Determination of Fault Probabilities for ARAIM

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Two critical parameters for advanced receiver autonomous integrity monitoring are the probability of satellite fault and the probability of constellation fault. This paper provides specific definitions for each of these fault types. We describe how these faults are evaluated and how to estimate their probability occurrence. Providing a precise definition of what constitutes a fault is essential so that all observers are able to agree on whether or not one has occurred.

Manuscript received February 14, 2018; revised December 27, 2018; released for publication March 17, 2019. Date of publication April 9, 2019; date of current version December 5, 2019.

DOI. No. 10.1109/TAES.2019.2909727

Refereeing of this contribution was handled by Y. Wu.

This work was supported by the Federal Aviation Administration Satellite Product Team under MOA contract number DTFAWA-15-A-80019.

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I. INTRODUCTION

The aviation community is pursuing advanced receiver autonomous integrity monitoring (ARAIM) in order to obtain global provision of horizontal and vertical guidance [1]–[3]. ARAIM is an extension of existing receiver autonomous integrity monitoring (RAIM) [4], [5], which performs a consistency check among GPS L1 C/A measurements to provide horizontal navigation for aircraft. ARAIM extends RAIM by adding four elements: multiple constellations, dual frequency, a deeper threat analysis to support vertical guidance, and the possibility to update key integrity parameters used by the aircraft. ARAIM will be developed in two phases [3]: first, a horizontal-only service (H-ARAIM) that will be used to validate the overall concept; and later, a service that will also provide vertical guidance (V-ARAIM).

The ARAIM integrity parameters are based upon constellation service provider (CSP) commitments and observational history. An updatable parameter set allows the performance to adapt to the changing global navigation satellite system (GNSS) environment. In particular, it will allow air navigation service providers (ANSPs) to include new constellations as they become available, and to improve the integrity parameters as they establish a history of good performance. It is important to observe and verify the actual constellation performance, in order to determine whether or not it is consistent with the commitment from the CSP. This evaluation requires a careful and continuous observation of the satellite signals to ensure that faults do not go undetected. Failure to observe actual faults will lead to an optimistic assessment of the true fault rate. Furthermore, it is important to recognize that the observed fault rate may be smaller than the true fault rate, due to the limited sampling size and the statistical nature of fault occurrence. We present conservative methods to estimate the true fault probabilities given the number of observed faults over a given time period.

This paper presents a rigorous approach to the understanding of the CSP commitments and weighing them against the actual observed fault rates. Our goal is to create requirements and fault rates that can be mutually agreed upon. Having a concrete set of definitions and methodologies for determining these important ARAIM parameters will help facilitate international agreement toward a globally agreed upon set of values.

The purpose of this paper is to propose certain key definitions and assertions that are foundational to the design of ARAIM architectures, algorithms, and integrity support messages. These definitions and assertions are based on a current perspective of ARAIM, with special emphasis on integrity. It is expected that they will be amended or revised as the ARAIM concept evolves over time.

This paper is an expanded version of an earlier conference publication [6], including an additional definition, revisions of the analytical fault rate derivation, an update of the GPS fault probability evaluation that now accounts for the past ten years of data, and a new assessment of GLONASS fault probabilities.

II. INTEGRITY PARAMETERS

ARAIM has four safety critical parameters that must be provided to the aircraft:

1) the probability of satellite fault, P_{sat};
2) the probability of constellation fault, P_{const};
3) an overbound of random ranging errors, σ_{URA};
4) an overbound of the ranging bias errors, b_{nom}.

It is important that these parameters conservatively describe the true satellite behavior in order for the airborne ARAIM algorithm to maintain integrity.

The next section provides precise definitions of signal-in-space (SIS) faults and their associated probabilities of occurrence. We have chosen to use a deterministic definition for the fault such that there is no ambiguity about whether or not one exists. Furthermore, this definition is consistent with the one specified by GPS [5]. The ARAIM protection level equations are based on an expected statistical distribution of the errors. Thus, one might expect to use a corresponding statistical definition of errors (i.e., fault-free errors are drawn from a Gaussian distribution). Unfortunately, such a definition is largely impractical [7]. Under a probabilistic definition, it is not always possible to know whether a fault occurred without consideration of the surrounding data. We prefer the deterministic definition because it is instantaneous and unambiguous. Fortunately, in the case of GPS, the two approaches have led to a consistent selection of faults. We have yet to identify any significant threats through statistical selection [7] that were not identified by the definitions in the next section.

Following the definitions, we have a series of assertions. These assertions are a set of hypotheses used in the analysis of system safety. Each assertion undergoes a process of evaluation and modification resulting in a statement that is asserted to be true. Assertions are frequently tested and re-evaluated to ensure they remain correct given the current state of knowledge. Sometimes, new evidence or a new analysis may either strengthen the statement that can be made or force a modification. It is important that these assertions be open and available for discussion.

Many times, analyses may rely on hidden assumptions that are not fully recognized. By uncovering these assumptions and then carefully evaluating them, we elevate those that survive to assertions. Statements in which we have confidence must replace assumptions whose veracity cannot be determined.

III. DEFINITIONS

DEFINITION 1 An SIS *fault state* is said to exist on satellite i in constellation j when the magnitude of the instantaneous SIS ranging error $e_{i,j}$ is greater than $k_{f,j} \times \sigma_{\mathrm{URA},i,j}$ at the worst user location.

Note 1—For the purpose of this definition, the values of $k_{f,j}$ and $\sigma_{\mathrm{URA},i,j}$ are to be interpreted as known quantities. These parameters will be defined in the assertions later.

Note 2—It is expected that all usable constellations will broadcast parameters that are equivalent in purpose to the GPS user range accuracy (URA).

DEFINITION 2 The probability that, at any given time and due to a common cause, any subset of two or more satellites within constellation j is in a fault state is no greater than $P_{\mathrm{const},j}$.

Note—Common cause satellite faults are also known as wide faults (WFs). One example is blundered navigation data broadcast by multiple satellites, with a common cause originating at the CSP ground segment.

DEFINITION 2A The probability that, at any given time and due to a common cause, any subset of two or more satellites within constellation j and at least two in view of user u are in a fault state is no greater than $P_{\mathrm{const},j,u}$.

Note—$P_{\mathrm{const},j,u}$ depends on how many (and possibly which) satellites the user is tracking and may vary with user location and time.

DEFINITION 3 The probability that, at any given time, satellite i in constellation j is in a fault state, excluding the multiple-satellite faults covered by Definition 2, is no greater than $P_{\mathrm{sat},i,j}$.

Note—Such faults are called independent satellite faults—also known as narrow faults (NFs)—and can be caused by erroneous satellite navigation data or anomalous satellite payload events. The probability that satellites i and k are simultaneously affected by independent fault modes is no greater than $P_{\mathrm{sat},i,j} \times P_{\mathrm{sat},k,j}$.

DEFINITION 4 The probability that satellite i in constellation j will transition from an unfaulted state, $|e_{i,j}| \leq k_{f,j} \times \sigma_{\mathrm{URA},i,j}$, to a faulted one, $|e_{i,j}| > k_{f,j} \times \sigma_{\mathrm{URA},i,j}$, within a period of time is called the satellite fault rate, R, and is typically expressed as a probability per hour.

Note—Faults have a finite duration either before they are corrected or before the user is notified that the satellite is unhealthy. In this paper, we will use the term mean time to notify (MTTN) to denote the expected average fault duration. Faults that affect a user may have initiated at the current time or at a prior time. On average, faults that initiated further in the past than the MTTN are no longer capable of affecting the user at the current epoch. Therefore, the probability that a user is currently in a faulted state, P, is related to R through $P = \mathrm{MTTN} \times R$ (see Assertions 5, 8, and 9).

IV. ASSERTIONS

ASSERTION 1 When using constellation $j = \mathrm{GPS}$ for H-ARAIM, it is acceptable to use $P_{\mathrm{const,GPS}} = 0$.

Rationale:

1) Misleading information during *en route*, terminal, or nonprecision approach navigation is designated a *major*

failure in Federal Aviation Administration (FAA) AC 20-138B [8].
2) Existing RAIM (RTCA DO-229E) [9] operates with GPS only, and has been certified and used for these aviation applications for over 20 years with $P_{\text{const,GPS}} = 0$.
3) H-ARAIM will be used for the same applications as existing RAIM.
4) H-ARAIM will use GPS satellites for the same function as they are used in existing RAIM.
5) FAA AC 23.1309-1E states that "similarity" arguments are acceptable in the analysis of major failure conditions [10] (see notes later).
6) Therefore, it is acceptable to use $P_{\text{const,GPS}} = 0$ for H-ARAIM.

Note 1—Relevant text from FAA AC 23.1309-1E (Sec. 17c, p. 29):

"c. Analysis of major failure conditions. An assessment based on engineering judgment is a qualitative assessment, as are several of the methods described below:

1) Similarity allows validation of a requirement by comparison to the requirements of similar certified systems. The similarity argument gains strength as the period of experience with the system increases. If the system is similar in its relevant attributes to those used in other airplanes and if the functions and effects of failure would be the same, then a design and installation appraisal and satisfactory service history of either the equipment being analyzed or of a similar design is usually acceptable for showing compliance. It is the applicant's responsibility to provide data that is accepted, approved, or both, and that supports any claims of similarity to a previous installation."

Note 2—The major significant difference from existing RAIM is the use of L5 ranging and accompanying intersignal correction in the Civil NAVigation (CNAV) message. The impact of these additional elements must be included in the similarity analysis.

ASSERTION 2 When using constellations other than GPS for H-ARAIM, it is not initially acceptable to assume $P_{\text{const},j} = 0$. However, as operational experience with H-ARAIM is gained over time, RAIM "similarity" arguments may eventually also support the use of $P_{\text{const},j} = 0$ for other constellations.

Rationale:

1) H-ARAIM will also use other constellations.
2) This is initially dissimilar to existing RAIM, which uses only GPS.
3) Therefore, a similarity argument following FAA AC 23.1309-1E cannot be used at the onset of service.

ASSERTION 3 For V-ARAIM, it is not acceptable to assume $P_{\text{const},j} = 0$ for any constellation, including GPS.

Rationale:

1) The existence of misleading information during precision approach navigation is designated a *hazardous* failure in FAA AC 20-138B [8].
2) FAA AC 23.1309-1E (Sec. 17d, p. 30) states that a detailed safety analysis is required for each hazardous failure [10].

ASSERTION 4 Each CSP j, for each space vehicle (SV) i in constellation j, shall make $\sigma_{\text{URA},i,j^*}$, or its equivalent, available to ANSPs and airborne users, by means of broadcast navigation data or written specification.

Note 1—During fault-free operation, the SIS ranging error is intended by CSP j to follow a normal distribution with zero mean and standard deviation of less than or equal to $\sigma_{\text{URA},i,j^*}$.

Note 2—Constellation subscript j^* is used for parameters defined by CSP j, whereas the constellation subscript j is used for parameters defined, validated, or adjusted by an ANSP.

Note 3—It is expected that all usable constellations will broadcast parameters that are equivalent in purpose to the GPS URA.

ASSERTION 5 Each CSP j will provide to ANSPs, by means of written specification or broadcast navigation data, sufficient information to compute values of state probabilities $P_{\text{sat},i,j}$ and $P_{\text{const},j}$ for faults in Definitions 1–3.

Note—There are many possible ways to convey such information. Parameters 1, 2, and 3 are the ones currently used by GPS in the standard positioning service (SPS) Performance Specification [5]. It is possible that in the future GPS may choose to specify the two parameters in list item 4 (instead of parameter 3) to individually define NF and WF rates. Parameters 1 through 4 are used as the basis for Assertions 7–9. However, other CSPs (or GPS in the future) may choose different parameter sets. For example, it is possible that in the future some CSPs could provide $P_{\text{sat},i,j}$ and $P_{\text{const},j}$ directly, instead of parameters 3 or 4. In this case, parameter 2 would still be needed to assess continuity (not yet addressed in these assertions). Parameter 1, used in Assertion 6, is applicable in all cases.

1) $k_{f,j}$—positive scalar chosen by CSP j to define the fault state via Definition 1.
2) MTTN_{j^*}—mean (or maximum) time for CSP to notify users of a fault, and either parameter 3 or 4.
3) R_{TF,i,j^*}—total fault (TF) rate for satellite i in constellation j, including both NF and WF events.

Note—R_{TF,i,j^*} may be specified to be the same for all satellites in constellation j (as it currently is for GPS: $R_{\text{TF},i,j^*} = R_{\text{TF},j^*} = 10^{-5}/\text{h/SV}$).

4) R_{NF,i,j^*} and R_{WF,i,j^*}—the NF rate for satellite i in constellation j and the rate of occurrence for the set of all WFs affecting satellite i in constellation j, respectively.

Note—R_{NF,i,j^*} and R_{WF,i,j^*} may each be specified to be the same for all satellites in constellation j.

ASSERTION 6 ANSPs will implement ground-based offline monitoring of current and archived satellite measurements to compute parameters $b_{\mathrm{nom},i,j}$ and $\alpha_{\mathrm{URA},i,j}$, such that

1) $\alpha_{\mathrm{URA},i,j} \geq 1$ and $\sigma_{\mathrm{URA},i,j} = \alpha_{\mathrm{URA},i,j} \times \sigma_{\mathrm{URA},i,j^*}$.
2) The CDF of the instantaneous SIS range error is left- and right-CDF overbounded using the distributions $N(-b_{\mathrm{nom},i,j}, \sigma^2_{\mathrm{URA},i,j})$ and $N(b_{\mathrm{nom},i,j}, \sigma^2_{\mathrm{URA},i,j})$ over the range $[\Phi(-k_{f,j} \times \sigma_{\mathrm{URA},i,j}), 1 - \Phi(-k_{f,j} \times \sigma_{\mathrm{URA},i,j})]$, where Φ is the standard normal CDF.
3) The following additional effects are accounted for in the computation of $b_{\mathrm{nom},i,j}$ and $\alpha_{\mathrm{URA},i,j}$:
   a) Repeatable or persistent biases in receiver-observed SIS errors—for example, due to signal deformations originating at the satellite. Biases common to all satellites in a constellation are excluded.
   b) Statistical uncertainty due to limited sample sizes available to the offline monitor function.
   c) The possibility that satellite SIS ranging errors may not be stationary over long periods.
   d) SIS ranging errors from different satellites will be combined linearly by aircraft with the assumption of statistical independence.

ASSERTION 7 ANSPs will implement ground-based offline monitoring to observe operational performance of the satellites and validate or, if necessary, adjust the parameters MTTN_{j^*} and R_{TF,i,j^*}, or R_{NF,i,j^*} and R_{WF,i,j^*}, specified by the CSPs in Assertion 5. The validated or adjusted parameters are denoted by MTTN_j and $R_{\mathrm{TF},i,j}$, or $R_{\mathrm{NF},i,j}$ and $R_{\mathrm{WF},i,j}$, and together with $\sigma_{\mathrm{URA},i,j}$ are subject to the following constraints:

$$R_{\mathrm{TF},i,j} \geq R_{\mathrm{TF},i,j^*} \quad \text{or}$$
$$\left\{R_{\mathrm{NF},i,j} \geq R_{\mathrm{NF},i,j^*} \quad \text{and} \quad R_{\mathrm{WF},i,j} \geq R_{\mathrm{WF},i,j^*}\right\}$$
$$2\Phi\left(-k_{f,j} \times \sigma_{\mathrm{URA},i,j}\right) = \begin{cases} R_{\mathrm{TF},i,j} \times \mathrm{MTTN}_j \\ \text{or} \\ R_{\mathrm{NF},i,j} \times \mathrm{MTTN}_j \end{cases}.$$

Note 1—The ANSP-adjusted fault rates $R_{\mathrm{TF},i,j}$, $R_{\mathrm{NF},i,j}$, and $R_{\mathrm{WF},i,j}$ should not be reduced below the CSP-provided values R_{TF,i,j^*}, R_{NF,i,j^*}, and R_{WF,i,j^*}, but may be increased by the offline monitor in case of elevated observed fault rates or statistical uncertainty due to limited sample sizes.

Note 2—The adjusted MTTN_j could potentially be reduced relative to the CSP-provided value MTTN_{j^*}, but only if the latter is a specified maximum time to notify and the former is the actual MTTN determined from long-term observation by the offline monitor.

ASSERTION 8 From Definition 3, $P_{\mathrm{sat},i,j} := \mathrm{Prob}\{\mathrm{NF}_{i,j}\}$, where $\mathrm{NF}_{i,j}$ is an NF on satellite i in constellation j. If $R_{\mathrm{NF},i,j}$ is available, then $P_{\mathrm{sat},i,j} = R_{\mathrm{NF},i,j} \times \mathrm{MTTN}_j$. If only $R_{\mathrm{TF},i,j}$ is available, $R_{\mathrm{TF},i,j} \times \mathrm{MTTN}_j \geq P_{\mathrm{sat},i,j}$ may be used as an upper bound.

Proof of upper bound:

Recall that the TF rate $R_{\mathrm{TF},i,j}$ includes *both* NF and WF events for SV i in constellation j, and consider an NF on SV i in constellation j.

$$\begin{aligned} P_{\mathrm{sat},i,j} :&= \mathrm{Prob}\left\{\mathrm{NF}_{i,j}\right\} \leq \mathrm{Prob}\left\{\mathrm{NF}_{i,j} \cup \mathrm{WF}_{i,j}\right\} \\ &= R_{\mathrm{TF},i,j} \times \mathrm{MTTN}_j \end{aligned}$$

where $\mathrm{WF}_{i,j}$ is the set of all WFs affecting satellite i in constellation j.

ASSERTION 9 If $R_{\mathrm{WF},i,j}$ is available, then the upper bound $\sum_{i=1}^{n_j} R_{\mathrm{WF},i,j} \times \mathrm{MTTN}_j \geq P_{\mathrm{const},j,u}$ may be used. If only $R_{\mathrm{TF},i,j}$ is available, then the looser upper bound $\sum_{i=1}^{n_j} R_{\mathrm{TF},i,j} \times \mathrm{MTTN}_j \geq P_{\mathrm{const},j,u}$ may be used.

Proof of upper bound:

$$\begin{aligned} P_{\mathrm{const},j,u} &\leq \sum_{i=1}^{n_j} \mathrm{Prob}\left\{\mathrm{WF}_{i,j}\right\} = \sum_{i=1}^{n_j} R_{\mathrm{WF},i,j} \times \mathrm{MTTN}_j \\ &\leq \sum_{i=1}^{n_j} \mathrm{Prob}\left\{\mathrm{NF}_{i,j} \cup \mathrm{WF}_{i,j}\right\} \\ &= \sum_{i=1}^{n_j} R_{\mathrm{TF},i,j} \times \mathrm{MTTN}_j. \end{aligned}$$

ASSERTION 9A In place of $P_{\mathrm{const},j}$, ARAIM users may apply $P_{\mathrm{const},j,u}$.

Note 1—ANSPs will not be aware of which satellites from constellation j are in view of an arbitrary ARAIM user u. Therefore, in the case where only $R_{\mathrm{TF},i,j}$ is available, the integrity support message (ISM), instead of defining $P_{\mathrm{const},j}$ directly, may (via a flag or other indicator) inform users that they may use $P_{\mathrm{const},j} := \sum_{i=1}^{n_j} P_{\mathrm{sat},i,j}$, rather than using the larger value from Definition 2.

Note 2—Alternatively, when selecting a value for $P_{\mathrm{const},j}$ in the ISM, it is sufficient for ANSPs to select a value greater than or equal to maximum value of $P_{\mathrm{const},j,u}$ over all users, rather than using the larger value from Definition 2.

Note 3—Tighter upper bounds may be found in subsequent analysis.

ASSERTION 10 The GNSS core constellations are sufficiently independent such that the only potential source of common mode error between them comes from incorrect Earth orientation prediction parameters (EOPPs).

Rationale:

1) Each GNSS core constellation provides vital strategic national functionality and each has a stated requirement for independence from the others.
2) Each constellation has been independently developed and is independently operated.
3) The only common information used by all core constellations is physical constants, coordinate reference frame definitions, and timing standards.
   a) Physical constants do not change with time.

b) Each constellation uses its own state's implementation of the International Terrestrial Reference Frame (ITRF), which is consistent to within centimeters of each other.

c) Timing offsets between the different constellations are directly estimated by the user.

ASSERTION 11 The likelihood that incorrect EOPPs lead to consistent and harmful errors on more than one constellation at a time is negligible.

Rationale:

1) Each CSP has a separate entity for computing and disseminating the EOPPs.
2) The true Earth orientation parameters (EOPs) change very slowly over time.
3) The satellite orbit estimation errors are not dependent on a constant rotation offset (see the next section for a more complete description).
4) Broadcast navigation data are not updated on all satellites in all constellations at the same time.
   a) The airborne algorithm can detect most scenarios where not all satellites are affected.
   b) After all satellites are updated, EOPP errors are undetectable at the aircraft but only affect horizontal positioning.

V. EARTH ORIENTATION PARAMETERS

This section provides more details behind the brief rationale listed under Assertion 11. The EOPs define the angular rotation between the Earth-Centered, Earth-Fixed (ECEF) ITRF and the International Celestial Reference Frame (ICRF). The EOPPs are predicted values of the EOPs used by CSPs as part of their orbit estimation algorithms. The ICRF is an inertial frame that is useful for orbital estimation. The orbital elements of the GNSS satellites are estimated in this inertial frame. Incorrect EOPPs could lead to the wrong position estimate in ITRF. In the worst case, the measurements for this incorrect position fix would all be consistent with one another and therefore not detectable by the aircraft algorithm.

The overall organization responsible for estimating and predicting EOP values is the International Earth Rotation and Reference Frame Service (IERS) [11]. Fig. 1 shows historical values from the IERS for the angular offset of Earth's axis of rotation toward 90° W (Polar X) and toward the prime meridian (Polar Y). Also shown is the excess length of day (LOD), which is the actual time taken to complete one rotation relative to the Sun, minus 86 400 s. Fig. 2 shows the changes in these parameters from one day to the next. Day-to-day LOD variations come from interactions between the solid Earth and the atmosphere, which tend to affect the rotation rate of the solid Earth more significantly than the orientation angle of the spin axis (delta X and delta Y). In the United States, the U.S. Naval Observatory coordinates with IERS to create and disseminate the EOPP values. These are downloaded by the National Geospatial-Intelligence Agency that then provides them to

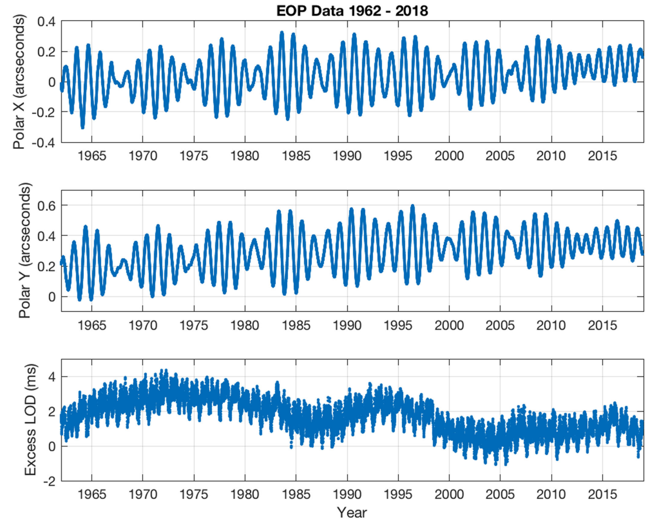

Fig. 1. Historical EOP values.

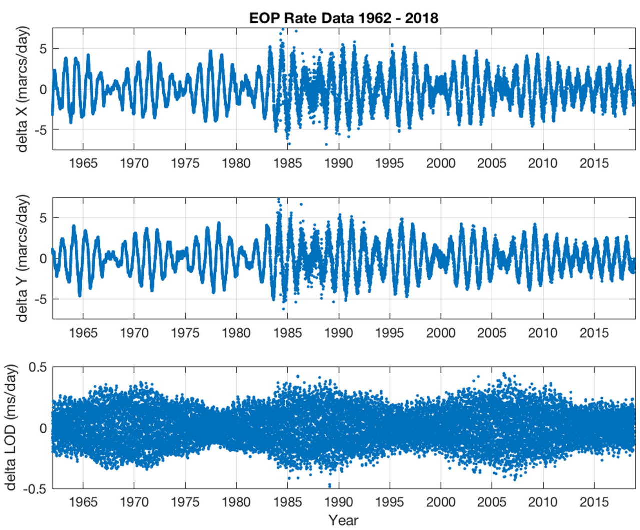

Fig. 2. Historical changes in EOP values from one day to the next.

the Air Force for use by GPS. Each organization has its own quality and consistency checking before accepting the EOPPs. The details of these checks, the time it takes to complete an update, and the frequency of update are not publicly described. Russia has its own Institute of Applied Astronomy that participates in the estimation of EOP values and that provides them to GLONASS. Europe has the Paris Observatory and other national observatories that participate in estimating EOPs and that can provide these values for Galileo. Finally, China also has its own national observatories to provide values for Beidou.

The orbit estimation process begins with pseudorange measurements made to the satellites from terrestrial reference stations. These stations are fixed to an ECEF reference frame. If the orbits were determined instantaneously, an inertial frame would not be necessary. However, because measurements over several days may be used in the estimation process, they are combined with a dynamic model that is best represented in an inertial frame. The EOPPs are used to rotate the measurements into this frame and are then again used to rotate the satellite position estimates back out to the ECEF frame. Erroneous EOPP values that are closely aligned to the true axis of rotation, but that have a constant offset about the axis of rotation, will have a negligible impact on the final position estimates, since both the rotation and its inverse are used. It would take a significant misalignment of the rotation axis or an inconsistent set of rotational values (several milliseconds change to the LOD) over the course of a couple of days for bad EOPP values to create an appreciable satellite positioning error. Such errors would be much larger than historical variations.

The EOPs are predictable to the centimeter level over days and to the meter level over months [12]. The solid Earth exchanges angular momentum with the atmosphere and the hydrosphere, which are the dominant sources of EOP variation. However, these variations are measured in milliarcseconds (mas), which corresponds to one thousandth of 1/3600 of one degree of rotation. One mas corresponds to a 3.1 cm horizontal shift at Earth's surface. Incorrect EOPPs can arise from erroneous reported values or theoretically from sudden changes to the true values. However, the true EOPs do not change very quickly. Historically, the largest observed pole motion is less than 25 cm per day and the largest observed change in the LOD is under half a millisecond (also of order 25 cm) per day (see Fig. 2). Thus, erroneous changes in EOPPs leading to a meter or larger effect are readily apparent and can be very effectively screened out by any GNSS CSP.

A consistent and harmful EOPP fault common to all constellations would require a sudden EOP change well outside all historical observation or a common mode prediction error at multiple centers. This error would have to fail to be caught at multiple EOP centers and multiple CSPs. Even in such an event, the fault would spread to satellites over an extended timescale rendering it initially observable to the airborne algorithm. The CSPs would need to continue to fail to observe the error for an extended time in order to ultimately reach the state where all satellite measurements were consistently wrong. Even in this final state, the error would be exclusively in the horizontal direction and likely small. Any EOPP error greater than 1 m should be readily observable through simple consistency checks. Thus, a multimeter error or larger would be exceedingly unlikely to escape detection for long enough to be broadcast to multiple satellites.

VI. VERIFICATION OF FAULT RATES

Definition 1 is consistent with the deterministic definition of a GPS fault provided in the GPS performance commitment [5]. It is possible to use this fault definition to determine when faults have occurred on any GPS satellite. Also provided are commitments for an upper bound on the potential satellite fault rate. Historical information on the occurrence of observed satellite faults may be used to determine a range of possible true underlying fault rates. These estimated rates may then be compared against the specified fault rate. Several assumptions are used to infer a fault rate base upon the observed faults:

1) The probability of a fault occurring within a time interval is proportional to the length of that time interval.
2) A fault occurring in one time interval does not affect the probability of it occurring in other time intervals (when the SV is set healthy).
3) The probability of a fault occurring does not change over time.

These assumptions are uncertain, which is why they are listed as assumptions rather than assertions. Without knowing the cause of a fault or what actions were taken to restore service, it is difficult to know whether or not the fault is likely to reoccur within a short time span. However, no GPS satellite has faulted twice in the last 11 years [13] let alone twice in rapid succession. On the other hand, we have seen GLONASS faults that correct themselves and then reoccur [14]. If these are treated as separate faults rather than one long continuous one, then assumption 2 is violated. If they are treated as a single fault, then the mean duration is significantly increased.

Operation of the constellations does change over time. New satellites are launched and old satellites are retired. Satellite designs and capabilities are changed, leading to new blocks of satellites. The master control segment software is updated and the staffing changes from year to year. It is impossible to claim that any satellite system is truly stationary. However, all evidence points to overall GPS performance improving with time. The accuracy has improved [13], [15], the fault rates appear to be decreasing [16], and the time to identify a fault and set the satellite unhealthy appears to be decreasing [16]. If the system truly is improving over time, assuming it is constant provides a conservative estimate of the future fault rates.

These three assumptions require further discussion; if the majority of the community accepts them as true, they may be elevated to assertions. Otherwise, they should be refined or replaced until there is consensus on a workable set of assertions. For now, we will work with these assumptions to see what we can learn from them. Sufficiently rare events that are described by the aforementioned three assumptions are expected to follow a Poisson distribution. The probability $P(k|R)$ of observing exactly k events over interval T for a given rate R is

$$P(k|R) = \frac{(RT)^k \, e^{-RT}}{k!}.$$

Instead, we need the probability density $f(R|k)$ of the rate R given k events observed over time interval T, which can be found using the Bayes rule

$$f(R|k) = \frac{P\,(k|R)\, f\,(R)}{P\,(k)}.$$

Unfortunately, we know neither $f\,(R)$ nor $P\,(k)$. A variety of options for the *a priori* probability of R can be found in the literature [17]. A preferred option is one where the result is invariant under the choice of parameterization (e.g., fault rate, R, versus mean time between faults, $1/R$). For the Poisson distribution and our choice to parameterize by R, the reference prior is one that is proportional to $1/\sqrt{R}$ [18]. Following [18], we obtain

$$f(R|k) = \frac{T(RT)^{k-1/2}\, e^{-RT}}{\Gamma\,(k+1/2)}$$

where Γ is the Gamma function. The expected value of R, given k events in time T, is then (see the Appendix for derivations and a discussion of outcomes using different prior distributions)

$$\hat{R} \equiv E\,(R|k) = \left(k + \frac{1}{2}\right) \Big/ T.$$

Before using this formula to estimate the expected fault rate, we need to make sure that it properly accounts for the distribution of possible values for R. Our goal is to properly characterize the probability of hazardously misleading information (HMI) given the uncertainty in rate. According to Definition 4 and Assertions 5, 8, and 9, the probability of fault F is equal to the fault rate multiplied by the mean time to notify the user (MTTN).

$$P\,(F \mid R) = (\text{MTTN}) \cdot R.$$

The probability of HMI given the possibility of a particular fault F is given by

$$P\,(\text{HMI},\, F|R) = P\,(\text{HMI}|F,\, R)\; P(F|R).$$

However, since the fault rate R itself has a distribution $f(R)$, the aforementioned risk can be rewritten as

$$P\,(\text{HMI},\, F) = \int_0^\infty P\,(\text{HMI}|\; F,\, R)\; P(F|R) f\,(R)\; dR\,.$$

The risk of HMI only depends on whether or not the fault is present. If the fault is already known to be present (or not), the rate at which it occurs does not impact whether or not it leads to HMI. The first term in the integral therefore does not depend on the rate and can be pulled out of the integral, leading to

$$\begin{aligned} P\,(\text{HMI},\, F) &= P\,(\text{HMI}|F) \int_0^\infty P\,(F|R)\, f\,(R)\, dR \\ &= P\,(\text{HMI}|F)\; \bar{P}_F \end{aligned}$$

where the integral is simply the expected value of $P(F|R)$

$$\bar{P}_F = E\,(P\,(F|R))$$

and therefore the expected value of the rate preserves the HMI calculation.

The conditional distribution for fault rate may be substituted into the aforementioned equation to obtain

$$\begin{aligned} \bar{P}_{F|k} &= E(\text{MTTN} \cdot R|k) = (\text{MTTN}) \cdot \widehat{R} \\ &= (\text{MTTN}) \left(k + \frac{1}{2}\right) \Big/ T. \end{aligned}$$

This result demonstrates that our estimate for the fault rate is consistent with a fault probability that incorporates uncertainty of the true fault rate given that k faults were observed over the interval T.

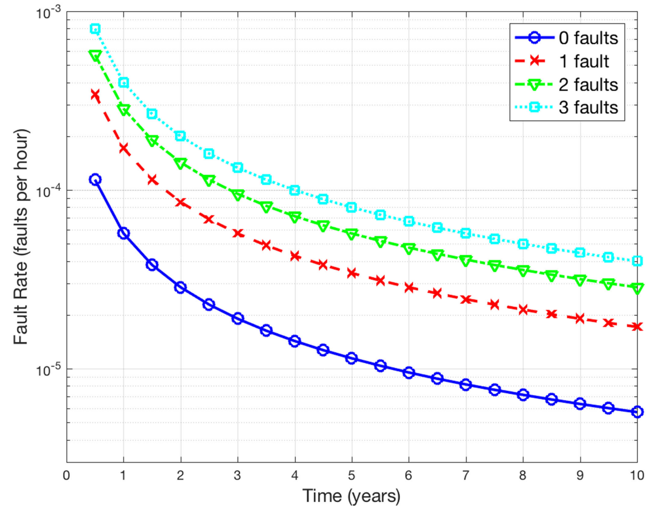

Fig. 3. Estimated wide satellite fault rate values, $\hat{R}_{\text{WF},i,j}$, for different numbers of observed constellation faults.

VII. VERIFICATION OF THE WF RATE

Let us now specifically examine the WF rate using our formula for the expected fault rate. We begin with GPS, which has not experienced any known constellation WFs that would affect dual-frequency performance [13]. Fig. 3 shows different values for the estimated WF rate, $\hat{R}_{\text{WF},i,j} = \bar{R}_{\text{WF},i,j|k}$, given a fixed number of observed faults ($k = 0$, 1, 2, or 3) over the time period T (which ranges from six months to ten years on the x-axis). For fixed values of k, the estimated rate is inversely proportional to the observation time. As can be seen, when there are no observed faults, the expected fault rate goes below 10^{-4}/h within one year of observation.

Although no faults have been observed on GPS, we do not want to reactively change our broadcast value of P_{const} should a fault be observed subsequently. Therefore, it is prudent to use a curve corresponding to at least one more fault than has actually been observed. By following this practice, one should wait to use the value of 10^{-4}/h, until at least 1.5×10^4 h (~1.7 years) have elapsed, if no constellation faults are observed, or 2.5×10^4 h (~3 years) if one constellation fault is observed, etc. Notice that it is very difficult to validate significantly smaller values of R_{WF,i,j^*}. It requires a minimum six years to verify a value of 10^{-5}/h. If, as suggested earlier, the curve corresponding to one constellation fault is used, even though none have actually been observed, it would then require more than 17 years of observation to validate that rate.

For GPS, we only count the last 11 years as having operated in a manner consistent enough to be treated as quasi-stationary [16]. Given 11 years with no observed constellation faults, we recommend following the second line from the bottom in Fig. 3 ("x"-markers designating the "one fault" case). This would indicate a verifiable rate for $\hat{R}_{\text{WF},i,j}$, below 2×10^{-5}. Note that the conservative approach makes this recommended rate three times as large as the expected rate given zero faults. The GPS performance standard [5] does not yet specifically provide a WF rate. Instead, a TF rate per satellite is given. One can infer a WF rate ranging between 10^{-4} and 10^{-5}/h given this information. For vertical operations, we recommend using a WF rate of 10^{-4}/h for GPS combined with an MTTN of 1 h [13] to yield a value for $P_{\text{const,GPS},u}$ of 10^{-4}. If future specifications provide lower committed values for R_{WF,i,j^*}, we can consider lowering the recommended value of $P_{\text{const,GPS},u}$ to take advantage of this commitment. Other constellations will require at least ~1.75 years of operation with zero observed constellation faults, before we should consider validating WF rates of 10^{-4}/h for them. It would be prudent to use an initial time period of at least twice that length.

GLONASS has recently specified a draft upper limit on the WF rate of 10^{-4}/h [19]. It has suffered from two constellation WFs since January 1, 2009 [14] [20]. Two such faults over nine years indicate an expected rate of 3.2×10^{-5}/h ("two faults" curve with triangle markers in Fig. 3), or 4.5×10^{-5}/h when, following our recommendation, we increase the fault count by one ("three faults" curve with square markers in Fig. 3). These numbers support the specification. However, the duration of each fault was approximately 10 h resulting in an estimated value for $P_{\text{const,GLONASS},u}$ of 3.2×10^{-4}. A claim is made that due to changes in operation, the cause of the first constellation fault has been eliminated and that assessment of the WF should not include the first event from 2009 [21]. It is argued that only data after that year should be included.

Thus, for GLONASS, using our recommendation of increasing the fault count by one leads to two faults over an eight-year period that corresponds to a conservative rate of 3.6×10^{-5}/h (triangle markers in Fig. 3), well below the draft specification. Unfortunately, the 2014 event lasted for 9.5 h resulting in an estimated value for $P_{\text{const,GLONASS},u}$ of 3.4×10^{-4}. The current draft specification only describes the fault rate and does not include values of the maximum or mean times to notify.

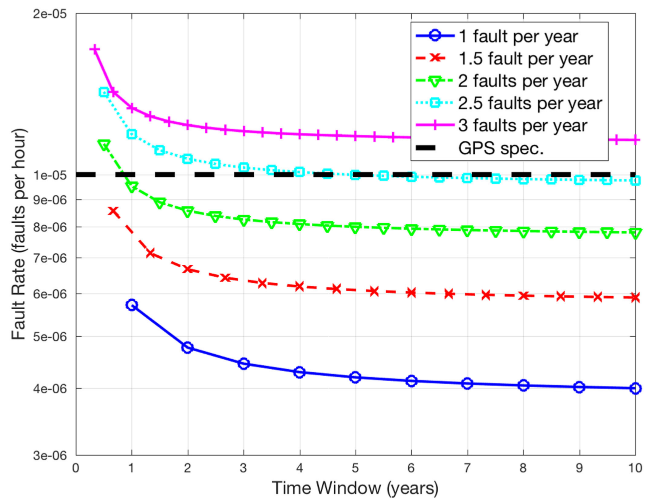

Fig. 4. Estimated narrow satellite fault rate values, $\hat{R}_{\text{NF},i,j}$, for 30 satellites in the constellation.

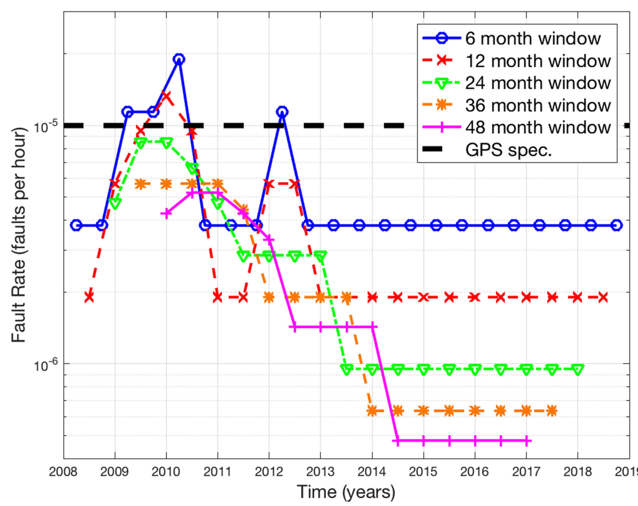

Fig. 5. Estimated narrow satellite fault rates, $\hat{R}_{\text{NF},i,j}$, for GPS with varying averaging window lengths.

VIII. VERIFICATION OF THE NF RATE

The NF is different from the WF in that multiple satellites are operating simultaneously within each constellation and the total number of faults observed is not only likely to be greater than zero, but also likely to increase with longer observational periods. Recently, GPS has had at least 30 healthy satellites on orbit at any given time. This means that for each hour that passes, 30 satellite-hours will be observed. We will assume that all satellites are equally likely to experience a fault (this assumption can be refined by grouping satellites by their blocks or any other suitable grouping). Under our assumptions, the observation period used for the rate estimate is the number of operational satellites multiplied by the actual time duration.

Fig. 4 shows a plot of the estimated NF rates, given observed fault rates of between one and three observed individual faults per year. The horizontal axis is the time window used for the calculation. As an example, for one fault per year, the time window matches the number of faults k. In one year, one fault is observed, in two years, two faults are observed, etc. For two faults per year, the number k is increased by one every six months. The estimated rates in the figure assume that there are 30 healthy satellites at any given time and that all satellites have an equal value of R_{NF,i,j^*}.

Fig. 4 shows that the GPS spec can be validated for up to 2.5 observed faults per year with a five-year evaluation time window. However, three faults per year, on average, are not consistent with the spec number. Multiple passing time windows should elapse before attempting to validate rates approaching the GPS spec of 1×10^{-5}/sat/h. It is also advisable to pad the initially observed fault count so that the estimated rate is not invalidated by subsequently occurring events.

We next examined the estimated NF rates given the observed GPS fault history using different time windows. Five faults have been observed over the last 11 years [13], [20] (no faults have been observed since 2012). Fig. 5 shows

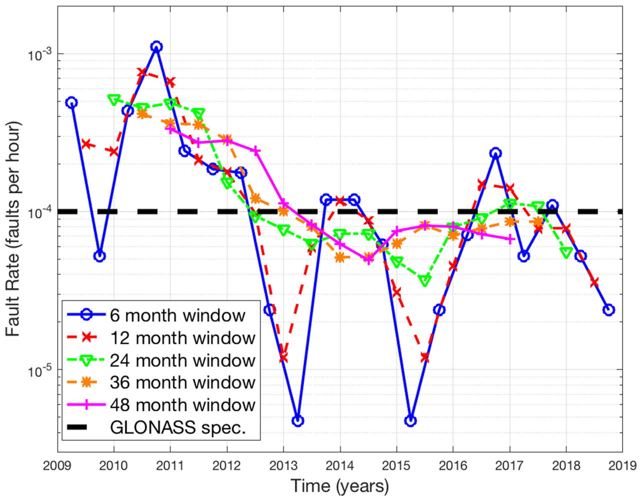

Fig. 6. Estimated narrow satellite fault rates, $\hat{R}_{\mathrm{NF},i,j}$, for GLONASS with varying averaging window lengths.

the results for window lengths from six months to four years. Given the observed set of faults, the GPS spec rate can be verified at every time step when the window length is two years or longer. We advocate using multiple time windows instead of using all of the data to create a single estimate as is done in [22]. The sliding windows allow us to evaluate possible variations in behavior over time. The shorter windows are likely more consistent with the constant rate assumption of the Poisson distribution. The data in Figs. 5 and 6 clearly show that there is variation in the behavior of both GPS and GLONASS over time.

When evaluating R_{NF,i,j^*}, we recommend using a sliding window that spans no more than a third of the total observation period. This window length is a compromise between maximizing the observation time in order to support smaller fault rates and providing multiple independent evaluations of the fault rate in order to evaluate consistency over time. The maximum rate value obtained over the different sliding window evaluations should be used. This approach adds conservatism to the calculation and may provide evidence either for or against stationarity (or continuous improvement). For example, in Fig. 5 prior to 2011, a time window of less than one year should have been used. A maximum value of $\sim 2 \times 10^{-5}$/sat/h would have then been obtained. After 2011, the one-year window would have been appropriate with a corresponding maximum value of $\sim 1.3 \times 10^{-5}$/sat/h. In 2019, with 11 years of GPS data, the 36-month window is appropriate with a corresponding maximum value of $\sim 5.7 \times 10^{-6}$/sat/h. With another year of data, we would advocate using a 48-month window. However, this would only lower the estimated rate to $\sim 5.2 \times 10^{-6}$/sat/h. As stated under Assertion 7, we do not advocate using a rate below the specification provided by GPS. Therefore, the obtained value is sufficient to validate the specified rate of $R_{\mathrm{NF},i,j^*} = 1 \times 10^{-5}/\mathrm{sat/h}$. Combined with an MTTN of 1 h, we obtain a value of $P_{\mathrm{sat},i,j} = 1 \times 10^{-5}/\mathrm{sat}$ for GPS.

The aforementioned process is only conservative if the true performance is either stationary or improving over time. The data from 2008 to 2012 do not necessarily make it obvious that GPS performance was improving. However, the data both before 2008 and after 2012 are consistent with a general trend of improvement. If the data before 2008 are discounted, then prior to ∼2014 it would have been prudent to increase the fault count used to estimate the fault rate. For example, in 2010, after observing four six-month periods with at most one fault per any given six months, it would have been better to assume two faults per six months were possible. Indeed, that higher rate was observed during the next six-month period. In the first several years of operation, it is best to be conservative and pad the observed fault count. For other constellations, rates approaching 1×10^{-5}/sat/h should only be used after many years with sufficiently few observed faults.

We also examined the estimated fault rates given the observed GLONASS fault history [14], [23]. The draft specified GLONASS upper limit on the NF rate is 10^{-4}/h [19]. Fig. 6 shows the estimated narrow satellite fault rates for GLONASS with varying averaging window lengths. On average, there were 36.6 NFs per year. The number has been going up and down over the years with a general downward trend over the time period evaluated. There are fairly sizable differences from one year to the next. 2015 was the best year with only two faults, but the subsequent years had 31, 16, and 7 faults. Such large variations in performance over time make it difficult to claim stationary behavior or continuous improvement. Furthermore, some GLONASS satellites that have faults are much more likely to fault again shortly afterward. Therefore, the fault rate methodology described in this paper is not truly applicable to the observed GLONASS behavior.

The estimated fault rates for GLONASS shown in Fig. 6 only apply if we neglect the concerns raised in the previous paragraph. As can be seen, the general trend is one of improvement; however, the years 2016 and 2017 reversed this trend. Starting in 2013, the longer term windows (24 months and greater) yield results that are largely consistent with the draft specification. Given ten years of data in the figure, we advocate using the 36-month estimates that have a maximum value of $\sim 4.1 \times 10^{-4}$/sat/h. The MTTN over the period is 1.2 h; however, there is quite a bit of variability in this number as well. Individual fault durations range from above 29 h to below 15 min and yearly MTTN varies from below 15 min to 2.5 h. Given the variability in the observed performance and the violation of the assumptions associated with use of the Poisson distribution, we do not feel confident in validating the draft specification number at this time. We are not confident that the past data are necessarily a suitable predictor of future performance.

IX. CONCLUSION

The assertions put forward in this paper provide guidelines for offline determination of the ARAIM integrity parameters. GPS data have been previously studied to determine the observational fault rates and the α_{URA} and b_{nom} parameters [13], [20]. The data indicate that the commitments described in [5] have been met. In fact, the data of last several years indicate that the commitments were very conservative relative to actual operation. However, it remains an open question how much trust to put into either the commitments or the data regarding future performance.

This question is an important one for ANSPs. Those that fundamentally trust the U.S. Air Force and its operation of GPS will likely feel very confident that the historical level of performance will continue to be met going forward. Less trusting ANSPs may feel that worse behavior is possible. ARAIM should not be pursued if one cannot believe that there is a safe set of parameters that will sufficiently describe future behavior. The FAA and many other ANSPs have trusted GPS with RAIM for many years. However, many other nations have yet to approve RAIM.

The requirements of ARAIM vertical operations are more stringent than those for operations currently supported by RAIM. We have provided a set of definitions and assertions to highlight the critical elements of a safety proof. This set is not yet complete. We have further demonstrated methods for validating the specified fault rates given actual observations. Our hope is to expand the discussion on these elements and ultimately foster a common international agreement on the relevant integrity parameters for each constellation. ARAIM is a very promising method for achieving global horizontal and vertical navigation. Much work remains to achieve its promise.

APPENDIX

Here, we examine *a priori* probabilities of R that are proportional to R^m between 0 and some $R_{\max}$ (i.e., $f(R) = (m+1)R^m/R_{\max}^{m+1}$), where m is an exponent that can take on any value. We can then find $P(k)$ from

$$P(k) = \int_0^{R_{\max}} P(k|R)\, f(R)\, dR$$

$$P(k) = \int_0^{R_{\max}} \frac{(RT)^k\, e^{-RT}}{k!} \frac{(m+1)\, R^m}{R_{\max}^{m+1}} dR$$

$$= \frac{(m+1)}{T^m R_{\max}^{m+1}} \int_0^{R_{\max}} \frac{(RT)^{k+m}\, e^{-RT}}{k!} dR$$

$$= (m+1) \frac{\gamma\,(k+m+1,\, R_{\max}T)}{k!\,(R_{\max}\, T)^{m+1}}$$

where $\gamma(k+m+1,\ R_{\max}\,T)$ is the lower incomplete gamma function (provided $k+m+1>0$). As $R_{\max}T$ approaches infinity, the lower incomplete gamma function approaches the gamma function.

$$\lim_{R_{\max}T\to\infty} \gamma\,(k+m+1,\, R_{\max}T) = \Gamma\,(k+m+1)\,.$$

The conditional fault rate probability density is then given by

$$f(R|k) = \frac{(RT)^k\, e^{-RT}}{k!} \frac{(m+1)\, R^m}{R_{\max}^{m+1}} \frac{k!(R_{\max}T)^{m+1}}{(m+1)\,\Gamma\,(k+m+1)}$$

$$= T\,\frac{(RT)^{k+m}\, e^{-RT}}{\Gamma\,(k+m+1)}.$$

The mean conditional fault rate is then given by

$$E(R|k) = \int RT\,\frac{(RT)^{k+m}\, e^{-RT}}{\Gamma\,(k+m+1)} dR$$

$$= \int \frac{(RT)^{k+m+1}\, e^{-RT}}{\Gamma\,(k+m+1)} dR = \frac{\Gamma\,(k+m+2)}{T\;\Gamma\,(k+m+1)}$$

$$= \frac{(k+m+1)}{T}.$$

For the reference prior (which also corresponds to Jeffrey's prior [17], [18]), $m=-1/2$, which leads to the value

$$E(R|k) = \left(k + \frac{1}{2}\right) \Big/ T$$

as we found earlier. Other common options for *a priori* are [17] a uniform distribution in R ($m=0$) and a uniform distribution in $\log(R)$ ($m=-1$). These choices lead to $(k+1)/T$ and k/T, respectively. Note that these options bracket our preferred choice of $m=-1/2$.

For large values of k, all of these choices lead to very similar estimates of the expected fault rate. The most significant discrepancy occurs when $k=0$. Here, the choice for *a priori* has the biggest effect. When $m<0$, the prior probability, $f(R) \propto R^m$, gives more weight to smaller rates than to larger rates. The opposite is true for $m>0$. It may seem that the choice of $m=0$ that favors neither higher nor lower rate would be preferable. However, as mentioned earlier, we could have chosen to characterize the faults not by fault rate R, but by mean time between faults: $\tau = 1/R$. Under that parameterization, a prior uniform distribution in τ corresponds to $m=-2$, whereas using the corresponding reference prior instead yields the value $m=-1/2$. Thus, we see that the reference prior does result in a consistent output distribution across different parameterizations.

ACKNOWLEDGMENT

The opinions expressed in this paper are the authors' and this paper does not represent a government position on the future development of ARAIM.

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