Bounding Continuity Risk in H-ARAIM FDE

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BIOGRAPHY

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ABSTRACT

Future dual-frequency, multi-constellation advanced receiver autonomous integrity monitoring (ARAIM) is expected to bring significant navigation performance improvement to civil aviation. Among the two ARAIM schemes that are being investigated, horizontal ARAIM (H-ARAIM) aims at providing horizontal navigation for aircraft en-route, terminal, initial approach, non-precision approach (NPA) and departure operations. Because most of those operations cannot be easily aborted once started, H-ARAIM continuity requirements are stringent, and loss of continuity (LOC) for H-ARAIM is considered a serious safety event. In this paper, we address the impact of detected faults and unscheduled satellite outages (USO) on H-ARAIM continuity, develop new methods to quantify continuity risk, and investigate H-ARAIM overall availability performance accounting for both integrity and continuity. Using multi-constellation global navigation satellite systems (GNSS), the heightened likelihood of the user encountering a fault or USO could significantly increase continuity risk. In response, our previous work explored implementing an exclusion function to improve continuity after fault detection, and, separately, analyzed critical satellites to assess the impact of USO on H-ARAIM continuity. In this work, we derive a more rigorous continuity risk equation that quantitatively accounts for the LOC contributions from both measurement faults and USO. This new approach unifies the separate critical satellite analysis into one integrity risk equation, which characterizes the integrity threat under USO conditions. Moreover, this approach allows us to determine whether an exclusion function is still needed following a USO event. With this new method fully described and derived, the last part of the paper applies it to analyze H-ARAIM availability performance. The results indicate that dual-constellation H-ARAIM can provide high service availability, where both integrity and continuity requirements are met.

INTRODUCTION

Global navigation satellite system (GNSS) measurements are vulnerable to faults, including satellite and constellation failures, which can potentially lead to major integrity threats for users. To mitigate their impact, fault detection algorithms, such as receiver autonomous integrity monitoring (RAIM), can be implemented [1, 2]. The core principle of RAIM is to exploit redundant measurements to achieve self-contained fault detection at the user receiver [3]. With the modernization of GPS, the full deployment of GLONASS, and the emergence of Galileo and Beidou, a greatly increased number of redundant measurements have become available, which has recently led to a renewed interest in RAIM. In particular, due to its potential to achieve worldwide coverage with a reduced investment in ground infrastructure, dual-frequency, multi-constellation advanced RAIM (ARAIM) has attracted considerable attention in the European Union and the United States [4, 5].

Currently, two versions of ARAIM corresponding to two operational scenarios are being investigated: horizontal ARAIM (H-ARAIM) aims at providing horizontal navigation for aircraft en-route, terminal, initial approach, non-precision approach (NPA) and departure operations, and vertical ARAIM (V-ARAIM) is intended for aircraft approach [6]. ARAIM is scheduled to first provide horizontal service with improved availability performance as compared to existing RAIM [6]. Therefore, H-ARAIM is of primary interest and is the focus of this paper.

RAIM became operational in the mid-90s as a backup navigation tool to support aircraft en-route flight using GPS only [7]. H-ARAIM may be considered an evolution of RAIM that takes advantage of GNSS modernization and of newly deployed GNSS. H-ARAIM also serves for operations with more stringent navigation requirements. For example, horizontal alert limits (HAL) as low as 0.1 nautical miles are considered for H-ARAIM NPA operations; in this case, when H-ARAIM is used as primary navigation tool, loss of continuity (LOC) becomes a more serious safety event. These differences in target level of safety must be accounted for in the design of H-ARAIM, and motivate the reassessment of fault detection and exclusion (FDE) methods as compared to conventional RAIM.

Most ARAIM work to date has focused on developing theoretical methods to reduce integrity risk, and false alarms are typically regarded as the only continuity concern. However, for H-ARAIM operations, continuity requirements are stringent, and other sources that cause LOC must also be properly accounted for. This is because H-ARAIM missions cannot be easily aborted once started, and additional navigation means must be found when LOC occurs. As a consequence, pilots would face increased workloads and more stress would be brought to air traffic controllers (ATC). In particular, for the cases when other navigation methods are not available, LOC during H-ARAIM operations can lead the aircraft to be in a dangerous situation. In response, this work aims at precisely quantifying and bounding the overall H-ARAIM continuity risk. Future multi-constellation GNSS will provide greatly increased measurement redundancy, which improves ARAIM detection capability. However, the accumulated likelihood of the user encountering a fault or unscheduled satellite outage (USO) will also increase, leading to a rise in continuity risk. In addition, newly deployed constellations may be subject to larger prior probabilities of satellite fault and USO. Therefore, H-ARAIM LOC due to detected satellite faults and USO are the primary concerns of this paper.

To improve continuity after fault detection, an exclusion algorithm was designed, and the associated predictive FDE integrity risk was bounded in our prior work [8]. The exclusion function is called once an alarm is triggered, and it autonomously identifies and removes the cause of the alarm, thereby preserving continuity of service. However, the gain in continuity comes at the cost of increased integrity risk [9, 10]. This is due to the fact that (a) excluding satellites may weaken the satellite geometry, and (b) the possibility of excluding the wrong satellite increases the integrity risk. Therefore, exclusion introduces a tradeoff between integrity and continuity. Our previous work also separately analyzed critical satellites to address the impact of single-satellite USO on H-ARAIM continuity. A critical satellite is one whose loss will result in the integrity risk exceeding requirements. To be more general, in this work, we extend the critical satellite approach to include USO conditions with more than one satellite. This approach presumes a required capability of still being able to perform detection and exclusion using the remaining satellites after satellite(s) is unexpectedly lost. However, it will be shown that the resulting H-ARAIM continuity risk bound is overly conservative, especially when an exclusion function is implemented.

In response, we develop a new method that rigorously quantifies the impact of measurement faults and USO on H-ARAIM continuity, and unifies the FDE integrity evaluation and the critical satellite analysis in one step. In this new approach, the overall H-ARAIM continuity risk equation is first expressed in terms of mutually exclusive (and exhaustive) USO scenarios, and then accounts for all fault modes under the corresponding USO conditions. As a result, the LOC contributions from those events can be controlled by setting FDE thresholds in order to meet the continuity requirement. In this paper, we investigate two ways to allocate the overall continuity budget into each specific USO and fault mode scenario. First, the same budgets are used to set FDE thresholds over all USO conditions. Therefore, to execute the FDE functions during operation, the receiver does not need to know whether there is a USO present or not because of the same requirements when computing thresholds. In contrast, if the users are always aware of the current USO status including the time epochs when USO occurs and ends, different continuity budgets can be applied under USO condition. Accordingly, the performance requirements on the FDE functions may change depending on whether there are unexpected satellite losses. Since the prior probabilities of USO also need to be accounted for, to meet the same allocated budget, the FDE thresholds will become tighter under USO conditions than the outage-free (OF) case, thereby resulting in smaller contributions in the integrity risk equation. Moreover, this approach enables us to identify whether an exclusion function is even needed or not after USO has occurred. Using baseline dual-constellation GPS/Galileo, there are cases when H-ARAIM operation only requires the detection function to be available after a satellite loss, since the accumulated joint probabilities of simultaneously encountering measurement faults and USO are well below the continuity risk requirement.

To quantify the integrity risk associated with this new approach, integrity threats under USO conditions also need to be properly accounted for. For example, the user may be exposed to a hazardous situation after a satellite is lost and no alarm occurs using the remaining satellites. In response, we derive a predictive integrity risk equation that characterizes all the conditions that the user may be in, including both USO and satellite fault. As in our previous work in [8], a computationally efficient upper bound is derived to help evaluate integrity risk. We will show that overall continuity risk can be controlled by setting FDE thresholds under OF and USO conditions, and that the H-ARAIM operation is available if the resulting integrity risk meets its requirement.

The last part of the paper compares the critical satellite approach and new method in regard to addressing the impact of USO on H-ARAIM continuity, and presents a performance analysis. Required navigation performance (RNP) 0.1 and RNP 0.3 operations are used as examples to show the achievable H-ARAIM performance. (RNP 0.1 is the most stringent navigation requirement for H-ARAIM operations.) Two separate analyses corresponding to both approaches are carried out using a baseline GPS/Galileo combined constellation [6]. In addition, we analyze the two ways to allocate the continuity budget using the new method. Using the critical satellite approach that conservatively accounts for the USO impact on H-ARAIM continuity, the results show limited availability for both RNP 0.1 and 0.3, especially when critical satellite pairs are considered. In comparison, by implementing the new method introduced in this work, the results indicate that high availability can be achieved, where both the continuity and integrity requirements are met.

IMPACT OF FAULT AND USO ON H-ARAIM CONTINUITY

In aviation navigation, continuity measures the capability of the system to perform its function without unscheduled interruptions during the intended operation. Continuity risk, or probability of LOC, is the probability of a detected but unscheduled navigation function interruption after an operation has been initiated. The occurrence of H-ARAIM LOC is regarded as a *major* failure condition when backup navigation systems are not available [11]. Our prior work has interpreted and discussed the H-ARAIM continuity risk requirement $\boldsymbol{C_{REQ}}$ [8], which is specified on a per-hour basis in a range from 10^{-8}/hour to 10^{-4}/hour by the International Civil Aviation Organization (ICAO) [12]. The range of $\boldsymbol{C_{REQ}}$ accounts for the number of aircraft that simultaneously use the same navigation service. Therefore, the actual H-ARAIM $\boldsymbol{C_{REQ}}$ used during operation may be variable, and is highly dependent on the traffic density and airspace complexity. In this paper, we use $\boldsymbol{C_{REQ}} = 10^{-6}$/hour as an example to investigate H-ARAIM performance. This value corresponds to the conservative assumptions that 100 aircraft are simultaneously using the same GNSS navigation service, and possible mitigation means are available if LOC occurs.

Measurement fault is one of the main sources that leads to H-ARAIM LOC, because most faults will be instantaneously detected by the detection function. According to the commitment of the GPS constellation service provider (CSP), GPS is expected to have less than 3 faults per year [13]. This commitment corresponds to a failure rate of 10^{-5}/hour/space vehicle (SV), which already exceeds H-ARAIM $\boldsymbol{C_{REQ}}$. New constellations may be subject to larger fault probabilities than GPS, especially in the early stage of their deployments. In addition, for H-ARAIM operation, a conservative value 10^{-4}/hour/constellation must be assumed for constellations other than GPS to account for the constellation-wide fault [14]. Therefore, using multi-constellation GNSS, the accumulated likelihood of fault occurrences must be mitigated to meet the H-ARAIM continuity requirement.

The exclusion function can identify and exclude the faulted measurement after a detection event occurs so that the user can use remaining satellites for positioning. We have designed an exclusion algorithm in prior work to reduce H-ARAIM continuity risk due to fault detection [8]. With an exclusion function being implemented, LOC occurs only if a fault is detected but cannot be excluded. The probability of such event occurring is controllable by setting exclusion thresholds, which ensure the post-exclusion LOC probability is smaller than the allocated continuity budget. In other words, the exclusion function should be designed to lower the LOC probability due to measurement fault to the desired level.

Satellite outage is another source that interrupts operation, and can significantly increase H-ARAIM continuity risk. There are two types of outage: scheduled satellite outage (SSO) is announced at least 48 hours in advance to the user, and USO typically results from sudden system malfunctions or maintenance occurring outside the scheduled period [13]. Since SSO is known before the operation, it impacts availability rather than continuity. Therefore, we only account for the impact of USO on H-ARAIM LOC. GPS standard positioning service performance standard (GPS SPS PS) has specified that the prior probability of USO occurrence is less than 2×10^{-4}/hour/SV [13]. Assuming the other constellations can achieve the same probability as GPS, the total probability of the H-ARAIM user undergoing USO exceeds $\boldsymbol{C_{REQ}}$ significantly. In response, we investigated single critical satellite to quantify this impact in prior work [8], and develop a new method in this paper by accounting for USO in both the continuity and integrity risk equations. These two methods will be described in the following sections. As comparison, both methods are used to analyze the overall H-ARAIM availability performance.

CRITICAL SATELLITE APPROACH

This section describes a modified critical satellite approach from the one in our prior work [8], and addresses its drawbacks. In particular, we account for the continuity risk contributions of more than one satellite USO. A single critical satellite is one whose loss will result in the integrity risk exceeding the requirement during an operation. In other words, after one satellite is unexpectedly lost, if the remaining satellites cannot support the FDE functions, the lost satellite is a critical satellite. Similarly, if the simultaneous loss of multiple satellites causes LOC, then those satellites are considered a critical satellite group. At one specific location and time epoch, the contribution of USO to H-ARAIM continuity risk P^*_{USO} can be evaluated by:

$$P^*_{USO} = n_c \cdot P_{out} + n_p \cdot P^2_{out} + P_{rare} \tag{1}$$

where

- n_c : number of single critical satellites at one snapshot.
- P_{out} : prior probability of single-satellite USO occurrence, $P_{out} = 2\times10^{-4}$ /hour/ SV .
- n_p : number of critical satellites pairs, i.e., the simultaneous loss of the corresponding two satellites results in LOC.
- P_{rare} : the sum over all critical satellite groups of the probabilities of losing more than two critical satellites simultaneously. Since their prior probabilities are very small, there is no need to go over all the combinations.

It is worth clarifying that the superscript '*' in equation (1) corresponds to the critical satellite approach. To ensure the overall H-ARAIM continuity requirement, a separate analysis is needed to compute n_c and n_p. In this work, we use $P^*_{USO,REQ} = 10^{-7}$/hour as an example allocation of the continuity budget to the impact of USO. The requirement of $P^*_{USO} < P^*_{USO,REQ}$ can be expressed in terms of n_c and n_p as follows:

$$n_c < \frac{P^*_{USO,REQ} - P_{rare}}{P_{out}} = 5\times10^{-4}\,\text{SVs} \text{ , and } n_p < \frac{P^*_{USO,REQ} - P_{rare}}{P^2_{out}} = 2.5\,\text{Pairs} \tag{2}$$

Since n_c and n_p are integers, $\boldsymbol{C_{REQ}}$ can only be met when there are no single critical satellites ($n_c = 0$), and the number of critical satellite pairs is less than 2 ($n_p < 2$). The following algorithm can be applied to determine n_c and n_p, in order to assess H-ARAIM continuity:

Step 1 : Evaluate the integrity risk P_{HMI} using all-in-view satellites. If P_{HMI} is smaller than the integrity requirement I_{REQ}, then go to the next step. Otherwise, set $n_c = 0$ and $n_p = 0$.

Step 2 : Remove one single SV (or SV pair) and reevaluate the integrity risk. If the reevaluated integrity risk exceeds I_{REQ}, then the removed SV (or SV pair) is regarded as a critical one. Otherwise, it is not critical.

Step 3 : Repeat step 2 for all the SVs and SV pairs. Count all the critical SVs and SV pairs to obtain n_c and n_p.

Using the critical satellite approach, we can conservatively account for the impact of USO on H-ARAIM continuity risk. An operation is declared to be available only if the all-in-view integrity risk obtained in step 1 is smaller than I_{REQ} and $P_{USO}^{*} < P_{USO,REQ}^{*}$. However, there are three main issues with this approach. First, this method requires a separate analysis to get n_c and n_p, which significantly increases the computational load and complicates H-ARAIM service. Second, in step 2, the reevaluated integrity risk is conditioned on the loss of SV(s), but the prior probabilities of those events are neglected when comparing with the requirement I_{REQ}. Therefore, the availability results of this approach are overly conservative. Third, when reevaluating the post-outage integrity risk, the same requirements on FDE function performance as in the OF case are implicitly assumed. However, since the USO prior probabilities also need to be accounted for to set the post-outage FDE thresholds, the requirements on the FDE functions under USO conditions can be less stringent.

OVERALL H-ARAIM CONTINUITY RISK

In response to the drawbacks of the critical satellite approach, a new method that rigorously quantifies and tightly bounds the impact of USO on H-ARAIM continuity is developed in this work. The principle of this approach is unifying all the LOC contributions from both measurement fault and USO in one continuity risk equation, and limiting those contributions by setting FDE thresholds. Accordingly, the associated predictive integrity risk also accounts for the threats under USO conditions, and the operation is available when the overall P_{HMI} meets the requirement.

With the new method, the total H-ARAIM continuity risk is first divided into two main groups: (a) LOC due to not excluded (NE) detection events, and (b) all other contributions including ionospheric scintillation and radio frequency interference, i.e.:

$$P_{LOC} = P_{D,NE} + P_{Other} \tag{3}$$

In this paper, we employ an example allocation of the overall C_{REQ} to account for the two terms in equation (3), and the requirements are $P_{D,NE,REQ} = 9\times10^{-7}$/hour and $P_{Other,REQ} = 10^{-7}$/hour, respectively. The mitigations of the causes of LOC in P_{Other} are beyond the scope of this paper, so we assume the contribution of P_{Other} is always smaller than the allocated continuity budget. To address the impact of USO on H-ARAIM continuity, the first component of equation (3) can be expressed as:

$$P_{LOC} = \sum_{k=0}^{H} P\left(D_k, \overline{E}_k \mid O_k\right) P_{O_k} + P_{Other} \tag{4}$$

where

O_k : outage occurs on satellite subset k, where $k = 0, 1 \ldots H$, used to denote all possible USO combinations including the OF condition (O_0), single-satellite USO, dual-satellite USO, etc.

P_{O_k} : prior probability of the event 'O_k'.

D_k : detection occurs using the remaining satellites when there is a USO event 'O_k', '$k = 0$' denotes all satellites are in view.

$\overline{E}_k$: no exclusion can be made using the remaining satellites under the condition 'O_k'.

Moreover, under each USO condition 'O_k', all the fault hypotheses for the remaining satellites need to be characterized. By accounting for the fault modes of each specific scenario, equation (4) becomes:

$$P_{LOC} = \sum_{k=0}^{H}\left(\sum_{i=0}^{H_k} P\left(D_k, \overline{E}_k \mid H_i, O_k\right) P_{H_i}\right) P_{O_k} + P_{Other} \quad (5)$$

where

H_i : multiple fault hypotheses for the remaining satellites under the USO condition 'O_k', i = 0, 1 … H_k, which accounts for all faulty SV combinations including fault-free (FF) (i = 0), single-satellite faults, dual-satellite faults, etc.

P_{H_i} : prior probability of the fault hypothesis 'H_i'.

Equation (5) expresses $P_{D,NE}$ over all the USO conditions and associated fault modes, which enables us to limit those contributions. Under the scenario 'O_k', a detection event is a false alarm (FA) when there is no fault present, i.e., i = 0. The probability of its occurrence can be limited by setting the detection thresholds. In addition, when the user is exposed to a fault condition ($i \neq 0$), the probability of not successfully excluding the fault can be controlled by setting the exclusion thresholds. Therefore, using this method, the contributions of LOC from both measurement fault and USO can always be reduced until they meet their corresponding continuity budget allocations.

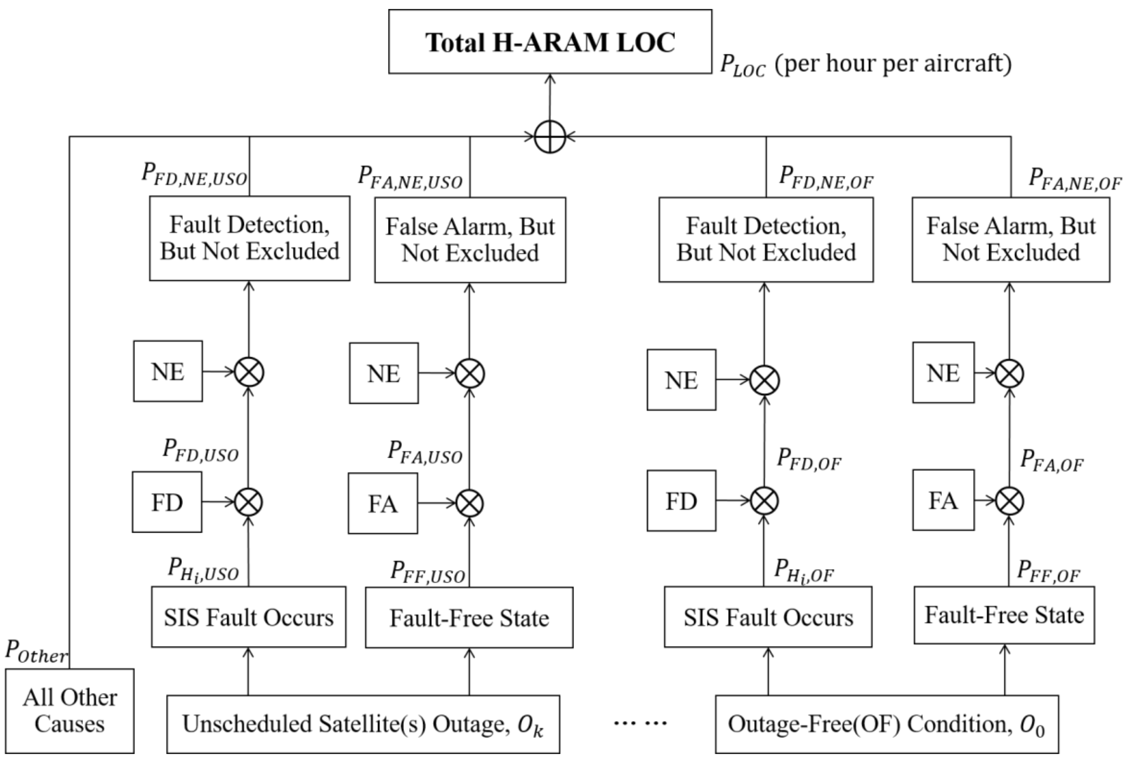

Fig. 1 H-ARAIM LOC Tree

Figure 1 provides a H-ARAIM continuity tree that visually expresses equation (5). To further address the method for controlling each contribution of the branches in this LOC tree, the FDE algorithm that operates in real time needs to be specified.

H-ARAIM FDE ALGORITHM

This paper employs a solution separation (SS) FDE algorithm designed in our prior work [8], and extends its application to USO conditions. Figure 2 shows the flow diagram of the real-time H-ARAIM FDE process, and this diagram is the same as the one in [8] under OF case, i.e., k = 0. However, during operation, the user may be exposed to an outage condition 'O_k' where $k \neq 0$. In this case, the evaluation of the real-time integrity risk only depends on the visible satellite geometry. If the integrity risk meets the requirement, the FDE functions perform in the same way as if the lost satellite(s) were not visible in the first place. That is, the remaining satellites post-outage are regarded as a new 'all-in-view' satellite set.

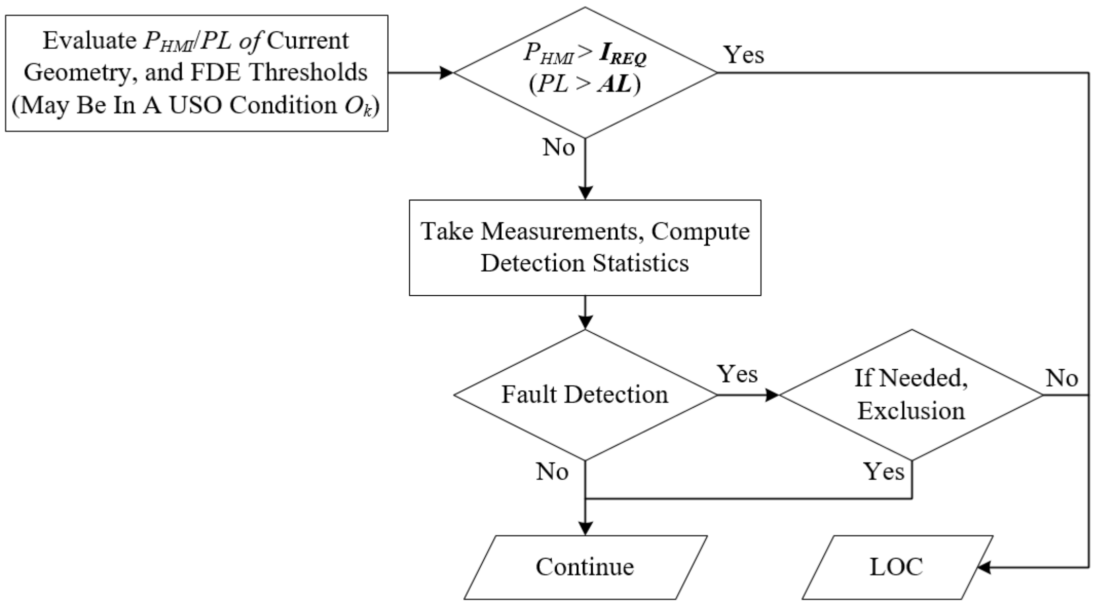

Fig. 2 Flow Diagram of Real-Time H-ARAIM FDE Process

The detection test statistics for SS ARAIM is defined as the differences between the all-in-view position solution and the subset solutions for all the monitored fault hypotheses [3]. In addition, since the USO conditions are also considered in this work, the test statistics are expressed under the outage event 'O_k':

$$\Delta_{d_k} = \hat{x}_{0_k} - \hat{x}_{d_k} = \varepsilon_{0_k} - \varepsilon_{d_k} \text{, for } d = 1 \ldots h_k. \tag{6}$$

where

- d : subscript of the number of detection test statistics under 'O_k', from $1 \ldots h_k$; h_k is the total number of monitored fault modes.
- $\hat{x}_{0_k}$: least squares position estimate using all visible satellites after 'O_k'.
- $\hat{x}_{d_k}$: least squares position estimate using post-outage satellites without the one(s) in fault mode *d*.
- ε_{0_k} : estimation error using all post-outage visible satellites, i.e., the difference between the estimated position and true position.
- ε_{d_k} : estimation error using the post-outage satellite subset without the one(s) in fault mode *d*.

In the detection step of Figure 2, all the statistics in equation (6) are evaluated and compared with their corresponding thresholds $T_{\Delta_{d_k}}$, which can be obtained by limiting the FA probability. In the scenario 'O_k', a detection event occurs (D_k) if *any* of the statistics exceed its threshold, i.e., $\bigcup_{d=1}^{h_k} \left|\Delta_{d_k}\right| > T_{\Delta_{d_k}}$. Otherwise, there will be a no detection event ($\overline{D}_k$) when *all* the statistics are smaller than $T_{\Delta_{d_k}}$, i.e., $\bigcap_{d=1}^{h_k} \left|\Delta_{d_k}\right| \le T_{\Delta_{d_k}}$.

The exclusion function is a follow-up step once an alarm is triggered, and the mechanism for determining which satellite(s) to exclude is the core of the FDE algorithm. In our design, the exclusion function is composed of two main sub-steps, where the detection statistics Δ_{d_k} are first normalized by their standard deviations $\sigma_{\Delta_{d_k}}: q_{d_k} = \frac{\Delta_{d_k}}{\sigma_{\Delta_{d_k}}}$. Then the exclusion option order is determined according to the magnitudes of the absolute values of the normalized detection statistics, and the first exclusion option corresponds to the hypothesis that results in the maximum $|q_{d_k}|$. This order will be followed when making the exclusion attempts. As a result, the exclusion function always first tries to exclude satellites in the mode of maximum $|q_{d_k}|$, and then the second maximum, and so forth. The basis of designing this order is based on the distributions of q_{d_k} under a faulted condition. If the user has encountered an actual fault, it is most likely that the test statistic corresponding to that fault mode is much larger than the others, because it accounts for the difference between faulted and FF position estimates. This design will significantly simplify the exclusion process, and also increase the probability of correct exclusion. One may argue that the statistic under a multi-fault hypothesis may not be the maximum due to the various relative fault magnitudes within the mode. However, the goal of defining this order is only to provide a clue so that the algorithm knows which satellite(s) it should try to exclude. To ensure safety, we also employ a second layer detection test after each exclusion attempt.

The second layer detection test confirms that no alarm exists in the satellite subset after exclusion. The normalized second layer detection statistics under 'O_k' are defined as:

$$q_{e_k,l_k} = \frac{\hat{x}_{e_k} - \hat{x}_{e_k,l_k}}{\sigma_{\Delta_{e_k,l_k}}} = \frac{\varepsilon_{e_k} - \varepsilon_{e_k,l_k}}{\sigma_{\Delta_{e_k,l_k}}}, \text{ for } l = 1 \ldots h_{e_k}. \qquad (7)$$

where

- e : subscript of the fault mode being excluded, $e = 1\ldots h_k$.
- l : subscript of the second layer detection test statistics, from $1\ldots h_{e_k}$; h_{e_k} is equal to the number of overall fault modes in the new post-outage satellite subset excluding e.
- $\hat{x}_{e_k}$: least squares position estimate using the post-outage satellite subset excluding e.
- $\hat{x}_{e_k,l_k}$: least squares position estimate using new satellite subset after exclusion, except the one(s) in the second layer fault mode l.
- ε_{e_k} : estimation error using the post-outage satellite subset excluding e.
- ε_{e_k,l_k} : estimation error using the new satellite subset after exclusion, except the one(s) in the second layer fault mode l.
- $\sigma_{\Delta_{e_k,l_k}}$: standard deviation of the second layer detection test statistic Δ_{e_k,l_k}.

This step of the exclusion function goes through the exclusion options following the order determined in the previous step. For each exclusion option, the second layer detection test is performed by comparing each statistic in equation (7) with its corresponding threshold T_{e_k,l_k}, which is obtained by limiting the continuity risk. According to the design, in the scenario of USO event 'O_k', two conditions will result in a satellite subset being finally excluded (E_{j_k}): (a) there is no second layer detection after excluding this subset ($\overline{D}_{j_k}$), i.e.: $\bigcap_{l=1}^{h_{e_k}} |q_{j_k,l_k}| \le T_{j_k,l_k}$; and (b) this subset corresponds to the maximum detection statistic among the subsets that pass the second layer detection test. No exclusion ($\overline{E}_k$) can be made if there are always second layer detections after testing all options: $\bigcap_{e=1}^{h_k}\left(\bigcup_{l=1}^{h_{e_k}} |q_{e_k,l_k}| > T_{e_k,l_k}\right)$.

SETTING FDE THRESHOLDS USING THE NEW METHOD

With the FDE algorithm fully described, the joint event of detection and no exclusion ($P_{D,NE}$) can be characterized by the test statistics and their thresholds, which enables us to derive the equations for computing FDE thresholds. In this work, we introduce two ways to allocate $P_{D,NE,REQ}$ among all the USO conditions, depending on whether or not we are allocating the same continuity budget for FA,NE and FD,NE over 'O_k'.

Same Budget for FA,NE and FD,NE over USO Conditions

Recall the overall H-ARAIM LOC equation (5). The first component can be further bounded by:

$$P_{D,NE} < \sum_{k=0}^{h} \left(P\left(D_k, \overline{E}_k \mid H_0, O_k\right) P_{H_0} + \sum_{i=1}^{h_k} P\left(D_k, \overline{E}_k \mid H_i, O_k\right) P_{H_i} + P_{NM,Fault} \right) P_{O_k} + P_{NM,USO} \tag{8}$$

In equation (8), the last term $P_{NM,USO}$ accounts for the cases when USO occurs on multiple satellites. It is similar to P_{rare} in equation (1): since their prior probabilities are very small, this term is regarded as a not monitored component. The LOC sources under 'O_k' are further grouped into three categories: (1) FA,NE, (2) monitored FD,NE, and (3) not monitored fault modes. In this paper, we employ the following allocations in Table 1 to account for the LOC contributions in equation (8), and the sum of the values is equal to $P_{D,NE,REQ}$.

Table 1. H-ARAIM Continuity Requirement Allocation (Same for OF and USO Conditions)

$P_{FA,NE,REQ}$	4×10^{-7}/hour	$P_{FD,NE,REQ}$	3×10^{-7}/hour
$P_{NM,Fault,REQ}$	10^{-7}/hour	$P_{NM,USO,REQ}$	10^{-7}/hour

It can be observed that the values in Table 1 do not depend on USO conditions, i.e., 'O_k' for $k = 0, 1 \ldots h$. Therefore, there is only one requirement $P_{FA,NE,REQ}$ to set the detection thresholds, and one requirement $P_{FD,NE,REQ}$ to set the exclusion thresholds. Operationally, this allocation does not require the user to know whether there is a USO or not.

The first term of equation (8) can be bounded by eliminating the knowledge of no exclusion ($\overline{E}_k$), and can be written as:

$$P_{FA,NE,O_k} < P\left(\bigcup_{d=1}^{h_k} \left|\Delta_{d_k}\right| > T_{\Delta_{d_k}} \mid H_0, O_k \right) P_{H_0} \tag{9}$$

$$< \sum_{d=1}^{h_k} P\left(\left|\Delta_{d_k}\right| > T_{\Delta_{d_k}} \mid H_0, O_k \right) P_{H_0} \le P_{FA,NE,REQ} \tag{10}$$

Therefore, the first layer H-ARAIM detection thresholds $T_{\Delta_{d_k}}$ can be computed:

$$T_{\Delta_{d_k}} = \sigma_{\Delta_{d_k}} T_{d_k} \text{ , where } T_{d_k} = Q^{-1}\left\{ \frac{P_{FA,NE,REQ}}{2P_{H_0} \cdot h_k} \right\} \tag{11}$$

Q^{-1} is the inverse tail probability function in equation (11).

To evaluate the exclusion thresholds, the second term of equation (8) is bounded by eliminating the knowledge of first layer detection (D_k), and can be expressed as:

$$P_{FD,NE,O_k} < \sum_{i=1}^{h_k} P\left(\bigcap_{e=1}^{h_k} \left(\bigcup_{l=1}^{h_{e_k}} \left|q_{e_k,l_k}\right| > T_{e_k,l_k} \right) \mid H_i, O_k \right) P_{H_i} \tag{12}$$

$$< \sum_{i=1}^{h_k} P\left(\bigcup_{l=1}^{h_{e_k}} \left| q_{i_k,l_k} \right| > T_{i_k,l_k} \mid H_i, O_k \right) P_{H_i} \tag{13}$$

$$< \sum_{i=1}^{h_k} \sum_{l=1}^{h_{e_k}} P\left(\left| q_{i_k,l_k} \right| > T_{i_k,l_k} \mid H_i, O_k \right) P_{H_i} \leq P_{FD,NE,REQ} \tag{14}$$

The bound from equation (12) to (13) is worth mentioning, where only one exclusion option associated with the fault hypothesis is considered, i.e., $e = i$. Since the fault is excluded, the second layer detection statistics q_{i_k,l_k} in equation (12) are fault free, and they follow a zero-mean normalized Gaussian distribution. Thus,

$$T_{i_k,l_k} = Q^{-1}\left\{ \frac{P_{FD,NE,REQ,H_i}}{2 \cdot h_{e_k}} \right\}, \text{ where } P_{FD,NE,REQ,H_i} = \frac{P_{FD,NE,REQ}}{h_k \cdot P_{H_i}} \tag{15}$$

In summary of this sub-section, by allocating the same continuity budget to FA,NE and FD,NE over all USO conditions, equations (11) and (15) have been derived to compute FDE thresholds. This approach provides significant benefit for real-time operation because it does not require the receiver to be aware of whether or not USO has occurred. However, this allocation is not optimal and it may result in a large predictive integrity risk. This is due to the fact that using same budget in Table 1 does not distinguish the OF and true USO states, and the resulting FDE thresholds under USO condition are as large as OF condition. Therefore, we also explore different allocations among all USO events by accounting for their prior probabilities P_{O_k}.

Different Budget for FA,NE and FD,NE over USO Conditions

If the receiver always knows whether there is a USO present at *any* time epoch, the allocated continuity budget into 'O_k' may vary. Depending on the USO conditions during operation, the receiver can use different budgets to compute FDE thresholds. Therefore, the total budget $P_{D,NE,REQ}$ can be allocated optimally among the events 'O_k' in equation (5) to minimize the associated integrity risk. The optimization process is beyond the scope of this paper, and we only employ an example case in this part to demonstrate the idea and to make comparison with the equal allocation in the last part.

Depending on whether there is a USO present or not, equation (5) is divided into (a) FA,NE and FD,NE under OF (k = 0) condition and (b) under USO ($k \neq 0$) cases:

$$P_{D,NE} = P_{FA,NE,OF} + P_{FD,NE,OF} + P_{FA,NE,USO} + P_{FD,NE,USO} \tag{16}$$

$$= P\left(D_0, \bar{E}_0 \mid H_0, O_0\right) P_{H_0} P_{O_0} + \sum_{i=1}^{H_0} P\left(D_0, \bar{E}_0 \mid H_i, O_0\right) P_{H_i} P_{O_0} + \sum_{k=1}^{H} \left(P\left(D_k, \bar{E}_k \mid H_0, O_k\right) P_{H_0} \right) P_{O_k} + \sum_{k=1}^{H} \left(\sum_{i=1}^{H_k} P\left(D_k, \bar{E}_k \mid H_i, O_k\right) P_{H_i} \right) P_{O_k} \tag{17}$$

Table 2 lists the requirements for the four components in equation (16). The equations to compute the FDE thresholds are very similar to equations (9) to (15), and they have been derived in Appendix A.

Table 2. H-ARAIM Continuity Requirement Allocation (Different under OF and USO Conditions)

$P_{FA,NE,OF,REQ}$	2×10^{-7}/hour	$P_{FD,NE,OF,REQ}$	2×10^{-7}/hour
$P_{FA,NE,USO,REQ}$	2×10^{-7}/hour	$P_{FD,NE,USO,REQ}$	3×10^{-7}/hour

Using this approach, the FDE thresholds under USO conditions can be set much tighter than under the OF conditions because the last two components in equation (17) account for the USO prior probabilities. Therefore, their corresponding contributions to the integrity risk can be reduced. In addition, in $P_{FD,NE,USO}$, the sum of the products of the fault and USO prior probabilities is generally very small. When the sum is already smaller than the requirement $P_{FD,NE,USO,REQ}$, there is no need to perform the exclusion function under USO conditions. However, this approach assumes that the receiver always knows the USO conditions during flight, and the assumption itself is worth discussing. Operationally, the user can recognize the cases in which lock on a particular satellite is suddenly lost due to USO, but it is questionable whether the user knows USO conditions at the starting point of the operation. In particular, it is more challenging to determine when the lost satellite is reinstated after suffering from a USO. These issues will be further investigated in our future work.

PREDICTIVE INTEGRITY RISK

Integrity is a measure of trust that can be placed in the correctness of the information supplied by the total system [12]. Integrity risk is defined as the probability that an undetected navigation error results in hazardous misleading information (HMI), which is the situation where the position error exceeds a predefined alert limit (AL). Since H-ARAIM only provides horizontal navigation service, only the horizontal AL needs to be considered. The predictive FDE integrity risk needs to characterize all possible situations that the aircraft may encounter. This is why all the exclusion options are accounted for in the predictive integrity risk equation [9, 10].

In this work, the overall predictive integrity risk equation associated with the new method also needs to account for the integrity threat when the user undergoes USO:

$$P_{HMI} = \sum_{k=0}^{H} P_{HMI|O_k} P_{O_k} < \sum_{k=0}^{h} P_{HMI|O_k} P_{O_k} + P_{NM,USO} \tag{18}$$

In equation (18) above, $P_{HMI|O_k}$ is the conditional integrity risk of the USO cases k = 0, 1 … h, and it consists of all the integrity risk contributions under 'O_k':

$$P_{HMI|O_k} = P(HI_k, \overline{D}_k \mid O_k) + \sum_{j=1}^{h_k} P(HI_{j_k}, E_{j_k}, D_k \mid O_k) \tag{19}$$

where

- HI_k : hazardous information exists in position estimate using post-outage all-in-view satellites for positioning: $\left|\varepsilon_{0_k}\right| > \ell$, where ℓ is AL.
- HI_{j_k} : hazardous information exists in position estimate using post-outage satellites except the one(s) being excluded: $\left|\varepsilon_{j_k}\right| > \ell$.
- E_{j_k} : satellite(s) within fault mode j is/are chosen to be excluded. There must be no second layer detection after excluding j ($\overline{D}_{j_k}$).

To evaluate equation (19), we have introduced a computationally efficient upper bound in [8, 9, 10], and the derivations will be briefly readdressed in this paper. By accounting for all the fault hypotheses under 'O_k', equation (19) becomes:

$$P_{HMI|O_k} < \sum_{i=0}^{h_k} P(HI_k, \overline{D}_k \mid H_i, O_k) P_{H_i} + \sum_{i=0}^{h_k} \sum_{j=1}^{h_k} P(HI_{j_k}, \overline{D}_{j_k}, D_k \mid H_i, O_k) P_{H_i} + P_{NM,Fault} \tag{20}$$

There is a change of notation from equation (19) to (20), where the exclusion event E_{j_k} is replaced by $\overline{D}_{j_k}$. This is a conservative step since the fact that j being finally excluded implies there is no second layer detection. Let P_{HMI,D,O_k} denote the integrity risk contributions of the detection function, i.e., the first term in equation (20). The following steps are typically used in SS ARAIM method [3]:

$$
\begin{aligned}
P_{HMI,D} &< P\left(HI_k \mid H_0, O_k\right)P_{H_0} + \sum_{i=1}^{h_k} P\left(HI_k, \overline{D}_k \mid H_i, O_k\right)P_{H_i} \\
&< P\left(\left|\varepsilon_{0_k}\right| > \ell \mid H_0, O_k\right)P_{H_0} + \sum_{i=1}^{h_k} P\left(\left|\varepsilon_{0_k}\right| > \ell, \left|\Delta_{i_k}\right| < T_{\Delta_{i_k}} \mid H_i, O_k\right)P_{H_i} \\
&< P\left(\left|\varepsilon_{0_k}\right| > \ell \mid H_0, O_k\right)P_{H_0} + \sum_{i=1}^{h_k} P\left(\left|\varepsilon_{i_k}\right| + T_{\Delta_{i_k}} > \ell \mid H_i, O_k\right)P_{H_i}
\end{aligned} \tag{21}
$$

Define P_{HMI,E,O_k} as the total integrity risk contributions from the exclusion function, i.e., the second term in equation (20). It can be bounded and evaluated by the following [8]:

$$
\begin{aligned}
P_{HMI,E,O_k} &< \sum_{j=1}^{h_k} P(HI_{j_k} \mid H_0, O_k)P_{H_0} + \sum_{i=1}^{h_k}\sum_{j=1}^{h_k} P(HI_{j_k}, \overline{D}_{j_k} \mid H_i, O_k)P_{H_i} \\
&< \sum_{j=1}^{h_k} P\left(\left|\varepsilon_{j_k}\right| > \ell \mid H_0, O_k\right)P_{H_0} + \sum_{i=1}^{h_k}\left(\sum_{\substack{j=1 \\ S_i \subset S_j}}^{h_k} P\left(\left|\varepsilon_{j_k}\right| > \ell \mid H_i, O_k\right)P_{H_i} + \sum_{\substack{j=1 \\ S_i \not\subset S_j}}^{h_k} P\left(\left|\varepsilon_{j_k,i_k}\right| + \sigma_{\Delta_{j_k,i_k}} T_{j_k,i_k} > \ell \mid H_i, O_k\right)P_{H_i}\right)
\end{aligned} \tag{22}
$$

A correct exclusion (CE) occurs when the faulted satellite subset S_i belongs to the excluded subset S_j, and the resulting position estimate is fault-free. Otherwise, excluding j under H_i will result in wrong exclusion (WE). Those two events are bounded differently in equation (22). Therefore, the overall FDE integrity risk associated with the new method can be evaluated by plugging equations (21), (22) into (20). All the FDE thresholds in those equations have been specified in the previous section, and an operation is available when P_{HMI} in equation (18) meets the integrity requirement I_{REQ}.

H-ARAIM AVALABILITY PERFORMANCE

Availability is defined as the fraction of time the navigation system is usable before the operation is initiated. This section investigates the H-ARAIM FDE availability performance by using the critical satellite approach and the new method. In addition, both ways of allocating the continuity budget using the new method are applied in the analysis. Dual-frequency baseline GPS/Galileo constellations under nominal simulation conditions [6] are used as an example for two intended operations: RNP 0.1 and 0.3. Table 3 lists some key parameters.

Table 3. Baseline H-ARAIM Simulation Parameters

$\boldsymbol{I_{REQ}}$	10^{-7} / hour	Constellation	24GPS + 24GAL
$\boldsymbol{C_{REQ}}$	10^{-6} / hour	P_{sat}	10^{-5}
HAL	RNP 0.1: 0.1nm (185m) RNP 0.3: 0.3nm (556m)	P_{const}	GPS: 10^{-8} / GAL: 10^{-4}
VAL	N/A	σ_{URA}	2.4m
Coverage Range	Worldwide	b_{nom}	0.75m

Figure 3 shows the overall H-ARAIM availability performance for RNP 0.1, in which only single-satellite USO is investigated. The result reveals that only a low coverage level can be achieved using the critical satellite approach, and the limited availability performance is dominated by the cases when $n_c \neq 0$. In contrast, the availability performance can be significantly improved using the new method.

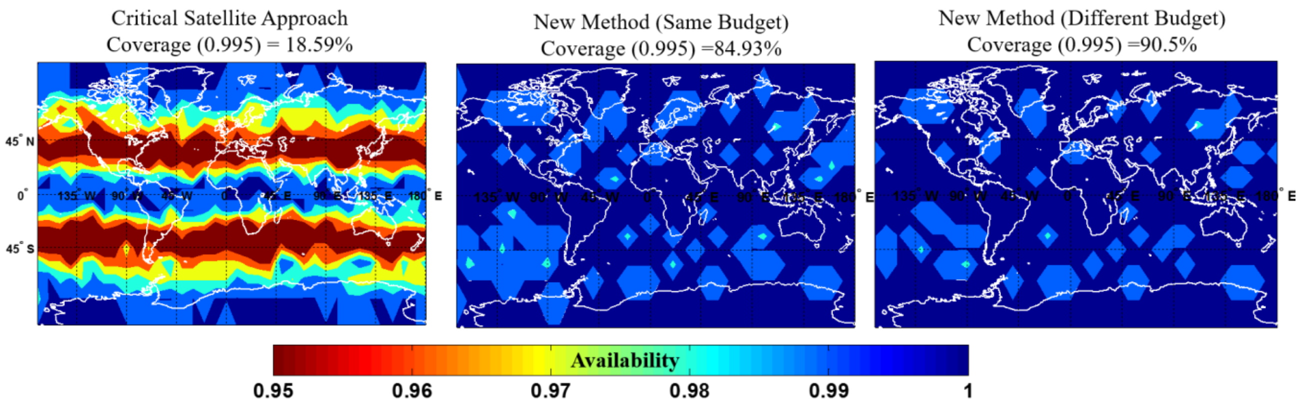

Fig. 3 Overall H-ARAIM Availability for RNP 0.1 by Only Accounting for Single SV USO

In comparison with Figure 3, the availability results in Figure 4 account for multiple-satellite USO. The availability is completely destroyed using the critical satellite approach because of the impact of critical satellite pairs, i.e., $n_p > 3$ at many snapshots. However, the same coverage level as in Figure 3 can still be maintained with the new method.

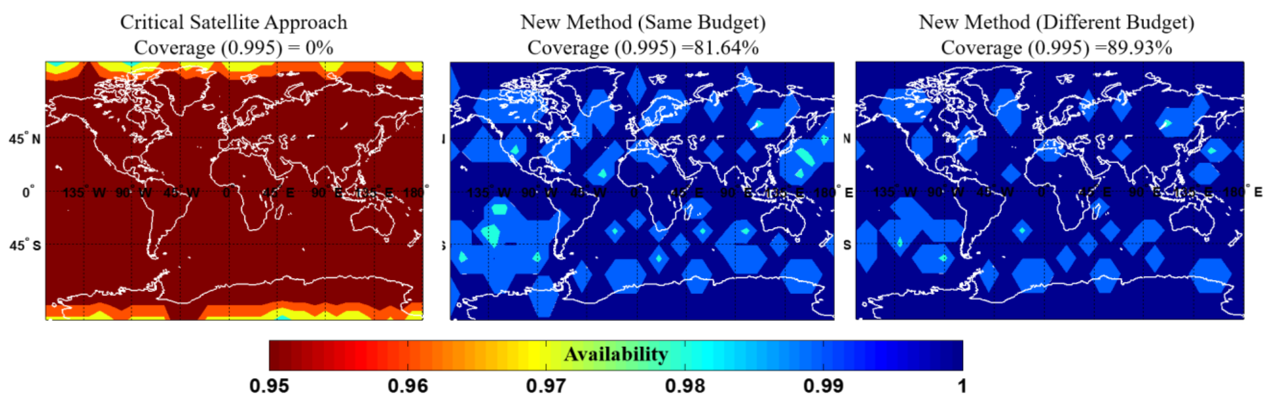

Fig. 4 Overall H-ARAIM Availability for RNP 0.1 by Accounting for *Multiple SV USO*

Table 4. Availability Coverage for RNP 0.3

	Critical Satellite Approach	New Method (Same Budget)	New Method (Different Budget)
Only Single SV USO	46.38%	95.03%	95.71%
Multiple SV USO	0.30%	94.39%	95.34%

Table 4 summarizes the availability coverage for RNP 0.3, in which the same trend has been observed using the three approaches as for RNP 0.1. As mentioned in prior sections, the main reason for the significantly different performance level is the over conservativeness of the critical satellite approach. Since the conditional integrity risk $P_{HMI|O_k}$ is used to compare with I_{REQ} and the prior probabilities of USO are eliminated, the results of the critical satellite approach reflect a "worst-case" performance. In contrast, the overall integrity risk of the new method is obtained by properly weighting $P_{HMI|O_k}$ over different scenarios, so the corresponding results are more reasonable for predicting H-ARAIM availability. Moreover, the performance evaluated with different budgets over USO conditions can be further improved by optimally allocating the continuity requirement, even though it may come at a cost in terms of computational load.

CONCLUSION

In this work, we address and compare two approaches to quantify and bound the impact of USO on H-ARAIM continuity: the critical satellite approach and a new method. In particular, there are three advantages of the newly derived approach. First, both the continuity and integrity risk equations explicitly include USO events, so the overall impact of USO can be assessed in one analysis. Second, different FDE thresholds are set for specific scenarios to limit H-ARAIM LOC, allowing for the USO continuity impact to be more precisely quantified. Third, this approach enables us to rigorously account for all the possible USO conditions including multi-satellite outages, and to determine whether an exclusion function is still needed after a USO event. In the performance analysis, availability results using new method are compared to those obtained using the previously developed critical satellite approach. The new results show that dual-constellation H-ARAIM can provide high availability for RNP 0.1 and 0.3, where both integrity and continuity requirements are met.

ACKNOWLEDGEMENT

The authors would like to thank the Federal Aviation Administration for sponsoring this work. However, the views and opinions expressed in this paper are those of the authors and do not necessarily reflect those of any other organization or person. The authors would also like to thank Mr. Ryan Cassel from the IIT navigation lab for his careful reviews of this paper.

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APPENDIX A

This appendix derives the equations to compute the FDE thresholds when different continuity budgets are allocated to USO conditions. The first term of equation (16) in the text can be bounded by:

$$P_{FA,NE,OF} < P\left(\bigcup_{d=1}^{h_0}\left|\Delta_{d_0}\right| > T_{\Delta_{d_0}} \mid H_0, O_0\right)P_{H_0}P_{O_0} \quad \text{(A.1)}$$

$$< \sum_{d=1}^{h_0} P\left(\left|\Delta_{d_0}\right| > T_{\Delta_{d_0}} \mid H_0, O_0\right)P_{H_0}P_{O_0} \leq P_{FA,NE,OF,REQ} \quad \text{(A.2)}$$

Based on the allocated $P_{FA,NE,OF,REQ}$ in Table 2, the first layer H-ARAIM detection thresholds $T_{\Delta_{d_0}}$ under OF condition can be computed:

$$T_{\Delta_{d_0}} = \sigma_{\Delta_{d_0}} T_{d_0}\text{, where } T_{d_0} = Q^{-1}\left\{\frac{P_{FA,NE,OF,REQ}}{2P_{H_0}\cdot P_{O_0}\cdot h_0}\right\} \quad \text{(A.3)}$$

The second term of equation (16) can be bounded by:

$$P_{FD,NE,OF} < \sum_{i=1}^{h_0} P\left(\bigcap_{e=1}^{h_0}\left(\bigcup_{l=1}^{h_{e_0}}\left|q_{e_0,l_0}\right| > T_{e_0,l_0}\right) \mid H_i, O_0\right)P_{H_i}P_{O_0} + P_{NM,Fault}P_{O_0} \quad \text{(A.4)}$$

$$< \sum_{i=1}^{h_0} P\left(\bigcup_{l=1}^{h_{e_0}}\left|q_{i_0,l_0}\right| > T_{i_0,l_0} \mid H_i, O_0\right)P_{H_i}P_{O_0} + P_{NM,Fault}P_{O_0} \quad \text{(A.5)}$$

$$< \sum_{i=1}^{h_0}\sum_{l=1}^{h_{e_0}} P\left(\left|q_{i_0,l_0}\right| > T_{i_0,l_0} \mid H_i, O_0\right)P_{H_i}P_{O_0} + P_{NM,Fault}P_{O_0} \leq P_{FD,NE,OF,REQ} \quad \text{(A.6)}$$

Thus, the exclusion thresholds T_{i_0,l_0} under OF condition can be evaluated:

$$T_{i_0,l_0} = Q^{-1}\left\{\frac{P_{FD,NE,OF,REQ,H_i}}{2\cdot h_{e_0}}\right\}\text{, where } P_{FD,NE,OF,REQ,H_i} = \frac{P_{FD,NE,OF,REQ} - P_{NM,Fault}P_{O_0}}{h_0\cdot P_{H_i}\cdot P_{O_0}} \quad \text{(A.7)}$$

A similar approach to OF conditions can be applied to compute FDE thresholds after USO has occurred, except the prior probabilities of USO are much smaller than P_{O_0}. Therefore, the third term of equation (16) can be expressed as:

$$P_{FA,NE,USO} < \sum_{k=1}^{h} P\left(\bigcup_{d=1}^{h_k} \left|\Delta_{d_k}\right| > T_{\Delta_{d_k}} \mid H_0, O_k\right) P_{H_0} P_{O_k} + P_{H_0} P_{NM,USO} \quad \text{(A.8)}$$

$$< \sum_{k=1}^{h} \sum_{d=1}^{h_k} P\left(\left|\Delta_{d_k}\right| > T_{\Delta_{d_k}} \mid H_0, O_k\right) P_{H_0} P_{O_k} + P_{H_0} P_{NM,USO} \leq P_{FA,NE,USO,REQ} \quad \text{(A.9)}$$

So, the detection threshold $T_{\Delta_{d_k}}$ under USO conditions can be evaluated by:

$$T_{\Delta_{d_k}} = \sigma_{\Delta_{d_k}} T_{d_k} \text{, where } T_{d_k} = Q^{-1}\left\{\frac{P_{FA,NE,USO,REQ} - P_{H_0} P_{NM,USO}}{2P_{H_0} \cdot P_{O_k} \cdot h \cdot h_k}\right\} \quad \text{(A.10)}$$

The last term of equation (16) can be expressed as:

$$P_{FD,NE,USO} < \sum_{k=1}^{h} \sum_{i=1}^{h_k} P\left(\bigcap_{e=1}^{h_k}\left(\bigcup_{l=1}^{h_{e_k}} \left|q_{e_k,l_k}\right| > T_{e_k,l_k}\right) \mid H_i, O_k\right) P_{H_i} P_{O_k} + P_{NM,Fault} P_{NM,USO} \quad \text{(A.11)}$$

$$< \sum_{k=1}^{h} \sum_{i=1}^{h_k} P\left(\bigcup_{l=1}^{h_{e_k}} \left|q_{i_k,l_k}\right| > T_{i_k,l_k} \mid H_i, O_k\right) P_{H_i} P_{O_k} + P_{NM,Fault} P_{NM,USO} \quad \text{(A.12)}$$

$$< \sum_{k=1}^{h} \sum_{i=1}^{h_k} \sum_{l=1}^{h_{e_k}} P\left(\left|q_{i_k,l_k}\right| > T_{i_k,l_k} \mid H_i, O_k\right) P_{H_i} P_{O_k} + P_{NM,Fault} P_{NM,USO} \leq P_{FD,NE,USO,REQ} \quad \text{(A.13)}$$

Therefore, the exclusion thresholds T_{i_k,l_k} under USO conditions can be evaluated:

$$T_{i_k,l_k} = Q^{-1}\left\{\frac{P_{FD,NE,USO,REQ,H_i}}{2 \cdot h_{e_k}}\right\} \text{, where } P_{FD,NE,USO,REQ,H_i} = \frac{P_{FD,NE,USO,REQ} - P_{NM,Fault} P_{NM,USO}}{h \cdot h_k \cdot P_{H_i} \cdot P_{O_k}} \quad \text{(A.14)}$$