Data-Driven Turbulence Modeling: Physics-Informed Machine Learning Framework

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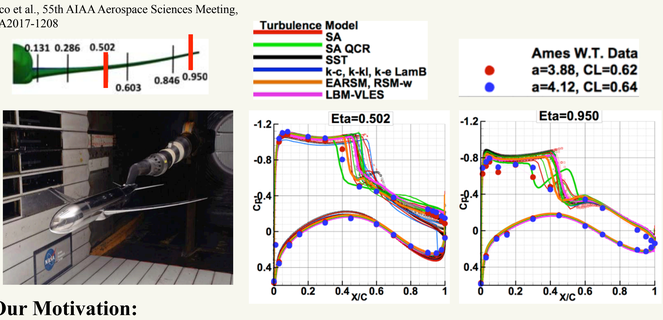
Motivation

Reynolds Averaged Navier-Stokes Equations:

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} - \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} = \nabla \cdot \tau \qquad \tau = -\overline{u_i' u_j'}$$

Hub of RANS models

Tinoco et al., 55th AIAA Aerospace Sciences Meeting, AIAA2017-1208



Our Motivation:

- ❑ RANS modeled Reynolds stresses are known to be unreliable for many flows.
- ❑ LES/DNS simulations are still infeasible for many industrial flows.
- ❑ Is it possible to employ existing LES/DNS database to enhance the RANS simulations?

Objective and Approach

The objective of this work is to demonstrate that the RANS simulated Reynolds stress of a new flow can be improved via our PIML framework with existing LES/DNS database. Three essential parts of our PIML framework is: (1) representation of Reynolds stresses discrepancies as responses; (2) identification of mean flow features as inputs and (3) construction of regression functions from training data.

(1) Representation of Reynolds Stresses as Responses

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} - \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} = \nabla \cdot (\tau_{RANS} + \Delta\tau)$$

$$\tau = 2k\left(\frac{1}{3}\mathbf{I} + \mathbf{A}\right) = 2k\left(\frac{1}{3}\mathbf{I} + \mathbf{V}\Lambda\mathbf{V}^T\right)$$

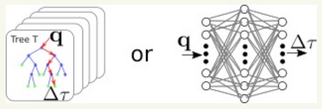
$$\Delta\tau \Rightarrow (\Delta \log_2 k, \Delta\xi, \Delta\eta, \Delta\varphi_1, \Delta\varphi_2, \Delta\varphi_3)$$

(2) Identification of Mean Flow Features

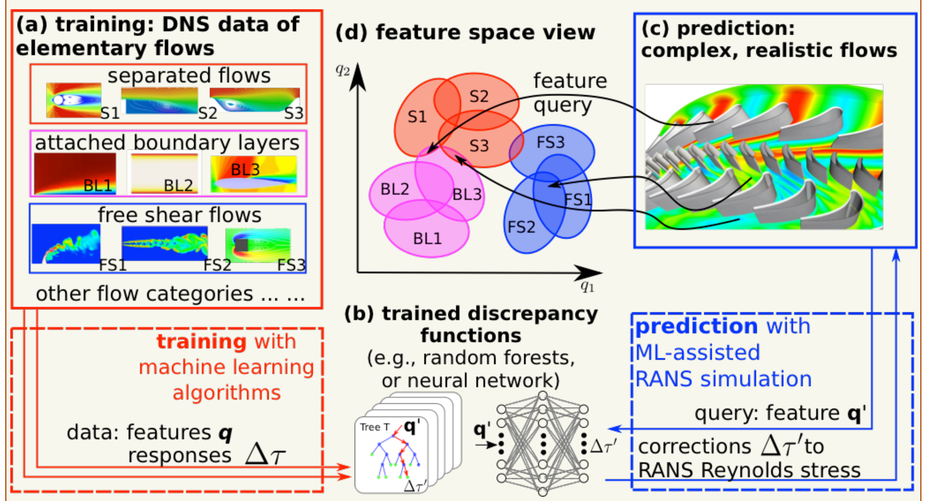
Integrity Basis of $\{\mathbf{S}, \mathbf{\Omega}, \nabla p, \nabla k\} \Rightarrow \mathbf{q}_i \ (i = 1, 2, ..., 47)$

(3) Construction of Regression Functions

Random Forest or Neural Network



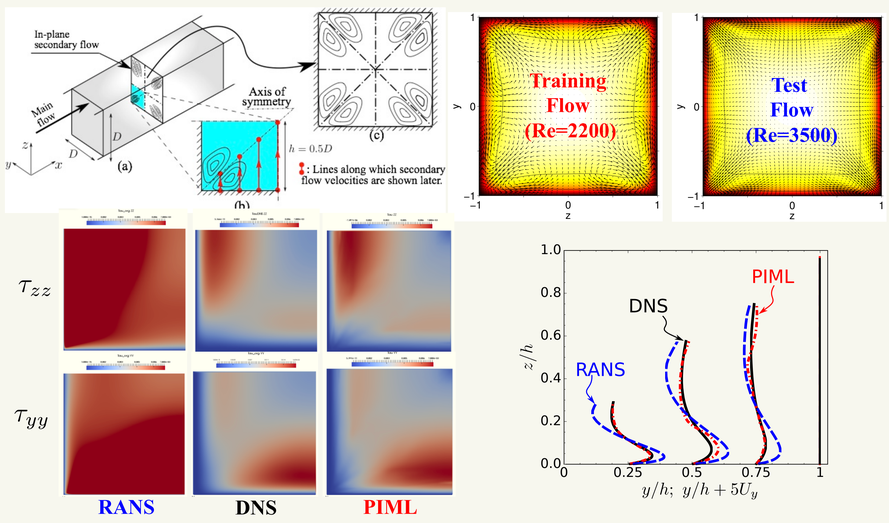
Physics-Informed Machine Learning Framework



Procedures of PIML Framework:

- ❑ Perform baseline RANS simulations on both the training flows and the test flow.
- ❑ Compute the mean flow feature vector $\mathbf{q}(\mathbf{x})$ based on the RANS simulations.
- ❑ Compute the discrepancies field $\Delta\tau_\alpha = \tau_\alpha^{RANS} - \tau_\alpha^{truth}$ between RANS modeled Reynolds stresses and the high-fidelity simulation data for the training flows.
- ❑ Construct regression functions $f_\alpha : \mathbf{q} \mapsto \Delta\tau_\alpha$
- ❑ Compute the Reynolds stresses discrepancies for the test set by evaluating the regression functions.

Application I. Flow in A Square Duct



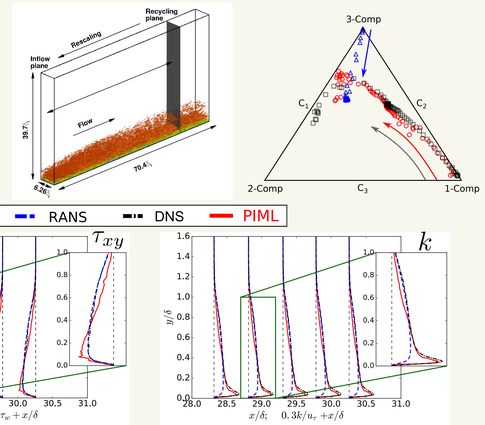
Application II. High Mach Number BL

Training Flow:

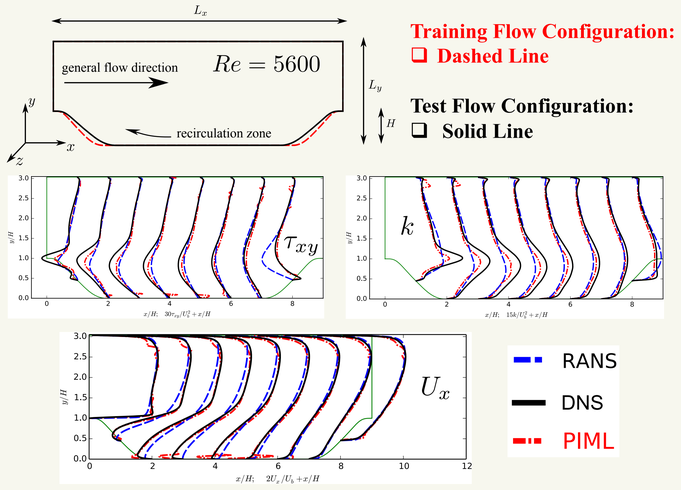
❑ $M = 2.5, T_w = 1$

Test Flow:

❑ $M = 8, T_w = 0.53$



Application III. Flow over Periodic Hills



Training Flow Configuration:

❑ Dashed Line

Test Flow Configuration:

❑ Solid Line

Conclusions

In this work, we proposed a physics-informed machine learning (PIML) approach to predict RANS modeled Reynolds stresses discrepancies by utilizing DNS database. The potential impacts include:

- ❑ Utilizing current high-fidelity simulations database to improve the accuracy of RANS simulations
- ❑ Assisting turbulence modelers to derive better RANS models
- ❑ Inspiring other data-driven modeling approaches in computational mechanics

Bibliography

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J-X. Wang, J-L. Wu, J. Ling, G. Iaccarino, H. Xiao, "A Comprehensive Physics-Informed Machine Learning Framework for Predictive Turbulence Modeling", Submitted, available at: arXiv:1701.07102, 2017.

Further Information

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