Physics-Informed, Data-Driven Approach For Reducing Model Discrepancy

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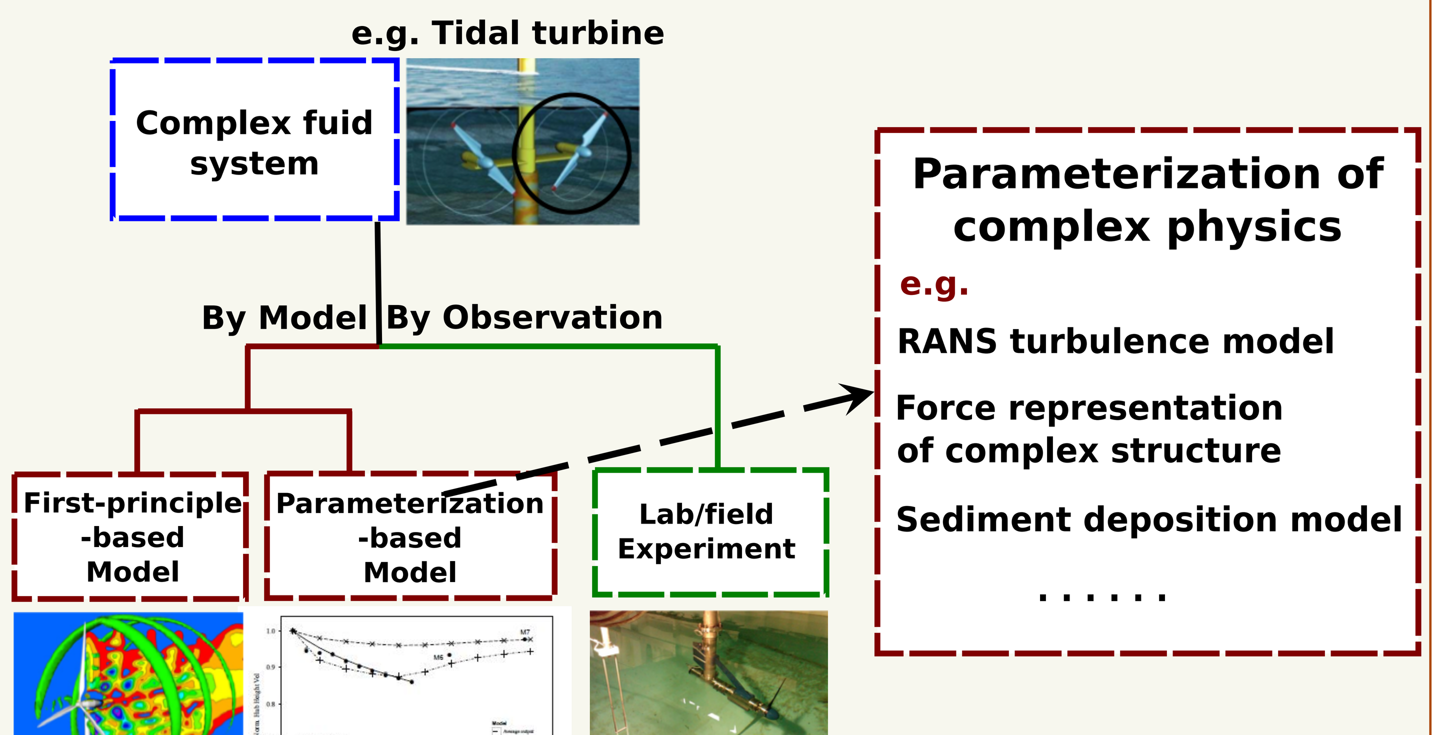
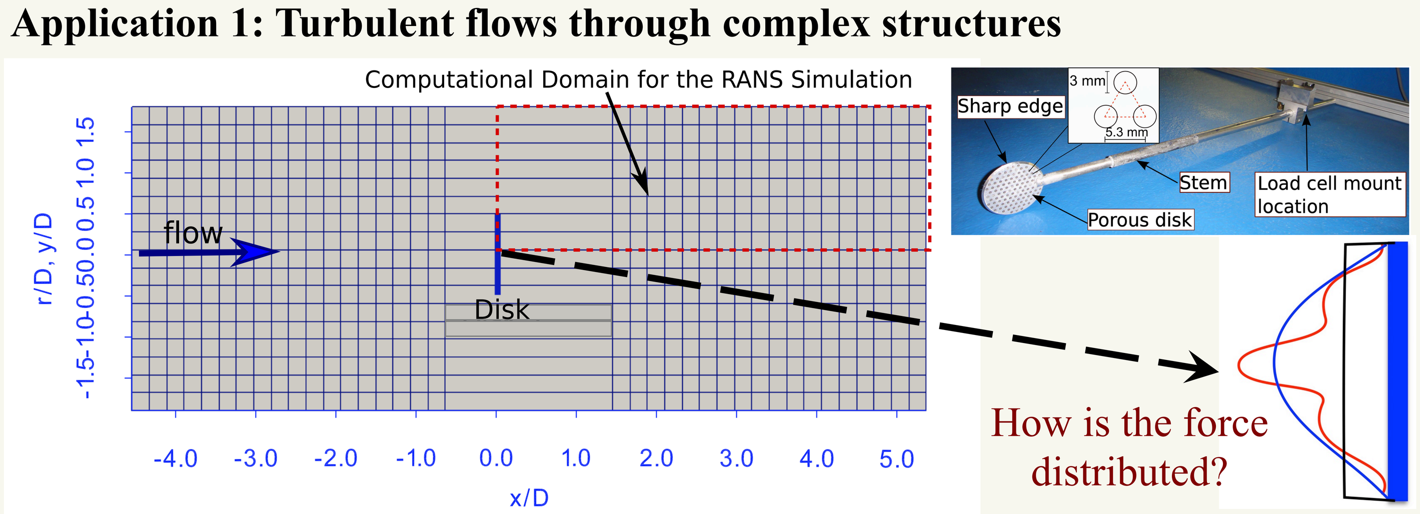
Motivation



How to use sparse noised data to improve the prediction of modeling of complex physical system?

Problem Formulation

Application 1: Turbulent flows through complex structures



$$\frac{\partial U_i}{\partial t}+\frac{\partial U_i U_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 U_i}{\partial x_i \partial x_j}-\frac{\partial \tau_{ij}}{\partial x_j}+\boxed{f_i}$$

Actuation disk model

Application 2: Model-form uncertainties reduction in RANS simulations

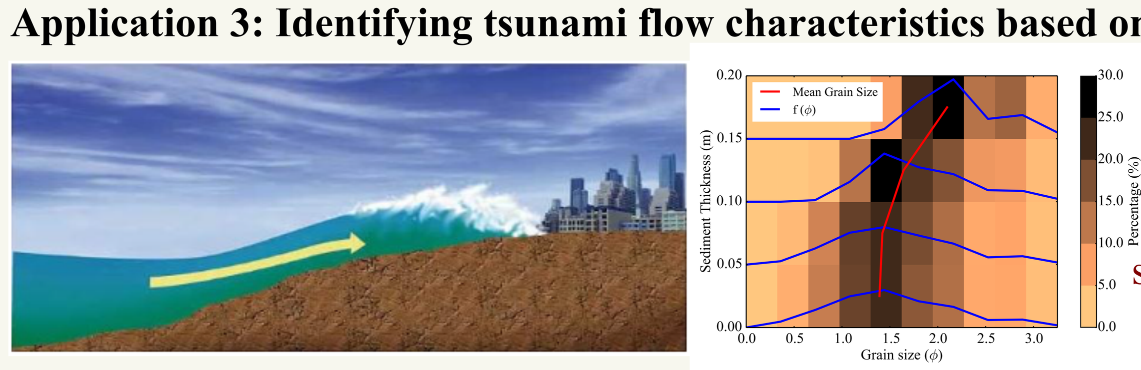
$$\boxed{\frac{\partial U_i}{\partial t}+\frac{\partial (U_i U_j)}{\partial x_j}+\frac{1}{\rho}\frac{\partial p}{\partial x_i}-\nu\frac{\partial^2 U_i}{\partial x_j \partial x_j}}=\boxed{\nabla\cdot\tau}$$

Divergence of Reynolds stress

Turbulence models

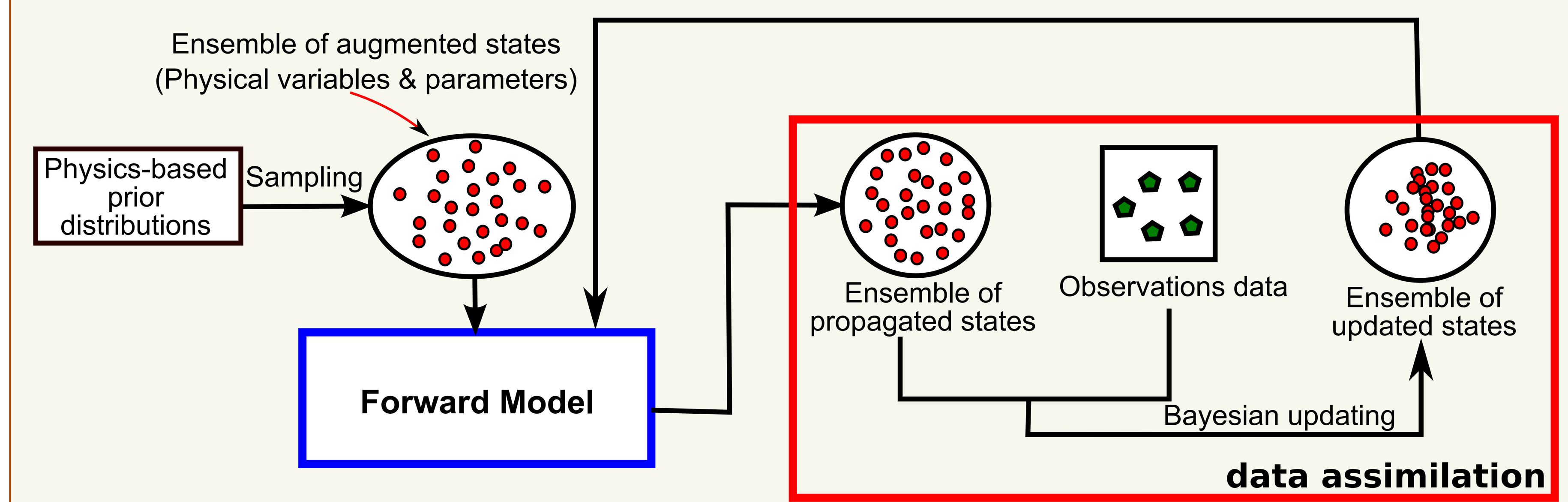
How to reduce the model uncertainties/ discrepancies in RANS?

Application 3: Identifying tsunami flow characteristics based on sediments



How to infer tsunami flow based on sediment deposit

Iterative Ensemble Kalman Method



Step. 1 Sampling of prior
Provide an initial guess of modeling discrepancy for complex physics, which is represented as a prior ensemble.

Step. 2 Propagation
Each sample of prior is propagated by solving the forward model.

Step. 3 Correction
The propagated states are corrected by comparing with the sparse observation data .

Iglesias et al. Inverse Problems 29 (4) 2013

Reduced Order Representation of Complex Physics

The unresolved physics are usually spatial fields with physical hard constraints. For example:

- Force field representing the hydrodynamic effects of the unresolved complex structure in fluids.
- Reynolds stress discrepancy fields in RANS simulations due to the potentially inaccurate assumptions.

This type of spatial field $\Phi(x)$ can be represented with a number of orthogonal basis $\phi_\alpha(x)$

$$\Phi(x)=\sum_{\alpha=1}^{M} w_\alpha \phi_\alpha(x)$$

Orthogonal Jacobi polynomial basis in polar coordinates is used in application 1.

Proper Orthogonal Decomposition (POD) basis is used in application 2.

Verification with Synthetic Data

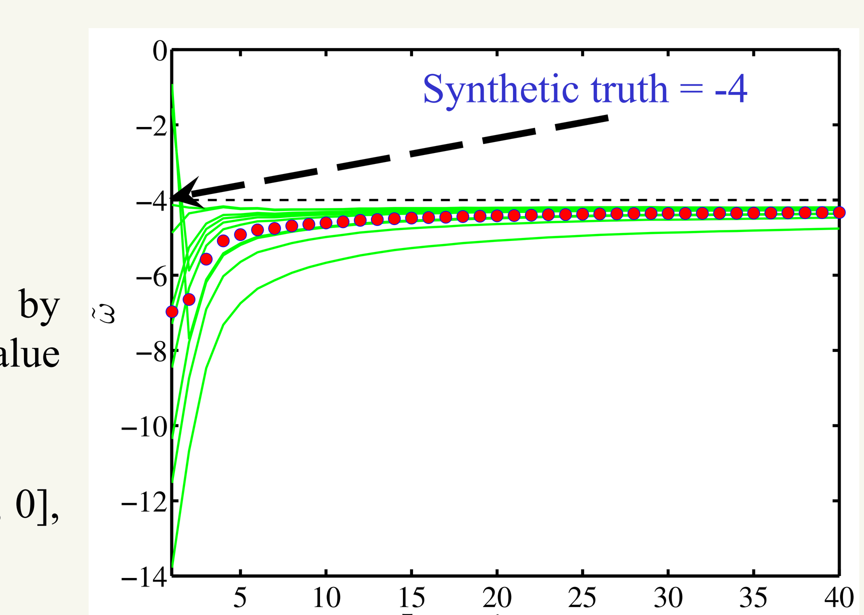
The performance of the inversion is verified in application 1, where the synthetic data are generated by the forward model with a specified force force distribution.

The synthetic true force distribution:

$$f(r)=f_t\lambda(1+\tilde{\omega}r^2),\quad \text{with } 0\le r\le 0.5D,$$

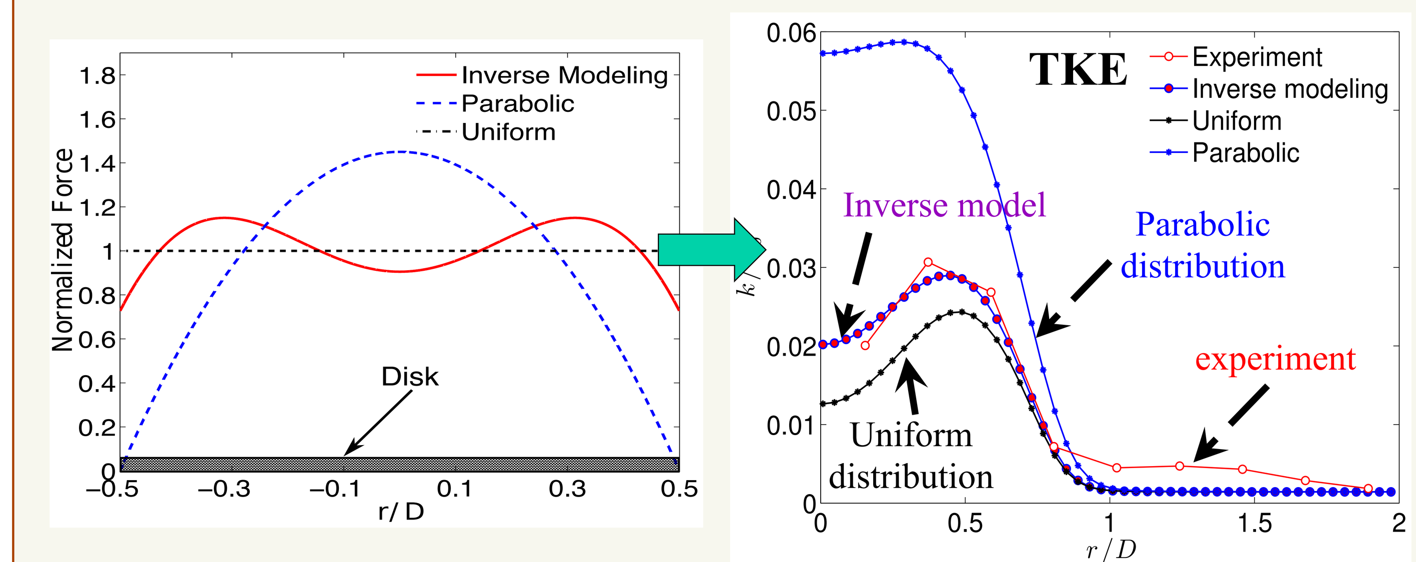
This distribution is the potential flow solution by assuming that porosity of the disk is zero. The true value of ω is -4.

Uniform distributed prior is given in the interval [-14, 0], which is with a biased mean of -7

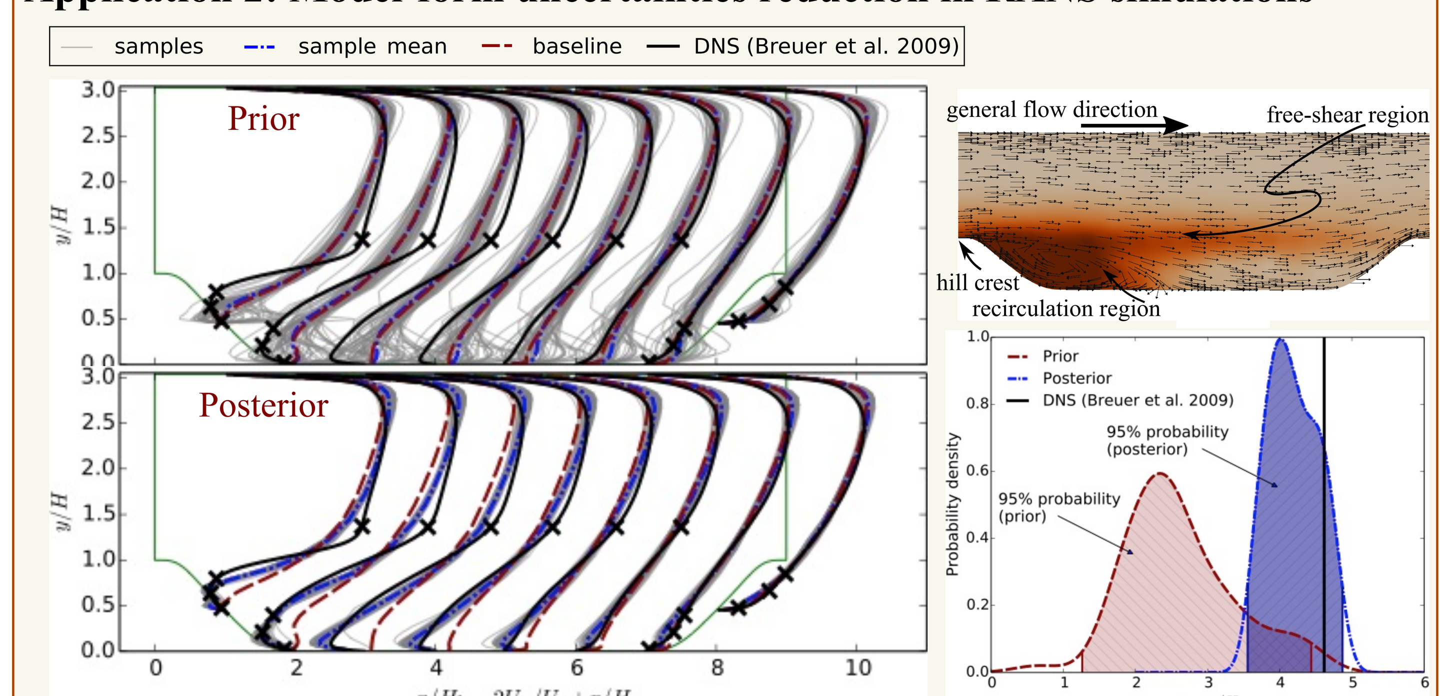


Inverse Modeling Results with Real Data

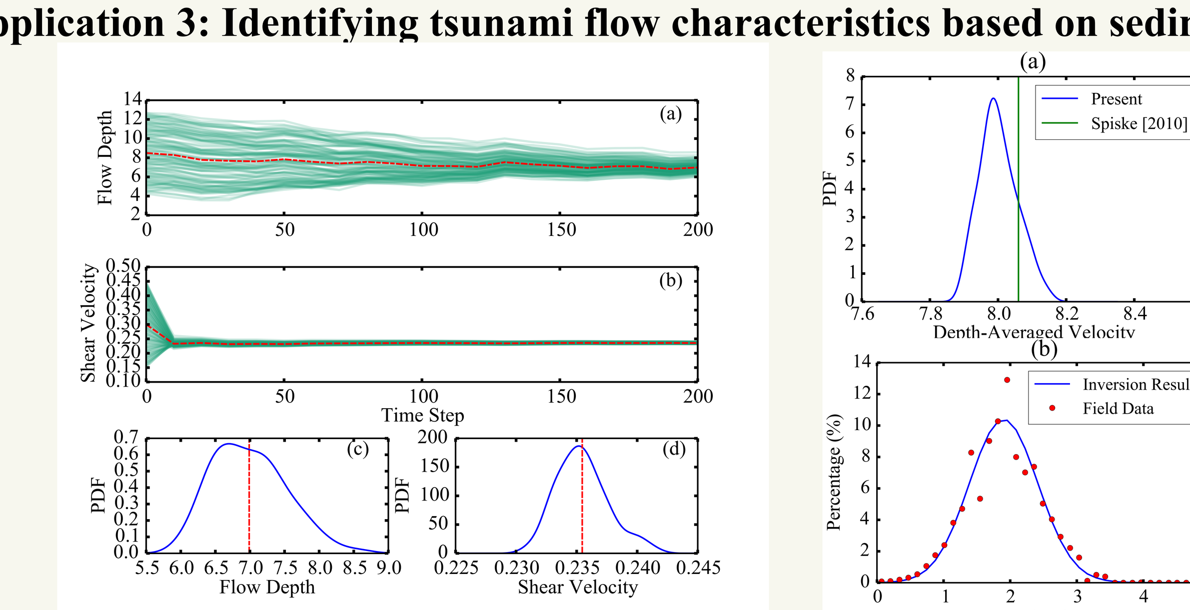
Application 1: Turbulent flows through complex structures



Application 2: Model-form uncertainties reduction in RANS simulations



Application 3: Identifying tsunami flow characteristics based on sediments



Conclusions

- Proposed a data-driven, physics-based approach for reducing model discrepancies due to unresolved physics based on sparse data.
- The modeling discrepancy of unresolved physics resort to full-field parameterization with compact representations.
- Iterative ensemble based Kalman method is used for performing approximate Bayesian inversion.

Bibliography

(1) J.-X. Wang, H. Xiao. Data-Driven CFD Modeling of Turbulent Flows Through Complex Structures. *International Journal of Heat and Fluid Flow*, 62 (B): 138-149, 2016.

(2) H. Xiao, J.-L. Wu, J.-X. Wang, R. Sun, and C. J. Roy. Quantifying and Reducing Model-Form Uncertainties in Reynolds Averaged Navier–Stokes Equations: An Data-Driven, Physics-Based, Bayesian Approach. *Journal of Computational Physics*, 324, 115-136, 2016.

Further Information

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