Sample Prelims. Problem

(i) Explain what are eigenvalues and eigenvectors of a matrix. Give at least two applications where eigenvalues and eigenvectors are used in Structural Mechanics.

(ii) Using the Power method, determine the largest eigenvalue of the following matrix. Using an appropriate starting vector, perform three iterations.

\[
\begin{bmatrix}
2 & -1 \\
-1 & 1
\end{bmatrix}
\]

(iii) Using the method of diagonalization, find the general solution of:

\[
\frac{d}{dt} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} -6 \\ 2 \end{bmatrix} e^{-2t}
\]