Continuity and Availability Evaluation in Horizontal ARAIM

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ABSTRACT

In this paper, we describe methods to account for loss of continuity (LOC) caused by receiver alerts and unusually high protection levels caused by satellite outages in Horizontal Advance Receiver Autonomous Integrity Monitoring (H-ARAIM). First, we derive receiver alert thresholds to limit the risk of LOC: the derivation starts from the definition of LOC and shows that detection and exclusion thresholds do not require separate continuity risk requirement allocations. Second, we develop a new computationally-efficient approach to account for the impact of outages on LOC, which is key for performance analyses under the assumption that H-ARAIM does not require pre-flight availability screening. Both of these contributions are implemented to predict worldwide integrity and continuity performance in the presence of satellite faults and outages.

1. INTRODUCTION

This paper aims at developing new assumptions and algorithms to enable fair comparison between dual-frequency (DF) multi-constellation (MC) Advanced Receiver Autonomous Integrity Monitoring (ARAIM) and existing single-frequency (SF) GPS-only Receiver Autonomous Integrity Monitoring (RAIM). The focus of the paper is on the continuity performance of horizontal ARAIM (H-ARAIM).

H-ARAIM is intended for safety critical aircraft navigation during en-route phases of flight [1-3]. Integrity is a measure of trust in sensor information and is therefore of primary concern for safety performance. However, continuity and availability must also be accounted for: without a false alert requirement, frequent alerts could be issued to improve integrity; such alerts would cause interruptions, thereby making the system impractical or even unsafe [4,5]. Thus, continuity and availability are essential to safety evaluation. Continuity is the probability of *unscheduled* mission interruptions. Availability is the *predicted* fraction of time where accuracy, integrity and continuity requirements are met [5].

The ARAIM Technical Subgroup of the E.U./U.S. Working Group C (WG-C) is seeking agreement between all ARAIM stakeholders, including GNSS receiver manufacturers, constellation service provides, and air navigation service providers, on a common interpretation of continuity and availability requirements. WG-C has been developing the ARAIM Continuity and Availability Assertions and Assumptions Document (C3AD) [6,7]. C3AD is a working document establishing definitions, assertions, and assumptions with two main objectives:

- to agree on how to set detection and exclusion thresholds at the aircraft receiver
- to agree on how to simulate, predict, and analyze continuity and availability performance

C3AD distinguishes *assertions* supported by strong evidence, such as historical data and precedents in existing operations using receiver autonomous integrity monitoring (RAIM) and space-based augmentation system (SBAS), from *assumptions* made with lower confidence but required for performance analyses. C3AD currently includes four definitions, four assertions, and nine assumptions, which are described in [6]. Rationales, justifications, and comments for these assertions and assumptions can be found in Appendix of [6].

DFMC ARAIM is intended for both en-route navigation *and* vertical guidance. It is therefore held to a higher level of scrutiny than SF GPS RAIM. Thus, in addition to the "*legacy assumptions*" that were used in RAIM and in early ARAIM documents [1-3], WG-C has been deriving an updated, more realistic set of "*current assumptions*". The current assumptions are required for fair comparison between RAIM and ARAIM and enable more realistic performance evaluations.

In prior work, we evaluated the impact of satellite outages on availability [6]. Outages are required, for example, while performing satellite station-keeping maneuvers. However, prior evaluations did not fully account for the fact that *ARAIM will*

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not require routine pre-flight availability predictions. This assumption implies that the predicted availability is 100%. Let P_{HMI} be the estimated integrity risk, or probability hazardously misleading information, and I_{REQ} be the integrity risk requirement. The assumption of *no pre-flight availability screening* implies that all transitions of P_{HMI} exceeding I_{REQ} are a source of loss of continuity (LOC), not of loss of availability (LOA) as in prior analyses. This motivates further analysis because the LOC risk requirement is more stringent than for LOA.

In this paper, under this new assumption, (1) we derive detection and exclusion threshold equations starting from the definition of LOC due to alerts at the user receiver; (2) we develop a computationally-efficient method to quantify the impact of satellite outages on LOC due to $P_{HMI} > I_{REQ}$ for performance analysis; (3) we implement these methods to evaluate SF GPS RAIM and DF GPS/Galileo H-ARAIM performance over a grid of worldwide locations.

The overall continuity risk, or probability of LOC, can be expressed as:

$$P(LOC) = P(LOC_{alert}) + P(LOC_{PL>AL}) + P_{other} \tag{1}$$

Section 2 of this paper shows how threshold setting and exclusion candidate selection can provide control over $P(LOC_{alert})$. In Section 3, we develop an efficient method to evaluate $P(LOC_{PL>AL})$ in the presence of outages. $P(LOC_{PL>AL})$ includes occurrences of LOC caused by $P_{HMI} > I_{REQ}$ transitions, or equivalently $PL > AL$ transitions, where PL is the protection level and AL is the alert limit. If the joint GPS/Galileo constellation is strong enough, then poor satellite geometries causing $P_{HMI} > I_{REQ}$ transitions should be rare. But, satellite outages can impact the probability of occurrence of such transitions. $P(LOC_{PL>AL})$ is also impacted by non-nominal error parameter values, such as unusually high user range accuracy (URA), but assessing $P(LOC_{PL>AL})$-sensitivity to URA is outside the scope of this paper (e.g., see [1-3] for example analyses). P_{other} is a place-holder for other sources of LOC such as RFI and ionospheric scintillation, over which ARAIM system designers have too little information for global analysis. P_{other} is outside the scope of this paper. Section 4 of this paper evaluates H-ARAIM performance using the tools developed in Sections 2 and 3. Section 5 presents concluding remarks.

2. LOSS OF CONTINUITY DUE TO ALERTS AT THE RECEIVER

This section focuses on the risk of LOC due to alerts at the receiver, $P(LOC_{alert})$. C3AD establishes conditions under which fault detection-only is sufficient to meet LOC risk requirements, e.g., for vertical ARAIM using GPS/Galileo. C3AD also explains that fault exclusion is required to meet H-ARAIM continuity risk requirements in [4,5]. Fault exclusion reduces the risk of LOC at the cost of an increased integrity risk caused by potential wrong exclusions.

Figure 1 gives an overview of events encountered using fault detection on the left-hand side (LHS), and using fault-detection-and-exclusion on the right-hand-side (RHS). These charts are inspired from Figure 1.3 in [5]. The top halves of both pie charts represent non-hazardous information where the positioning error is below the AL whereas the bottom half represents the complementary event. The angular sections are not to scale since normal operations typically occur more than 99% of the time. The diagonal quadrants are cases of no detection whereas the off-diagonal quadrants show detection events. No detection under faulted conditions cause missed alerts impacting integrity, which are rigorously accounted for in ARAIM but are not the focus of this paper.

For detection-only on the LHS, detection causes false or true alerts. Events caused by exclusion are captured by adding an intermediary ring on the RHS chart. When exclusion is implemented, normal operations resume with a subset of satellites removed if a false or true detection occurred and exclusion or correct exclusion was achieved. The outer ring's white angular section increased on the RHS as compared to the LHS. Operations may also continue if the navigation system misidentifies the faulty satellite subset and proceeds with a wrong exclusion, thus increasing the missed alert risk. Of primary interest in this paper are cases of false and true alerts occurring when detection occurred, but no post-exclusion subset could be validated, i.e., no subset could be found to be fault-free.

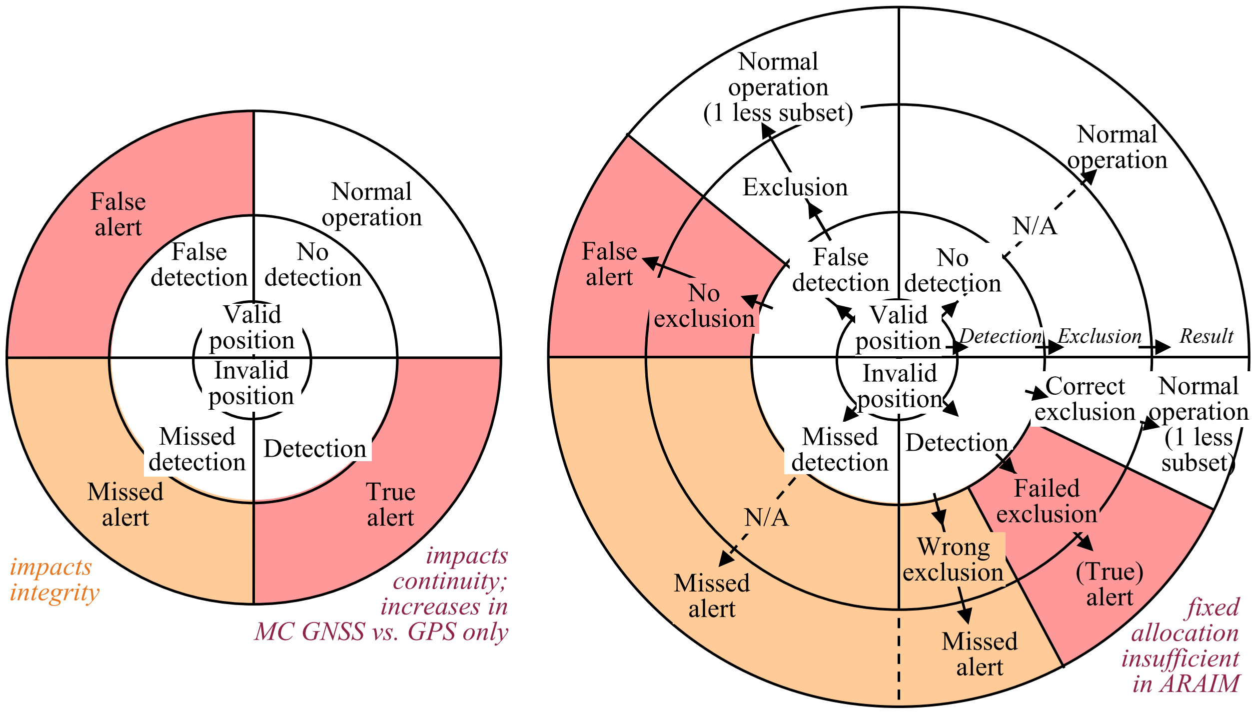

Figure 1. Overview of sources of loss of continuity (LOC) caused by alerts at the receiver [5]: (*LEFT*) when performing detection only, or FD; (*RIGHT*) when performing detection and exclusion, or FDE. Sections of the pie-charts are not to scale: normal operations (white-colored outer-ring) typically represent >99% of the total operational time. Loss of integrity is highlighted in orange, is rigorously accounted for in ARAIM, but is not the focus of this paper. LOC is highlighted in red: as compared to RAIM, ARAIM makes a more flexible and accurate account of LOC risks.

Exclusion can be considered a second layer of detection. *Does the continuity risk requirement need to be allocated between detection and exclusion tests?* Let us consider an illustrative example of solution separation (SS) detection of single space vehicle (SV) faults in an n SV geometry. For fault-detection only (FD), n SS tests would be performed to detect faults on SV1, or on SV2, …, or on SVn. For fault detection-and-exclusion (FDE), the full SV set includes n subsets of (n-1) SVs each, which constitute n post-exclusion candidates. Thus, in addition to the n first-layer detection tests, (n-1) second-layer detection tests can be carried out for each one of the n exclusion candidates: the total number of FDE tests can amount to n^2. Allocating the continuity risk requirement among n FD tests versus n^2 FDE tests could have a significant impact on threshold sizes. The following derivation will show, starting from a definition of LOC, that allocation between n tests is sufficient for FDE.

Fault Detection

For FD, the probability of LOC can be expressed as:

$$P(LOC_{alert,FD}) \leq P(D \cap H_0) + \sum_{i=1}^{2^n-1} \breve{P}(H_i) \tag{2}$$

where

D : is the detection event

n : is the number of satellites in view so that ($2^n - 1$) is the total number of potential single and multi-SV faults

H_i : for $i = 0,...,n$ are the fault-free hypothesis ($i = 0$) and fault hypotheses ($i \neq 0$)

$\breve{P}(H_i)$: is the prior probability of occurrence of hypothesis H_i , evaluated for continuity under the assumptions in C3AD.

In H-ARAIM, solution separations $\Delta_{SS,j} \equiv |\hat{x}_0 - \hat{x}_j|$ can be used as test statistics. They are defined as the difference between the full-set solution $\hat{x}_0$ and a subset solution $\hat{x}_j$, for $j = 1,...,h$, where h is the number of monitored hypotheses. The first term in the bound in equation (2) is the risk of false alert, which can be written as [3]:

$$\begin{aligned} P(D \cap H_0) &\le P(D \mid H_0) P_{H0} \\ &\le P\left(\bigcup_{j=1}^{h} \Delta_{SS,j} > T_j \middle| H_0 \right) \\ &\le \sum_{j=1}^{h} P\left(\Delta_{SS,j} > T_j \middle| H_0 \right) \end{aligned} \tag{3}$$

Under H_0, the term ($\hat{x}_0 - \hat{x}_j$) in $\Delta_{SS,j}$ is zero-mean normally distributed with standard deviation $\sigma_{SS,j}$. We can write the following equation: $P(\Delta_{SS,j} > T_j \mid H_0) = 2Q(T_j / \sigma_{SS,j})$ where $Q(\ )$ is the tail probability of the standard normal distribution. Let $C_{REQ,T}$ be the continuity risk requirement for threshold setting. $C_{REQ,T}$ can be equally allocated between the h detection thresholds. Thus, we want to achieve: $P(D, H_0) \le 2hQ(T_j / \sigma_{SS,j}) \le C_{REQ,T}$. The detection thresholds can therefore be expressed as:

$$T_j = Q^{-1}\left(\frac{C_{REQ,T}}{2h} \right) \sigma_{SS,j} \tag{4}$$

Using detection only, sources of LOC include true alerts whose risk of occurrence is bounded by the second term on the RHS in equation (2): $\sum_{i=1}^{2^n-1} \breve{P}_{Hi}$. This term increases as the number of satellites n increases, and becomes unacceptably large for H-ARAIM using multiple constellations. Exclusion is therefore needed in H-ARAIM [6].

Fault Detection and Exclusion

As illustrated on the RHS in Figure 1, when performing exclusion, the first layer of detection alone does not cause LOC. In this case, detection can be thought of as a trigger for attempting exclusion. (In a computationally-inefficient implementation, exclusion could be systematically attempted without first-layer detection.) Events of "no exclusion" and "failed exclusion" cause LOC.

The following LOC risk bound derivation can be used with different receiver algorithms. In one implementation, an exclusion candidate can be selected, e.g., by finding the largest measurement residual [3]. If the post-exclusion subset is found not to be fault-free, then an alert may be issued. Or, in a more sophisticated implementation, other exclusion candidates may be tested to find a fault-free subset until the exclusion candidates' list is exhausted, in which case an alert is triggered. Regardless of the approach, we will consider the exclusion candidate that maximizes the risk of LOC. We assume that the list of possible exclusion candidates matches the list of monitored fault hypotheses.

Let $\hat{x}_j$, for $j = 0,...,h$, be the post-exclusion satellite subset: for $j = 0$, no exclusion is attempted (no first-layer detection). Let $\Delta_{SS,j,k} \equiv |\hat{x}_j - \hat{x}_{j,k}|$ be the solution separation test statistics for subset j, for $k = 1,...,h$ and for excluded subset k not fully included in excluded subset j —we use the notation: $S_k \not\subset S_j$ — otherwise $\Delta_{SS,j,k} = 0$. The probability of LOC due to alerts of no-exclusion or failed-exclusion, noted $\overline{E}$, can be expressed as [8,9]:

$$
\begin{aligned}
P(LOC_{alert}) &\leq P(\bar{E}) \\
&\leq \sum_{i=0}^{2^n-1} P\left(\bar{E} \mid H_i\right) \breve{P}_{Hi} \\
&\leq \sum_{i=0}^{h} P\left(\bigcap_{j=0}^{h} \bigcup_{\substack{k=1 \\ S_k \not\subset S_j}}^{h} \Delta_{SS,j,k} > T_{j,k} \middle| H_i \right) \breve{P}_{Hi} + \underbrace{\sum_{i=h+1}^{2^n-1} \breve{P}_{Hi}}_{\equiv \breve{P}_{NM}}
\end{aligned}
\tag{5}
$$

where $\breve{P}_{NM}$ is the prior probability of occurrence for faults that are not monitored, computed as described in C3AD. Similar to FD in equation (2), threshold setting gives us control over the first RHS term. However, unlike FD, the h exclusion candidates in equation (5) let us control the second RHS term labeled $\breve{P}_{NM}$. This is represented in Figure 1 with the reduction in LOC risk for FDE as compared to FD. Building upon equation (5), because the probability of an intersection of events is bounded by the probability of any of the events occurring, we can match i and j to write that:

$$
P(LOC_{alert}) \leq \sum_{i=0}^{h} P\left(\bigcup_{\substack{k=1 \\ S_k \not\subset S_i}}^{h} \Delta_{SS,i,k} > T_{i,k} \middle| H_i \right) \breve{P}_{Hi} + \breve{P}_{NM}
\tag{6}
$$

Under H_i, $\hat{x}_{i,k}$ and $\hat{x}_i$ are fault free, and the distribution of $\Delta_{SS,i,k}$ is therefore known. We obtain the following bound:

$$
\begin{aligned}
P(LOC_{alert}) &\leq \sum_{i=0}^{h} P\left(\bigcup_{\substack{k=1 \\ S_k \not\subset S_i}}^{h} \Delta_{SS,i,k} > T_{i,k} \middle| H_0 \right) \breve{P}_{Hi} + \breve{P}_{NM} \\
&\leq \sum_{i=0}^{h} \sum_{\substack{k=1 \\ S_k \not\subset S_i}}^{h} P\left(\Delta_{SS,i,k} > T_{i,k} \middle| H_0\right) \breve{P}_{Hi} + \breve{P}_{NM}
\end{aligned}
\tag{7}
$$

We can then pull the sum of conditional probabilities over k out of the sum over i by taking the worst-case, *P*(*LOC*)-maximizing exclusion candidate over i. We then use the fact that $\sum_{i=0}^{h} \breve{P}_{Hi} \leq 1$, and that the maximum value of the sum over i is bounded by the sum of maximum values over i. These inequalities are expressed as:

$$
\begin{aligned}
P(LOC_{alert}) &\leq \max_{i=0,\dots,h} \left\{ \sum_{\substack{k=1 \\ S_k \not\subset S_i}}^{h} P\left(\Delta_{SS,i,k} > T_{i,k} \middle| H_0\right) \right\} \sum_{i=0}^{h} \breve{P}_{Hi} + \breve{P}_{NM} \\
&\leq \max_{i=0,\dots,h} \left\{ \sum_{\substack{k=1 \\ S_k \not\subset S_i}}^{h} P\left(\Delta_{SS,i,k} > T_{i,k} \middle| H_0\right) \right\} + \breve{P}_{NM} \\
&\leq \sum_{k=1}^{h} \max_{\substack{i=0,\dots,h \\ S_k \not\subset S_i}} P\left(\Delta_{SS,i,k} > T_{i,k} \middle| H_0\right) + \breve{P}_{NM}
\end{aligned}
\tag{8}
$$

To lighten notations, we define: $i_* \equiv \arg \max_{\substack{i=0,\dots,h \\ S_k \not\subset S_i}} P\left(\Delta_{SS,i,k} > T_{i,k} \middle| H_0\right) = \arg \max_{\substack{i=0,\dots,h \\ S_k \not\subset S_i}} 2Q\left(T_{i,k} / \sigma_{i,k}\right)$. The bound becomes:

$$P(LOC_{alert}) \leq 2\sum_{k=1}^{h} Q\left(T_{i*,k} / \sigma_{SS,i*,k}\right) + \breve{P}_{NM} \leq C_{REQ,T} \tag{9}$$

For equal continuity risk requirement allocations among thresholds, we obtain the following expression:

$$\frac{T_{i*,k}}{\sigma_{SS,i*,k}} \leq Q^{-1}\left(\frac{C_{REQ,T} - \breve{P}_{NM}}{2h}\right) \tag{10}$$

Threshold $T_{i*,k}$ can be set while taking into account its corresponding test statistic distribution with parameter $\sigma_{SS,i*,k}$. $\breve{P}_{NM}$ is subtracted from $C_{REQ,T}$ because even if faults are not monitored, they can trigger detection and prevent exclusion. The residual continuity risk requirement only needs to be allocated among h thresholds, considering the worst-case exclusion candidate.

It is worth noting that performance evaluations under "current ARAIM assumptions" account for the fact that multiple tests are performed over the aircraft navigation system's exposure period. The derivation of the number of effective samples n_{ES} and of their impact on $P(LOC_{alert})$ can be found in [10,11]. For a one-hour exposure period and a 10-second time to alert (TTA), $n_{ES} = 360$. The ARAIM detection and exclusion thresholds can be expressed as:

$$\text{under } \textit{current} \text{ H-ARAIM assumptions:} \quad \frac{T_{i*,k}}{\sigma_{SS,i*,k}} \leq Q^{-1}\left(\frac{C_{REQ,T} - \breve{P}_{NM}}{2\ h\ n_{ES}}\right) \quad \text{where } n_{ES} = 360 \tag{11}$$

3. LOSS OF CONTINUITY DUE TO OUTAGES

This section addresses a second source of LOC in equation (1) by developing a computationally-efficient method to evaluate $P(LOC_{PL>AL})$. $P(LOC_{PL>AL})$ is the probability of LOC caused by $P_{HMI} > I_{REQ}$ transitions. We focus on the impact of SV outages because the impact of nominal geometries and of error parameter values has already been analyzed in prior performance evaluations [1-3]. Accounting for outages is required for offline performance analysis. However, no extra calculation is required at the receiver, which uses all visible, healthy SVs regardless of whether or not an outage occurred.

In prior work [6], we used the method outlined in [12] to implement the constellation state probability model specified in [13]. This method evaluates the worst-case, P_{HMI} -maximizing outage at each location-and-time and accounts for the probability of this outage occurrence. The method is computationally expensive: it took two weeks to generate a global availability map for GPS/Galileo ARAIM. In response, in [6], we derived a bound on the risk of loss of availability (LOA) that accounted for outages. Terms were eliminated in the derivation because they were negligibly small as compared to an assumed availability requirement.

In contrast, in this paper, outages will impact LOC. Because the LOC risk requirement is more stringent than that of LOA, neglecting terms is no longer acceptable. We must identify these terms and analyze them.

This paper makes the assumptions and assertions listed in Table 1. Table 1 shows a subset of the assumptions that can be found in C3AD [7], and that are previewed in [6]. Rationales and justifications for these assumptions are also found in [6,7]. In particular, Assumption 3 states that, under "current ARAIM assumptions" the aircraft receiver may not be required to perform pre-flight H-ARAIM availability prediction. Thus, transitions of $P_{HMI} > I_{REQ}$ that caused LOA in [6] are now considered a source of LOC. Table 1 also shows Assertion 4 and Assumption 2, which specify agreed-upon values for the average rate of SV outage onsets for GPS (based on GPS commitments in [12] and on historical data in [14,15]). Assumption 6 specifies a tentative continuity risk requirement for use in H-ARAIM performance analyses. Table 1 finally includes Assumption 9, which lists the elements that should be accounted for in ARAIM continuity and availability performance evaluations. These elements include occurrences of satellite outages.

Table 1 H-ARAIM Continuity and Availability Assertions and Assumptions Relevant to this Paper.

Assertion 4	For continuity evaluation, the average rate of scheduled and unscheduled GPS satellite outages, R_{out}, is no greater than $2 \cdot 10^{-4}$/ h / SV.
Assumption 2	For continuity evaluation, and for GNSS constellations other than GPS, the average rate of effective scheduled and unscheduled GPS satellite outages, R_{out}, is expected to range from $1 \cdot 10^{-4}$ / h / SV to $2 \cdot 10^{-3}$ / h / SV. NOTE of Assumption 2 adds that: "*Preliminary ARAIM continuity performance evaluations assume a nominal rate of $2 \cdot 10^{-4}$ / h / SV for all constellations* […]"
Assumption 3	Under nominal ARAIM performance assumptions, the aircraft receiver may not be required to perform H-ARAIM availability prediction.
Assumption 6	H-ARAIM airborne algorithms should consider a continuity risk requirement ranging from 10^{-4} to 10^{-8} per hour. NOTE 1 of Assumption 6 in Appendix adds that: "*Preliminary H-ARAIM performance analyses and receiver algorithm design may assume a tentative value of 10^{-5} per hour for the continuity risk requirement*. […]".
Assumption 9	For ARAIM continuity performance simulation, prediction, and analysis, the following sources of LOC are considered: • false and true alerts in V-ARAIM when no exclusion function is implemented, • false alerts with no exclusion and true alerts with failed exclusion in H-ARAIM and V-ARAIM when exclusion is used, • occurrences of PPL > AL, where PPL is the predictive PL accounting for the impact of scheduled and unscheduled single-satellite outages

For performance analysis under Assumption 3, we account for the risk of LOC caused by transitions of $P_{HMI} > I_{REQ}$ in the presence of SV outages, which is expressed as:

$$P(LOC_{PL>AL}) \equiv P(\ \hat{P}_{HMI} > I_{REQ}\ ) \tag{12}$$

The objective of the following derivation is to find a bound on $P(LOC_{PL>AL})$ that can be evaluated in a computationally efficient manner, i.e., without having to quantify P_{HMI} under multiple simultaneous outage cases.

Using the law of total probabilities:

- for the mutually exclusive, exhaustive hypotheses of no outage O_0, a single outage O_1 on any one satellite, and two-or-more simultaneous outages $O_{\geq 2}$,
- for the mutually exclusive, exhaustive hypotheses of first-layer detection D, in which case exclusion is attempted E_j for $j = 1,...,h$, and no first-layer detection $\bar{D}$ E_0,

using conditional probability expressions, and using the inequalities $P(\bar{D} \cap O_j) \leq P(O_j)$ and $P(A \cap O_0) \leq 1$, we can derive the following bound:

$$\begin{aligned}
P(LOC_{PL>AL}) &= \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ} \cap O_j) + P(\ \hat{P}_{HMI} > I_{REQ}\ \ | O_{\geq 2})P(O_{\geq 2}) \\
&\leq \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ} \cap O_j \cap \bar{D}) + \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ} \cap O_j \cap D\ ) + P(O_{\geq 2}) \\
&\leq \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ} \,|\, O_j \cap \bar{D})P(\bar{D} \cap O_j) + \sum_{j=0}^{1} P(\ \hat{P}_{HMI} > I_{REQ} \cap O_j \cap D\ ) + P(O_{\geq 2}) \\
&\leq \sum_{j=0}^{1} \left[P(\ \hat{P}_{HMI|\bar{D}\cap Oj} > I_{REQ}\ )P(O_j) \right] + P(\ \hat{P}_{HMI|D\cap O0} > I_{REQ}\ ) \\
&\qquad + P(\ \hat{P}_{HMI|D\cap O1} > I_{REQ}\ )P(D \cap O_1) + P(O_{\geq 2})
\end{aligned} \tag{13}$$

where, to shorten notations, conditional events are included in subscripts because they have a deterministic impact on the computed bound and requirement , e.g., $P(\ \hat{P}_{HMI|\bar{D}\cap Oj} > I_{REQ,\bar{D}}\ ) \equiv P(\ \hat{P}_{HMI} > I_{REQ} \mid O_j \cap \bar{D})$. The ARAIM algorithm allocated the integrity risk requirement to cases of no detection $\bar{D}$ and exclusion D . The bound is a sum of four terms.

- Using the bound $\sum_{j=0}^{1} P(O_j) \le 1$, the first term on the RHS in equation (13) the following inequalities can be written:

$$\begin{aligned} \sum_{j=0}^{1}\Big[P(\ \hat{P}_{HMI|\bar{D}\cap Oj} > I_{REQ}\ )P(O_j)\Big] &\le \max_{j=0,1}\{P(\ \hat{P}_{HMI|\bar{D}\cap Oj} > I_{REQ}\ )\}\sum_{j=0}^{1}P(O_j) \\ &\le P(\ \max_{j=0,1}\{\hat{P}_{HMI|\bar{D}\cap Oj}\} > I_{REQ,\bar{D}}\ ) \\ &\le P(\ \max_{O1}\{\hat{P}_{HMI|\bar{D}\cap O1}\} > I_{REQ,\bar{D}}\ ) \end{aligned} \quad (14)$$

  where the notation in the last inequality captures the fact that the worst-case, $\hat{P}_{HMI}$ -maximizing single-SV outage O_1 must be considered (out of the *n* possible single-SV outages, where *n* is the number of SVs in view).

- Considering the mutually exclusive, exhaustive hypotheses of no fault H_0 and any single or multi-SV fault $H_{\ge 1}$, using the bound $P(D \cap H_{\ge 1}) \le P(H_{\ge 1})$, assuming that outages, faults, and receiver false alerts are independent events, the third term in (13) becomes:

$$\begin{aligned} P(\ \hat{P}_{HMI|D\cap O1} > I_{REQ,D}\ )P(D\cap O_1) &\le P(\ \hat{P}_{HMI|H_0\cap D\cap O1} > I_{REQ,D}\ )P(D\cap H_0)P(O_1) \\ &\quad + P(\ \hat{P}_{HMI|H_{\ge 1}\cap D\cap O1} > I_{REQ,D}\ )P(H_{\ge 1})P(O_1) \\ &\le P(D\cap H_0)P(O_1) + P(H_{\ge 1})P(O_1) \\ &\le C_{REQ,T}P(O_1) + P(O_1)(1-P_{H0}) \end{aligned} \quad (15)$$

  Thus, the third term's bound is the sum of a product of relatively small numbers as compared to the overall continuity risk requirement.

- The fourth term, $P(O_{\ge 2})$, is small, but will require additional evaluation. For example, for GPS, C3AD Assertion 4 demonstrates that the average rate of outage onset is no greater than $2{\cdot}10^{-4}$/ h / SV [6,7,13,14]. Thus, if single-SV outages are independent events, the probability of simultaneous dual-SV outages is small. In addition, the GPS and Galileo constellation service providers' ground segments may act, at least within their constellation, on limiting occurrences of dual-SV outages. However, the observed average duration of GPS SV outage is 37 hours. WG-C is working on how outage duration should be factored in, given the limited exposure period of individual receivers to such outages, and the fact that once LOC is experienced (e.g., due to a $P_{HMI} > I_{REQ}$ transition), the aircraft may not expect the ARAIM service to be immediately available. In this paper, we assume that the relevant LOC duration caused by $O_{\ge 2}$ can be expressed as:

$$P(O_{\ge 2}) \le 1-(1-p_{out})^n - n\ p_{out}\ (1-p_{out})^{n-1} \quad \text{where} \quad p_{out} = 2\cdot 10^{-4}$$

Substituting equations (14) and (15) into (13), we obtain the following inequality:

$$P(LOC_{PL>AL}) \le P(\ \max_{O1}\{\hat{P}_{HMI|\bar{D}\cap O1}\} > I_{REQ,\bar{D}}\ ) + P(\ \hat{P}_{HMI|D\cap O0} > I_{REQ,D}\ ) + \Big[C_{REQ,T}P(O_1) + P(O_1)(1-P_{H0}) + P(O_{\ge 2})\Big] \quad (16)$$

For nominal ARAIM parameters, the term in the square brackets is smaller than the overall continuity risk requirement C_{REQ} , which is assumed to be 10^{-5} according to Assumption 6 in C3AD and in Table 1. The dominating term is $P(O_{\ge 2})$, which is discussed above, and will be further analyzed. Although the LOC bound does not need to be implemented this way, in order to reduce the number of variables carried along, we can set aside a continuity risk requirement allocation expressed as:

$$C_{ALLOC} \equiv \left[C_{REQ,T} P(O_1) + P(O_1)(1 - P_{H0}) + P(O_{\geq 2}) \right] \tag{17}$$

where, for $n = 20$ and for nominal ARAIM assumptions, $C_{ALLOC} \approx 8 \cdot 10^{-6}$.

We can simplify the notation in equation (16) by introducing refined notations:

- The event of no first-layer detection $\bar{D}$ is equivalent to validating the exclusion of no satellite, which is noted E_0.
- The event of first-layer detection D triggers exclusion attempts noted E_j, for $j = 1,...,h$. We can assume that the exclusion candidates match the monitored fault hypotheses. We assume that exclusion candidates include all single-SV faults.

Equation (16) becomes:

$$P(LOC_{PL>AL}) \leq P(\max_{O1}\{\hat{P}_{HMI|E_0 \cap O1}\} > I_{REQ,E_0}) + P(\max_{j=1,...,h}\{\hat{P}_{HMI|E_j \cap O0} > I_{REQ,E_j}\}) + C_{ALLOC} \tag{16}$$

Assuming, as is the case in the nominal ARAIM algorithm [16], that that the integrity risk requirement is equally allocated among exclusion cases: $I_{REQ,E} = I_{REQ,E_j}$, for all $j = 0,...,h$, the following inequality can be written:

$$P(\max_{O1}\{\hat{P}_{HMI|E_0 \cap O1}\} > I_{REQ,E}) \leq P(\max_{j=1,...,h}\{\hat{P}_{HMI|E_j \cap O0}\} > I_{REQ,E}) \tag{17}$$

Equation (17) captures the fact that the risk bound is identical when a single satellite is unused, whether it is due to an outage or to an exclusion at the receiver. The inequality captures the fact that the set of single-SV outages is included in the set of exclusion candidates. Using the bound (17), we can rewrite (16) as:

$$P(LOC_{PL>AL}) \leq 2P(\max_{j=1,...,h}\{\hat{P}_{HMI|E_j \cap O0}\} > I_{REQ,E}) + C_{ALLOC} \tag{18}$$

It is worth noticing that the derivation started with the objective of accounting for SV outages in LOC risk evaluation. We obtained an expression of a "predictive P_{HMI}" $P\hat{P}_{HMI}$, which can be expressed as:

$$P\hat{P}_{HMI} \equiv \max_{j=1,...,h}\{\hat{P}_{HMI|E_j \cap O0}\} \tag{19}$$

$P\hat{P}_{HMI}$ is to be compared to $I_{REQ,E}$ for offline performance analysis. While different exclusion candidates must be considered, no outage needs be simulated, which is computationally much more efficient than prior methods [12]. Also, there is no need for the receiver to evaluate $P\hat{P}_{HMI}$: the receiver only needs to derive $\hat{P}_{HMI}$ using the satellites it chooses to use, whether or not it experiences an outage O_j or an exclusion E_j.

Thus, for an overall continuity risk requirement C_{REQ}, if we wanted to allocate a portion $C_{REQ,T}$ of the requirement to $P(LOC_{alert})$ considering $P(LOC_{PL>AL})$ in equation (18), we could defined $C_{REQ,T}$ as:

$$C_{REQ,T} = \frac{C_{REQ} - C_{ALLOC} - P_{other}}{2} \tag{20}$$

We would then want to show, considering nominal ARAIM error models, that the GPS/Galileo satellite geometry is strong enough to support high availability of $P\hat{P}_{HMI} < I_{REQ,E}$. We perform such an analysis in the next section.

4. CONTINUITY PERFORMANCE EVALUATION

Figure 2 shows worldwide availability maps evaluated assuming the presence of satellite outages using single-frequency GPS RAIM (in the left column) versus dual-frequency GPS/Galileo ARAIM (right column). The maps are derived for a required navigation performance: RNP 0.3. A list of key simulation parameters, requirements, and their values is given in Table 2. Availability is color-coded from red to blue, representing an availability range of ≤90% to 100%.

The top row shows availability evaluated under legacy assumptions versus the bottom row under more realistic current assumptions. As compared to legacy assumptions used in GPS RAIM and in past ARAIM analyses in [1-3], the current assumptions enforce the following steps:

- allocating integrity risk requirement among exclusion candidates [16]
- bounding integrity and continuity risks while accounting for the number of effective time-to-alert intervals over a one-hour exposure period [10,11].

The availability performance of DFMC H-ARAIM is expectedly greater than that of SF GPS RAIM. This is more apparent under the current ARAIM assumptions, which make a more accurate account of sources of loss of integrity (LOI), LOC, and LOA.

These results are highlighted again in Table 3, which shows coverage of 100% availability (and of 99% availability in parentheses), in the cases where outages are accounted for and when they are not. Coverage values show that 100% availability is not achieved at all locations using H-ARAIM, which will require further investigation by the ARAIM Technical Subgroup of Working Group C.

Table 2 Dual-frequency GPS/Galileo H-ARAIM availability and continuity analysis parameters

Parameter Description	**Parameter Value**
Constellations	nominal 24 GPS and 24 Galileo satellite constellations [3]
Overall performance criterion	coverage of 99.9% and 100% availability
RNP 0.3 Horizontal Alert Limit (HAL)	$HAL = 556$ m
Temporal resolution	24 hours with 600 second steps
Spatial resolution	10 by 10 degree user grid
Integrity risk requirement	$I_{REQ} = 10^{-7}$
Continuity risk requirement	$C_{REQ} = 10^{-5}$
User Range Accuracy (URA)	$URA_{GPS} = URA_{GAL} = 2.4$m
Prior probability of satellite fault for integrity	$P_{sat,GPS} = P_{sat,GAL} = 10^{-5}$
Prior probability of constellation fault	$P_{const,GPS} = 10^{-8}$, $P_{const,GAL} = 10^{-4}$
Rate of satellite outage onset	$R_{out,GPS} = 2 \cdot 10^{-4}$/h, $R_{out,GAL} = 2 \cdot 10^{-4}$/h
Nominal bias values (for integrity)	$b_{nom} = 0.75$ m
Time to alert (*TTA*), exposure time (T_E), mean failure duration (*MFD*)	$TTA = 10$ s, $T_E = 1$ h, $MFD = 1$ h

Table 3 Sensitivity of coverage of 100% availability of integrity and continuity, and in parentheses, coverage of 99% availability of integrity and continuity

	Assumption	**SF GPS RAIM**	**DF GPS/Galileo H-ARAIM**
No outage	legacy	93.26 % (98.57%)	100 % (100%)
	current	0.32% (0.90%)	62.34% (90.89%)
Accounting for outages	legacy	2.08 % (5.99%)	99.92 % (100%)
	current	0.04% (0.65%)	61.98% (90.71%)

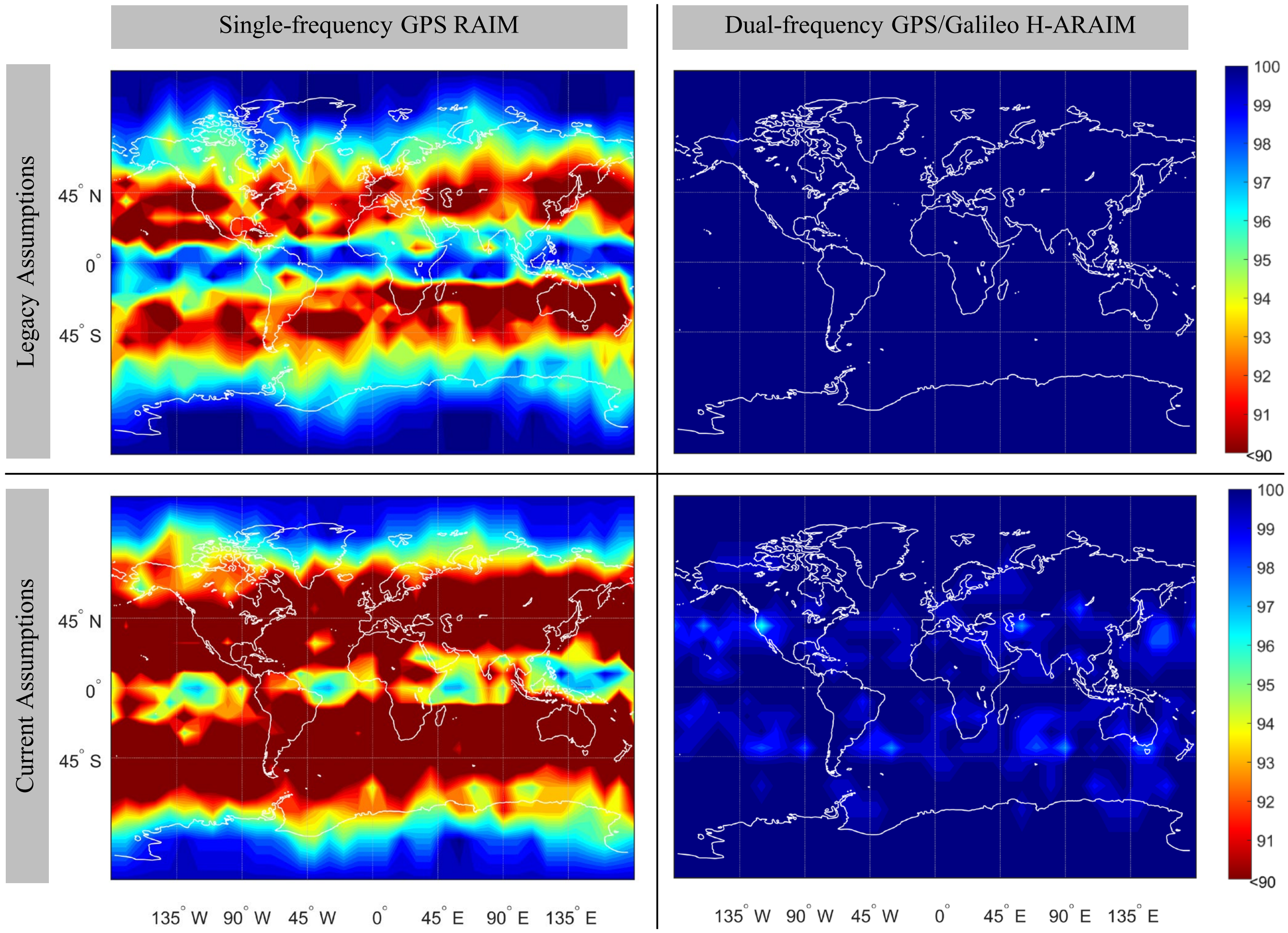

Figure 2. Worldwide maps of availability accounting for satellite outages for single-frequency GPS RAIM versus dual-frequency GPS/Galileo ARAIM, for legacy assumptions versus more realistic current assumptions. Current assumptions include an integrity risk requirement allocation for exclusion candidates [3] and an evaluation of integrity and continuity risks over time by accounting for the number of effective samples over a one-hour exposure period [10].

5. CONCLUSION

The ARAIM continuity assertions and assumptions set the foundations for threshold setting at the receiver, and for availability predictions used in ARAIM algorithm design and performance analysis. These foundations are required to quantify the performance improvement brought by ARAIM over RAIM, while making more realistic and more rigorous assumptions than in prior RAIM and early ARAIM evaluations. In this paper, we derived a bound on the loss of continuity due to alerts at the receiver, which we used to set detection and exclusion thresholds. We then analyzed the impact of not requiring routine pre-flight availability predictions in ARAIM: we developed a new computationally-efficient method to account for outages in continuity evaluation. Future work includes further performance evaluation, and validation of the assumptions on the mean satellite outage duration.

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