constant Focus of ACA Community
RANS Simulations of Complex Flows: Predictions Not Credible

Wide variation in data among state-of-the-art turbulence models!
Laminar-to-turbulent transition modeling: yet another challenge!

“All Models are False, Some are Useful!”

TetrUSS simulations by Frink et al, AIAA Journal of Aircraft, 2012

Source: Ref. 120
Credible Data: Critically Important for Aircraft Design

“Design is an iterative decision-making activity performed by team of engineers to produce plans by which resources are converted, preferably optimally, into systems or devices to meet human need.”

T.T. Woodson
Introduction to Engineering Design, 1966

- Success of design efforts strongly depends on quality and timeliness of decisions.
- On-time quality decisions depend on availability of credible data at the right time and the right cost.

![Graph showing relative cost of major aerodynamic configuration change over design phases: Start Conceptual Design to First Flight.]

Quality Decisions ➔ Better Quality Product at Lower Cost
Less than Satisfactory RANS Effectiveness Limits its Value

2005

The Aeronautical Journal, Vol. 109

Tinoco, Bogue, Kao, Yu, Li, and Ball

“The major impact of CFD, delivered to date at Boeing, has mainly been related to its application to high speed cruise.”

2014

NASA/CR-2014-218178

CFD Vision 2030 Study: A Path to Revolutionary Computational Aerosciences

Slotnick Boeing R&T
Khodadoust Boeing R&T
Alonso Stanford University
Darmofal MIT
Gropp NCSA
Lurie Pratt & Whitney
Mavriplis Univ. of Wyoming

“…the well-known limitations of RANS methods for separated flows have confined reliable use of CFD to a small region of the flight envelope or operating design space.”

Source: Refs. 115 & 116
Assessment of RANS Effectiveness: Drag Prediction

AIAA CFD Drag Prediction Workshops (DPWs)

- **Primary Goal:** Assess state-of-the-art CFD methods as practical aerodynamic tools for the prediction of forces and moments on industry-relevant geometries, with a focus on absolute drag.
- **Test Cases:** Variants of commercial transport wing-body configurations; transonic flows; many meshes and flow-solvers; multiple turbulence models
- **Interesting Findings from 6\(^{th}\) DPW:** Tinoco et al, *Journal of Aircraft*, 55 (4), 2018
  - NASA Common Research Model (CRM) Wing-Body: *Solutions exhibited “tighter” convergence of total drag with a spread of less than 10 counts*
  - NASA CRM Wing-Body-Nacelle-Pylon: *Drag increment predicted within the uncertainty of the test data…this is of significant importance to industry design processes*
  - NASA CRM Wing-Body Static Aeroelastic Effect: *Higher lift predicted at a given angle of attack, and more negative (nose down) pitching moment at a given lift coefficient than observed in test data.*
Assessment of RANS Effectiveness: Prediction of High-Lift Characteristics

AIAA High Lift Prediction Workshops (HiLiftPWs)

• Formally initiated in 2009; three (3) workshops to date: 2010, 2013, 2017; numerous publications

• **Primary Goal:** Assess the numerical prediction capability (mesh, numerics, turbulence modeling, high-performance computing requirements, etc.) of current-generation CFD technology for swept, medium/high-aspect ratio wings in landing/takeoff (high lift) configurations.

• **Test Cases:** Variants of commercial transport configurations; subsonic flows; variety of grid systems and flow solvers; multiple turbulence models

• **Interesting Findings from 3rd HiLiftPW:** Rumsey et al, AIAA 2018-1258
  - JAXA Standard Model High-lift Configuration with and without Pylon/Nacelle
    - Fairly tight clustering of results in the linear lift-curve range, and very large scatter in results near maximum lift
    - Differences between nacelle/pylon on and off were well predicted in general
    - Significant influence of grid for the solutions near maximum lift
    - Transition model results were inconsistent near maximum lift; reasonable results for the wrong reasons!

Source: Ref. 118
Two Key Factors Hamper RANS Effectiveness

1. Numerical Modeling

Angle of Attack = 16°

Example: Solution Sensitivity to Compression Factor in Limiter Function in Upwind Scheme

2. Turbulence Modeling

Example: Solution Sensitivity to Turbulence Modeling

“What We Simulate is Not Reality Itself, But Reality Determined by Our Models”
Numerical Modeling: Requires Spatial Discretization

Computed solutions differ from exact solutions due to discretization errors.

- **Structured Grid**
  - Boundary Conforming

- **Unstructured Grid**
  - Boundary Conforming

- **Hybrid Grid**
  - Boundary Conforming

- **Unstructured Cartesian Grid**
  - Non-Boundary Conforming

**Difficult to Assess Errors: Exact Solution Not Known a Priori**
Numerous Numerical Schemes to Solve Euler & RANS PDEs!

<table>
<thead>
<tr>
<th>Year</th>
<th>Developer(s)</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td>MacCormack</td>
<td>Two stage scheme for hyperbolic equations</td>
</tr>
<tr>
<td>1973</td>
<td>Boris &amp; Book</td>
<td>Flux Corrected Transport (FCT) oscillation control via slope limiters</td>
</tr>
<tr>
<td>1974</td>
<td>Van Leer</td>
<td>Higher-order Godunov scheme - MUSCL</td>
</tr>
<tr>
<td>1981</td>
<td>Steger &amp; Warming</td>
<td>Flux splitting</td>
</tr>
<tr>
<td>1981</td>
<td>Jameson, Schmidt, Turkel</td>
<td>Shock capturing via controlled diffusion – full convergence to steady state</td>
</tr>
<tr>
<td>1981</td>
<td>Ni</td>
<td>Multigrid Euler solver</td>
</tr>
<tr>
<td>1983</td>
<td>Roe</td>
<td>Approximate Riemann solver</td>
</tr>
<tr>
<td>1983</td>
<td>Harten</td>
<td>Theory of Total Variation Diminishing (TVD) schemes</td>
</tr>
<tr>
<td>1983</td>
<td>Jameson</td>
<td>Agglomeration multigrid full approximation storage (FAS) scheme for Euler equations</td>
</tr>
<tr>
<td>1985-86</td>
<td>Jameson, Baker, Weatherill</td>
<td>Airplane Code: 3D Euler equations on unstructured mesh – edge based data structure</td>
</tr>
<tr>
<td>1986-88</td>
<td>Yoon-Jameson</td>
<td>Lower-Upper Symmetric Gauss Seidel (LU-SGS) scheme</td>
</tr>
<tr>
<td>1987</td>
<td>Harten, Engquist, Osher, Chakravarthy</td>
<td>Essentially Non-Oscillatory (ENO) scheme</td>
</tr>
<tr>
<td>1990</td>
<td>Cockburn &amp; Shu</td>
<td>Local Discontinuous Galerkin (LDG) method</td>
</tr>
<tr>
<td>1991</td>
<td>Jameson</td>
<td>Multigrid dual time stepping scheme for unsteady flow</td>
</tr>
<tr>
<td>1993</td>
<td>Liou</td>
<td>Advection Upstream Splitting Method (AUSM) scheme</td>
</tr>
<tr>
<td>1994</td>
<td>Jameson</td>
<td>Theory of Local Extremum Diminishing (LED) scheme</td>
</tr>
<tr>
<td>1994-96</td>
<td>Liu, Osher, Chan, Shu</td>
<td>Weighted ENO (WENO) scheme</td>
</tr>
<tr>
<td>2001</td>
<td>Jameson-Caughey</td>
<td>Nonlinear Symmetric Gauss-Seidel (SGS) multigrid scheme</td>
</tr>
</tbody>
</table>

Minimize Truncation, Dispersive, and Dissipation Errors

Source: Ref. 88
No Shortage of Turbulence Models for RANS!

- **Zero-equation Models**
  - Cebeci-Smith (1967) and Baldwin-Lomax (1978): *two layer, algebraic*

- **Half-equation Models**
  - Johnson-King (1985): *ODE to specify shear stress level*

- **One-equation Models**
  - Baldwin-Barth (1990) and Spalart-Allmaras (1992): *turbulent kinetic energy*

- **Two-equation Models**
  - Jones-Launder (1972): *k-ε* (*turbulent kinetic energy and turbulent dissipation*)
  - Wilcox (1988): *k-ω*
  - Smith (1990): *k-kε*
  - Menter (1993): SST *k-ω*

- **Algebraic Stress Models**
  - Gatzki-Speziale (1993)
  - Girimaji (1996)

"It is quite clear that no model is universal, giving good results for all flows of interest."
– Peter Bradshaw, FRS
Imperial College & Stanford

"It is the mark of an educated man to look for precision in each class of things just so far as the nature of the subject admits."
– Aristotle

Source: Ref. 119
Why Turbulence Modeling is a Challenge?

Accurately Modeling the Complexity of Turbulence with a Few Free Parameters is an Extremely Long Shot Indeed

Source: Ref 121
"I am an old man now, and when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really rather optimistic."

Sir Horace Lamb

Address to British Association for the Advancement of Science
London, U.K., 1932

(1849-1934)

Turbulence has been the bane of fluid dynamicist’s existence—seemingly forever!

Leonardo da Vinci Flow behind obstacle, ca. 1510 – 1513, (from Royal Collection Trust, London, UK)
Turbulence in a Nutshell

Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity.

Lewis Fry Richardson
FRS, British Mathematician, Physicist, Meteorologist, Psychologist

11 October 1881 – 30 September 1953
“...we cannot calculate all flows of engineering interest to engineering accuracy. However, the best modern methods allow almost all flows to be calculated to higher accuracy than the best-informed guess, which means that the methods are genuinely useful even if they cannot replace experiments.

This reviewer’s opinion is that engineering calculations will have to be done by Reynolds-averaged methods for the foreseeable future, but that computer simulations of eddy motion can and will provide the detailed statistics—above all, the pressure fluctuation statistics—that cannot be adequately measured.”

Peter Bradshaw

TURBULENT SECONDARY FLOWS


Optimists Rule the World!
What’s the Dominant Contributor to Error in RANS Solutions?

Is it the Mesh, the Solver, or the Turbulence Model?


• **Approach:** Third High-Lift Prediction Workshop (39 datasets) and Fifth Drag Prediction Workshop (31 datasets) matched into groups based on three primary variables: mesh, flow solver and turbulence model

• **Qualitative Conclusions** based on crude statistical analysis due to sparse amount of data in each group
  
  o Mesh and turbulence model appear to have about equally large impacts on outputs. (✓)
    - Results of different mesh sets with the same flow solver and turbulence model differed about as much as the average results for the three groups varied from each other!
  
  o Even with relatively fine meshes used, there are still flow features resolved by some meshes and not others
  
  o Flow solver is at least as big a difference as other factors. (✓)
    - Community needs to do a better job of verification of numerical model and turbulence model implementations.
  
  o User selected input parameters can cause significant variation in output values. (✓)
    - Improved user training can help.

Source: Ref. 123
TiCTaC for Credible Aerodynamic Data: A Near Term Strategy

Tightly Coupled Tests and Computations

Devise the best way of *judiciously* coupling wind tunnel testing (WTT) with RANS CFD to deliver credible aerodynamic data—rapidly and affordably

*2002: First proposed* (Raj)

NASA/DOD Workshop on Aerodynamic Flight Predictions, Williamsburg, VA, USA, Nov 19-21, 2002

*2012: Revisited* (Raj)

5th Symposium on Integrating CFD and Experiments in Aerodynamics, JAXA, Tokyo, Japan, Oct 3-5, 2012

• 2016: Applied Aerodynamics Conference, Bristol, UK (Raj et al)
  o **Exploit technological advances** to develop and implement TiCTaC by leveraging complementary strengths of CFD and WTT
    ✓ WTT (Additive Manufacturing, Rapid Prototype Testing)
    ✓ CFD (Grid Adaption, High Performance Computing, Uncertainty Quantification)

---

A Near-term Alternative: TiCTaC
(Tightly Coupled Test and Computations)

• **Premise:** CFD Codes Will NOT Produce Credible Data for Your Application Unless Previously Validated on the "Same" Application
  - Too Many Potential Traps: Computing Systems, Coding Errors, Complexity of Mathematical and Numerical Methods, Modelling Approximations, Inappropriate Use, etc., etc., etc.
• **Approach:** Develop and Implement "Validation Plan" Targeted at Maximizing Prediction Credibility for Your Application
  - Identify the principal source(s) of uncertainty related to modeling of relevant flow physics and numerics
  - Perform dedicated tests for the sole purpose of "refining" modeling parameters
  - Utilize updated models to maximize credibility of CFD simulations

---

*Can We Realize its Enormous Potential in Practice?*

---

Source: Refs. 124-126
Progress of Wall-Modeled Large Eddy Simulation (WM-LES) closely tied to

- **Grids**: Methods for rapidly generating highly refined, truly boundary-conforming grids
- **Models**: Advanced near-wall flow models
- **Algorithms**: Higher-order numerical methods that minimize numerical dissipation
- **Software**: Development and implementation of effective strategies for designing computer software that exploits emerging computer hardware trends
- **V&V**: Approaches for verification and validation of complex software, and for uncertainty quantification
- **Data**: Approaches for efficiently managing large amounts of data, for fast processing of extremely large datasets to extract information of value to aerodynamicists

...
CFD Vision 2030: NASA Roadmap

Truly Effective ACA by 2050?

Source: Ref. 116
“Two workers at UNCAF (United Nations Computational Aerodynamics Facility) have recently made a **startling discovery…by building a small wooden model of an airplane and then blowing air past it in an enclosed tunnel, reasonably accurate predictions may be made of what the flow codes would compute.** Also, some factors, such as artificial viscosity (numerical diffusion), are neglected completely in wind tunnel modeling.”

“**While the wind tunnel may never fully replace the computer, it is almost certain to become the most useful engineering tool of the future.**”

**Will the Wind Tunnel Replace the Computer?**
*By BOB COOPERSMITH*
*AIAA Student Journal, Summer 1985*
Outline

• Introductory Remarks

• ACA Evolution and Pursuit of Effectiveness
  o 1900 – 1950: Shifting Landscape of Fluid Mechanics
  o 1950 – 1980: Infancy through Adolescence
    ▪ Level I: Linear Potential Methods
    ▪ Level II: Nonlinear Potential Methods
  o 1980 – 2000: Quest for Effectiveness
    ▪ Level III: Euler Methods
    ▪ Level IV: Reynolds-Averaged Navier-Stokes (RANS) Methods
  o 2000 – 2050: Challenges and Prospects for Effective ACA

• Closing Remarks
Pursuit of Effectiveness Started Long Ago…

“Both for engineering and for many of the less exact sciences, such as biology, there is a demand for rapid methods, easy to be understood and applicable to unusual equations and irregular bodies. If they can be accurate, so much the better; but 1 per cent, would suffice for many purposes.” – Richardson, 1910

“Prospective users…rarely interested in whether or not an accurate solution of an idealized problem can be obtained, but are concerned with how well the calculated flow agrees with the real flow.” – Hess and Smith, 1967

“The effectiveness of computational aerodynamics depends not only on the accuracy of the codes but to a very large degree—perhaps more than is generally appreciated—on their robustness, ease and economy of use.” – Miranda, 1982

…and Continues Today!

Source: Refs. 3, 14, 61
CFD Codes for ACA: A Word of Caution

• Developers Typically Offer ‘Validated CFD Code’
  o Implies that simulated results can be trusted to accurately predict the real flow characteristics.

• Traditional Code Validation Approach
  o Correlate computed and test results for a set of test cases.

• Limited Value of Traditional Code Validation
  o Even extensive correlations of computed and test results on geometries and flow conditions that differ substantially from those being considered for design are of limited value.
  o Too Many Potential Traps: Generation of grid-converged solutions; Availability of on- and off-surface data from the same test; Reynolds number scaling of test data; Accurate matching of boundary conditions; User proficiency; etc., etc., etc.

Beware of ‘Validated CFD Code’ – A Misnomer

Source: Refs. 91 & 92
Top 10 Takeaways from My Journey

1. ACA is a discipline, CFD is a tool (à la wind tunnels). CFD is to ACA as airplane is to air transportation.

2. If CFD data don’t match wind-tunnel data, ask why? If they do, most definitely ask why?

3. Effectiveness = Quality x Acceptance \( (E = Q \times A) \). It is ultimately assessed by design teams (who initiate the “Value Chain”), not developers.

4. Generating a ACA solution is easy; generating a credible solution is hard—REALLY hard.

5. **Effective ACA** is all about delivering credible solutions—on time, on budget—to meet engineering design needs.

6. Success requires communication & collaboration across all stakeholders to simultaneously improve quality of results and productivity of processes.

7. Converting a basic research concept into an effective capability is a long and arduous process marked by invention, initiative, and innovation.

8. A talented engineer can do wonders even with a poor tool.

9. Nothing—absolutely nothing—is worth compromising your integrity.

10. Life is akin to an unsteady system with unsteady boundary conditions—don’t expect a steady solution!
“It’s the Engineering, Stupid!”

A scientist discovers that which exists. An engineer creates that which never was.

Theodore von Kármán
1881-1963

Engineering is in the end about making something.

Eugene E. Covert, MIT
1926 - 2015

Don’t Lose Sight of the Big Picture!

Source: Internet
DEDICATED TO
LUIS R. MIRANDA

Aerodynamics Engineer par excellence
Father of $E = Q \times A$
A consummate professional, a model leader
My mentor, adviser, guide
ACKNOWLEDGMENTS

The author has been incredibly fortunate to have had many excellent opportunities to learn from a large number of high-caliber individuals over the past six decades. These individuals are too numerous to name, but they know who they are. He is eternally grateful to each of them for their generous guidance, help, and support.
Cited References

1. https://en.wikipedia.org/wiki/Andr%C3%A9_Gide
2. https://en.wikipedia.org/wiki/Lewis_Fry_Richardson
Cited References

Cited References


https://doi.org/10.2514/3.44974


Cited References


Cited References


