An Online Performance Calculator for Ramjet, Turbojet, Turbofan, and Turboprop Engines

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This paper documents the theory, equations, and assumptions behind an HTML-based online performance calculator for ramjet, turbojet, turbofan, and turboprop engines. Based on inputs of design parameters, flight conditions, and engine parameters, the specific thrust (ST), thrust specific fuel consumption (TSFC), propulsive efficiency (η_p) , thermal efficiency (η_{th}) , and overall efficiency (η_0) will be output for the engine of interest. Several sample cases are analyzed to verify the functionality of the calculator. These sample cases are derived from engines associated with Virginia Tech, which include: Pratt & Whitney's PT6A-20 and JT15D-1, Honeywell's TFE731-2B, and the Rolls Royce Trent 1000. With the help of this calculator, students will now be able to verify their air-breathing jet engine performance value calculations. This calculator is available here.

I. Nomenclature

M_a	=	Ambient Mach Number	c_{pb}	=	Specific Heat of the Burner
γ_a	=	Ambient Specific Heat Ratio	c_{pt}	=	Specific Heat of the Turbine
γd	=	Diffuser Specific Heat Ratio	c_{pf}	=	Specific Heat of the Fan
γ_c	=	Compressor Specific Heat Ratio	T_a	=	Ambient Static Temperature
γ_b	=	Burner Specific Heat Ratio	p_a	=	Ambient Static Pressure
γ_t	=	Turbine Specific Heat Ratio	p_e	=	Exit Static Pressure
γ_n	=	Nozzle Specific Heat Ratio	R_{air}	=	Gas Constant for Air
γ_f	=	Fan Specific Heat Ratio	R_{prod}	=	Gas Constant for the Products of Fuel/Air Mixture
γ_{nf}	=	Fan Nozzle Specific Heat Ratio	Q_R	=	Heat of Reaction of the Fuel
η_d	=	Diffuser Efficiency	f	=	Fuel to Air Ratio
η_c	=	Compressor Efficiency	p_{rc}	=	Compressor Pressure Ratio
η_b	=	Burner Efficiency	p_{rf}	=	Fan Pressure Ratio
η_t	=	Turbine Efficiency	\mathscr{B}	=	Bypass Ratio of the Fan
η_n	=	Nozzle Efficiency	T_{0i}	=	Stagnation Temperature (i = Station Number)
η_f	=	Fan Efficiency	p_{0i}	=	Stagnation Pressure (i = Station Number)
η_{nf}	=	Fan Nozzle Efficiency	ST	=	Specific Thrust
η_{prop}	=	Propeller Efficiency	TSFC	=	Thrust Specific Fuel Consumption
η_{pt}	=	Power Turbine Efficiency	η_p	=	Propulsion Efficiency
η_g	=	Gearbox Efficiency	η_{th}	=	Thermal Efficiency
c_{pc}	=	Specific Heat of the Compressor	η_0	=	Overall Efficiency

II. Station Nomenclature

- *a* = Ambient/Free Stream Conditions
- 1 = Entrance to Engine (Same as Ambient)
- 2 = Entrance to Compressor/Diffuser Exit
- 2.5 = Bypass/Fan Nozzle Exit

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- 3 = Compressor Exit/Burner Entrance
- 4 = Burner Exit/Turbine Entrance
- 4.5 = Free Turbine/Power Turbine Entrance
- 5 = Turbine Exit
- 6 = Nozzle Entrance
- e = Nozzle Exit

III. Introduction

 $U_{\rm will}^{\rm NDERSTANDING}$ the performance of air-breathing jet engines is critical to determining how a specific engine design will contribute to the flight of a modern commercial aircraft.

Air-breathing engines are preferred for large passenger aircraft over piston engines, as they allow for the proper maximization of the mass flow rate of the air entering the engine during takeoff. This is an important concept because the thrust produced by said engine is directly proportional to the mass flow rate. Thus, the maximum thrust produced by a given engine may be determined from air-breathing jet engine cycle analysis [1].

This online performance calculator was created as a tool for students in propulsion performance courses or aircraft design, so that they may verify their performance calculations for the air-breathing jet engine unit of the course.

This calculator is broken down into four sections: "Ramjet Engine", "Turbojet Engine", "Turbofan Engine", and "Turboprop Engine". The equations, theories, and assumptions followed for the analysis of each engine are based off of Chapter 5 of Hill and Peterson (1992) and the AOE 3164 course at Virginia Tech. The equations and theories shall be discussed later on, but the assumptions for each engine are as follows:

1) Engine cycle analysis with no afterburner is considered.

2) $R_{prod} = R_{air}$

- 3) $p_e = p_a$
- 4) Within any one component, a calorically perfect gas is assumed.

To verify the functionality of this HTML-based calculator, several performance calculations are performed for various real-life engines associated, in some respect, with Virginia Tech. The following air-breathing jet engines will be analyzed:

- 1) PT6A-20 [2]
- 2) JT15D-1 [3]
- 3) TFE731-2B [4]
- 4) Trent 1000 [5]

IV. Theoretical Background

A. Engine Performance

To understand the performance of air-breathing jet engines, one must first be familiarized with the equations and definitions of such parameters. The following control surface (Fig. 1), and the subsequent control volume analysis, is the basis for all performance value equations discussed in this section.



Fig. 1 Control Surface for Performance Calculations [1].

1. Specific Thrust

Derived from the thrust equation (Eq. (1)) resulting from the control volume analysis performed in Section 5.2 of Hill and Peterson (1992), ST is the thrust produced by the engine, \mathcal{T} , divided by the mass flow rate of the air, \dot{m}_a , through the engine control surface.

$$\mathcal{T} = \dot{m}_a [(1+f)u_e - u] + (p_e - p_a)A_e \tag{1}$$

Where the u_e is the exhaust velocity of the engine, u is the flight velocity, and A_e is the cross-sectional area of the exit of the nozzle exit. As mentioned in Section III, the analysis discussed in this paper assumes that flow exhausting from the engine is in the subsonic regime, which would result in $p_e = p_a$. Applying this assumption and dividing \dot{m}_a through Eq. (1), the following equation for ST may be derived:

$$ST = (1+f)u_e - u \tag{2}$$

Eq. (1) only applies to ramjet and turbojet engines, as turbofan and turboprop engines require slight modifications to count for the thrust contribution from the fan and propeller, respectively.

2. Thrust Specific Fuel Consumption

TSFC relates the mass flow rate of the fuel, \dot{m}_f , to \mathcal{T} . Simply put, TSFC is the ratio of \dot{m}_f to \mathcal{T} [1]. This means that a smaller TSFC indicates a larger thrust output for a given fuel injection. If one defines f as the ratio of \dot{m}_f to \dot{m}_a and understands that ST is defined by the ratio of \mathcal{T} to \dot{m}_a (as discussed above), the following equation for TSFC may be derived:

$$TSFC = \frac{\dot{m}_f}{\mathcal{T}} = \frac{f}{\mathcal{T}/\dot{m}_a} = \frac{f}{ST} = \frac{f}{(1+f)u_e - u}$$
(3)

Similar to Eq. (2), Eq. (3) only applies for ramjet and turbojet engines. Turbofan and turboprop engines require modifications.

3. Propulsion Efficiency

To determine how much kinetic energy of the flow is converted into thrust power, one can examine η_p . This is a popular value to be analyzed, as it indicates how a efficient a certain propulsion system is [1]. The equation for η_p is defined in Eq. (4) below.

$$\eta_p = \frac{\mathcal{T}u}{\dot{m}_a[(1+f)(u_e^2/2) - u^2/2]} = \frac{STu}{(1+f)(u_e^2/2) - u^2/2} \tag{4}$$

4. Thermal Efficiency

To determine how much chemical energy of the flow is converted into kinetic energy of the flow, one can examine η_{th} . The equation for ηth is defined in Eq. (5) below.

$$\eta_{th} = \frac{(1+f)(u_e^2/2) - u^2/2}{fQ_R}$$
(5)

5. Overall Efficiency

Typically, efficiency is defined as how well energy is utilized throughout a system. In the case of this analysis, η_0 is defined as how much of the chemical energy coming from the fuel is converted to thrust power. The equation for η_0 is defined in Eq. (6) below.

$$\eta_0 = \eta_p \eta_{th} = \frac{\mathcal{T}u}{\dot{m}_f Q_R} = \frac{STu}{fQ_R} \tag{6}$$



Fig. 2 Schematic Diagram and Station Numbers of a Ramjet Engine [1].

B. Ramjet Engine

The ramjet is the simplest air-breathing jet engine, as it only consists of a diffuser, combustion chamber, and exhaust nozzle [1]. Fig. 2 shows the schematic diagram, and corresponding station numbers, of typical ramjet engine.

Air enters the diffuser between stations (a) and (2), where it is compressed. Then, the compressed air is burned with the fuel injected in the combustion chamber between stations (2) and (4). Lastly, the gaseous products from the combustion reaction are expanded through the exhaust nozzle between stations (4) and (6) (Fig. 2) [1].

1. The Ideal Ramjet

To help understand how a ramjet engine performs, one may take a look at an idealized model. In this ideal case, one assumes that there are no stagnation pressure losses across the ramjet (i.e. $p_{0a} = p_{0e}$), as well as the static pressure at the exit being equal to the static pressure in the ambient/free stream (which, as mentioned in Section III, $p_e = p_a$ is valid for all engine analysis performed in this paper) [1]. Fig. 3 below represents the T-s diagram for an ideal ramjet under these assumptions.



Fig. 3 T-s Diagram for an Ideal Ramjet Engine [1].

Note that for the ideal ramjet case depicted in Fig. 3, the compression taking place in the diffuser isentropically transfers the static pressure at (a) to its stagnation state at (2). Internally, an isobaric process occurs within the combustion chamber, transferring the gas in its stagnation state from station (2) to the maximum temperature that occurs at (4). The air then travels through the exhaust nozzle, undergoing an isentropic process to arrive, again, at the ambient static pressure [1].

2. The Real Ramjet

In the case of real flow through a ramjet engine, these ideal assumptions do not hold. For a real ramjet, the flow experiences stagnation pressure loses across the diffuser, the combustion chamber, and the exhaust nozzle. The equations for each of these stagnation pressure loses are expressed below:

$$r_d = \frac{p_{02}}{p_{0a}}$$
(7)

$$r_c = \frac{p_{04}}{p_{02}} \tag{8}$$

$$r_n = \frac{p_{06}}{p_{04}} \tag{9}$$

Where r_d is the stagnation pressure loss across the diffuser, r_c is the stagnation pressure loss across the combustion chamber, and r_n is the stagnation pressure loss across the exhaust nozzle [1]. A visual representation of the aerodynamic loses experienced by the propellant flowing through a real ramjet engine is depicted in Fig. 4 below.

1



Fig. 4 T-s Diagram for a Real Ramjet Engine [1].

Note that in contrast to the ideal ramjet case depicted in Fig. 3, stagnation pressure losses are applied to shift the stagnation pressure of the flow to a lower isobar. For example, applying the stagnation pressure loss across the combustion chamber results in a transition between the isobar at (02) and the isobar at (04). This transition is represented by a curved, dotted line in Fig. 4.

C. Turbojet Engine

The turbojet engine is essentially a ramjet engine with modifications. If a compressor is incorporated directly after the diffuser, which is coupled with a turbine that provides the power required for said compressor, then a ramjet engine becomes a turbojet engine [1]. This addition of compressor and turbine significantly alters how this engine performs. Fig. 5 below depicts a typical schematic diagram of a turbojet engine, with corresponding station numbers.



Fig. 5 Schematic Diagram and Station Numbers of a Turbojet Engine [1].

Air coming from the free stream at (a), with velocity equal to the flight velocity, enters the engine at (1). The air is then decelerated as it goes through the diffuser between stations (1) and (2) before entering the compressor. Between stations (2) and (3), when the air is in the compressor, the flow experiences compression. The flow then enters the burner, between stations (3) and (4), where it is heated and mixed with the injected fuel. Expansion then occurs as the air passes through the turbine between stations (4) and (5). This process also provides the necessary power to the turbine, which, in turn, drives the compressor. Note that in the schematic shown in Fig. 5, an afterburner is included. This is an optional process where the flow may be heated even further by the combustion of more fuel (in addition to the combustion process that occurs in the burner). For this analysis, it is assumed that there is no afterburner, resulting in the conditions at station (5) being equal to the conditions at station (6). Lastly, the flow experiences acceleration as it exhausts through the nozzle between stations (6) and (7) [1].

1. The Ideal Turbojet

To understand the performance of a typical turbojet engine, one may take a look at an idealized model. This ideal turbojet model assumes that all of the aerodynamic processes occurring within the engine, except the heating processes that occur within the burners, are isentropic processes [1]. Fig. 6 depicts a T-s diagram showing this idealized turbojet case. Note that Hill and Peterson (1992) includes the cases for both non-afterburning and afterburning turbojet engines, but the afterburning case is not considered for this analysis.

In this ideal example, the pressure increases (slightly) from the ambient conditions at station (a) to the diffuser exit at station (2). Since the flow is decelerated through the diffuser, and the flow velocity is assumed to be zero at (2) for this ideal case, the pressure at station (2) represents the stagnation pressure for stations (a), (1), and (2). The flow then undergoes compression, leading to a rise in enthalpy $(h_3 - h_2 > 0)$ from station (2) to station (3) within the compressor. As the flow is heated in the burner, there is an isobaric temperature increase, depicted by the curve between (3) and (4). The expansion process experienced by the flow through the turbine, results in an enthalpy drop $(h_5 - h_4 < 0)$ from station (4) to station (5). Understand that, theoretically, since the power from the turbine is what drives the compressor, the magnitude of the enthalpy rise through the compressor should be equal to the magnitude of the enthalpy drop through the turbine $(h_3 - h_2 = h_4 - h_5)$ (assuming the mass-flow rate through the curbine exit (5)) are equal to the conditions at the entrance to the exhaust nozzle (6). The flow through the exhaust nozzle expands even furthur, leading to additional enthalpy drop between stations (6) and (7) [1]. Observe that since it is assumed that $p_e = p_a$, the point representing the exit of the engine, (7), is located on the same isobar as the ambient conditions.

2. The Real Turbojet

In the case of a real turbojet, it is no longer safe to assume that aerodynamic processes are isentropic, as there will be no reversible processes. Additionally, the heating process that occurs in the burners no longer simply raises the temperature of the flow, but changes the composition of the fluid during this combustion process [1]. Fig. 7 below depicts a T-s diagram showing the irreversible effects of the real turbojet case. Similar to the ideal case depicted in Fig. 6, Fig. 7 shows both the non-afterburning and afterburning cases, but the afterburning case is ignored for this analysis.



Fig. 6 T-s Diagram for an Ideal Turbojet Engine (based on sample case from text) [1].



Fig. 7 T-s Diagram for a Real Turbojet Engine (based on sample case from text) [1].

The flow through a real turbojet goes through the same aerodynamic processes as the ideal case, but due to irreversibilities, an increase in entropy occurs through each process that shifts the conditions of each station to the right. The amount of rightward shift of each station is related to the efficiency of each mechanism within the engine. The

efficiencies of the diffuser and compressor, η_d and η_c , respectively, are defined by the ratio of ideal enthalpy change to the real enthalpy change for their respective aerodynamic processes (Eqs. (10), (11)). The efficiencies of the turbine and exhaust nozzle, η_t and η_n , respectively, are defined by the ration of real enthalpy change to the ideal enthalpy change (Eqs. (12), (13)). The efficiency of the burner, η_b , is defined a bit differently than the efficiencies of the diffuser, compressor, turbine, and exhaust nozzle. The burner efficiency is defined by how much of the chemical energy produced by the fuel is released into the combustion chamber [1]. These efficiencies are typically defined as problem parameters, but the equations discussed in the above definitions (excluding η_b , as there is no explicit equation defined) are listed below. Note that any variable with a subscript "s" represents the enthalpy that would occur if the process was isentropic.

$$\eta_d = \frac{h_{02s} - h_a}{h_{02} - h_a} \tag{10}$$

$$\eta_c = \frac{h_{03s} - h_{02}}{h_{03} - h_{02}} \tag{11}$$

$$\eta_t = \frac{h_{04} - h_{05}}{h_{04} - h_{05s}} \tag{12}$$

$$\eta_n = \frac{h_{06} - h_7}{h_{06} - h_{7s}} \tag{13}$$

D. Turbofan Engine

If one were to adjust the design of a turbojet engine such that a high-bypass fan was incorporated before the entrance to the compressor, one would have created a turbofan engine. This fan improves the efficiency of a typical turbojet engine because the bypass flow now contributes to the thrust produced by the engine. This increases the propulsion efficiency, η_p , which, in turn, raises the overall efficiency of the engine. The performance analysis for the core of a turbofan engine is essentially the same as for a turbojet engine, but one must incorporate the bypass ratio of the fan, \mathcal{B} , which is the ratio of mass-flow rate from the fan to the mass-flow rate from the core [1]. Below is a typical schematic for a turbofan engine with the corresponding station numbers included.



Fig. 8 Schematic Diagram and Station Numbers of a Turbofan Engine [6].

Note that the turbofan engine includes both a high pressure turbine (HPT) and a low pressure turbine (LPT). The HPT is used to drive the high pressure compressor (HPC) and the LPT is used to drive the bypass fan. The flow from the ambient at station (a) enters the fan between stations (1) and (2), where it is split into two mass-flow contributions: mass-flow to the core and mass-flow to the fan exit. The mass-flow rate to towards the fan exit is defined as the bypass ratio, \mathcal{B} , multiplied by the mass-flow rate to the core, m_a . It is important to note that, for the analysis discussed in this paper, the flow through the fan and the flow through the core expand to ambient conditions separately [1, 6]. If the two exhaust streams were mixed, then analysis would need to be modified. Besides these changes, and the incorporation of \mathcal{B} to the performance value calculations (discussed later on), the engine cycle analysis for a turbofan engine is essentially the same as for a turbojet engine. In other words, once the flow separates into flow entering the core and flow exhausting from the fan, the cycle analysis within the core is treated as a turbojet.

The breakdown of this turbofan engine into ideal and real cases is unnecessary, as the analysis and T-s diagrams for each case would follow the same trend as a turbojet engine. While these two engines certainly vary in terms of performance, the base-level understanding of how flow is expanded, compressed, and combusted, as well how the corresponding T-s diagram changes from the ideal to real case, is covered in the turbojet engine section.

E. Turboprop Engine

Similar to the turbofan engine, if one were to modify the design of a typical turbojet engine such that a propeller is introduced prior to the engine entrance, one would obtain a turboprop engine. Fig. 9 below shows a typical schematic diagram for a turboprop engine.



Fig. 9 Schematic Diagram of a Turboprop Engine (unique turboprop station numbers included) [6].

Note that the only station numbers included are those in and around the unique components of a turboprop engine. The air through the core of the engine is analyzed in the same manner as a turbojet engine, but the air fed from the compressor turbine into the free turbine (between stations (4) and (4.5)) is what drives said free turbine, which provides power to the gearbox that drives the propeller blades. The free turbine may also be referred to as the "power turbine", as it supplies the power to the propeller blades. Much like a turbofan engine, the propeller contributes significantly to the total thrust generated by this type of engine. This type of propeller thrust results in a much higher propulsion efficiency, η_P , than that of a turbofan engine [6].

V. Equations Used

This section will comprise of the equations used for the HTML-based online propulsion calculator discussed in this paper. It will be broken down into four subsections which represent the equations that are used during the engine cycle analysis of the four air-breathing jet engines considered: ramjet, turbojet, turbofan, and turboprop. Note that all equations discussed are derived from Hill and Peterson (1992).

A. Ramjet Equations

Below is a screenshot of the "Ramjet Engine" section of this online propulsion calculator:

Note that the key assumptions, discussed in Section III, are displayed to the right of the section title for each engine, so the user understands that the performance values output have those specific assumptions associated with them. For a ramjet engine, the input design parameters needed to calculate the performance for a specific engine under question, are the maximum engine temperature located at the combustion chamber exit, T_{04} , flight mach number, M_a , and the heat of reaction of the fuel, Q_R . The input flight conditions are the ambient temperature, T_a , the ambient static pressure, p_a , the gas constant of air, R_{air} , and the specific heat ratio of air, γ_a . The user may then choose if they wish to analyze an ideal or real ramjet case drop down box. If they choose the ideal case, no additional input is required, and they may go ahead and click the "CALCULATE" button, which will output the ideal performance values for that engine design. Note that clicking the calculate button during an ideal case, automatically fills in the specific heat ratios to be 1. Conversely, if they choose the real case, they must input the necessary specific heat ratios and stagnation pressure ratios across each engine component. After inputting the necessary engine parameters for the real case, the user may now click the "CALCULATE" button, which will output the real performance values for that engine design. Below are the equations used at each specific station in Fig. 2. Note that only the equations for the real case shall be shown, as the ideal case does not change the structure of the equation, just the numbers for each variable.

1. Ambient

Equation used to calculate the flight velocity, u_a , which will be used for performance value calculations:

Ramjet Engine Assumptions: No Afterburner, R _{prod} = R _{air} , P _e = P _a
DESIGN PARAMETERS:
$T_{04}(K) = $ $M_a = $ $Q_R(J/kg) = $
FLIGHT CONDITIONS:
$T_a(K) =$ $p_a(Pa) =$ $R_{air}(J/kg/K) =$ $\gamma_a =$
IDEAL or REAL: IDEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
$Y_d =$ $r_d (p_{02}/p_{0a}) =$ $Y_b =$ $r_b (p_{04}/p_{03}) =$ $Y_n =$ $r_n (p_{0e}/p_{06}) =$
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = TSFC ((kg/s)/N) = \eta_p = \eta_h = \eta_0 = \eta_0 = 0$

Fig. 10 Screenshot of "Ramjet Engine" Section of Calculator.

$$u_a = M_a \sqrt{\gamma_a R_{air} T_a} \tag{14}$$

2. Diffuser Outlet/Entrance to Combustion Chamber

Equations used to calculate the stagnation conditions at the exit of the diffuser/entrance to the combustion chamber, as well the specific heat of the combustion chamber or burner, c_{pb} :

$$T_{02} = T_a \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)$$
(15)

$$p_{02} = r_d p_a \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)^{\frac{\gamma_a}{\gamma_a - 1}}$$
(16)

$$T_{03} = T_{02} \tag{17}$$

$$p_{03} = p_{02} \tag{18}$$

$$c_{pb} = \frac{R_{prod}\gamma_b}{\gamma_b - 1} \tag{19}$$

3. Exit to Combustion Chamber/Exhaust Nozzle Inlet

Equations used to calculate stagnation conditions at the exit of the combustion chamber/entrance to the exhaust nozzle, as well as the fuel-air ratio, f:

$$p_{04} = r_b p_{03} \tag{20}$$

$$f = \frac{\frac{T_{04}}{T_{03}} - 1}{\frac{Q_R}{c_{pb}T_{03}} - \frac{T_{04}}{T_{03}}}$$
(21)

$$T_{05} = T_{04} \tag{22}$$

$$p_{05} = p_{04} \tag{23}$$

4. Exhaust Nozzle Exit

Equations used to calculate the engine exit conditions necessary to perform desired performance calculations. Note that (6) is substituted by (e) to explicitly represent exit conditions:

$$T_{0e} = T_{05}$$
 (24)

$$p_{0e} = r_n p_{05} \tag{25}$$

$$M_e = \sqrt{\frac{2}{\gamma_n - 1} \left[\left(\frac{p_{0e}}{p_a}\right)^{\frac{\gamma_n - 1}{\gamma_n}} - 1 \right]}$$
(26)

$$T_e = T_{0e} \left(1 + \frac{\gamma_n - 1}{2} M_e^2 \right)^{-1}$$
(27)

$$u_e = M_e \sqrt{\gamma_n R_{prod} T_e} \tag{28}$$

Note that since a couple of the key assumptions made are that $p_e = p_a$ and $R_{prod} = R_{air}$, the ratio of p_a to p_e expressed in Eq. (26) and the R_{prod} expressed in Eq. (28) will be replaced by R_{air} . The solutions obtained from this engine cycle analysis are plugged into Eqs.(2-6) to obtain the desired output of performance values.

B. Turbojet Equations

Below is a screenshot of the "Turbojet Engine" section of this online propulsion calculator:



Fig. 11 Screenshot of "Turbojet Engine" Section of Calculator.

Similar to the "Ramjet Engine" section, the input design parameters include T_{04} , M_a , and Q_R . There is, however, an additional design parameter necessary for turbojet engine cycle analysis - the compressor pressure ratio, p_{rc} . p_{rc} is the ratio of the stagnation pressure at the exit of the compressor (station (3) in Fig. 5) to the stagnation pressure at the entrance to the compressor (station (2) in Fig. 5). This is a key design parameter because it aids in understand the effects of compression on the flow when travelling through the compressor for a certain turbojet engine design. One other difference to the input parameters for the "Turbojet Engine" section compared to the "Ramjet Engine" section are that the user shall have to input the specific heat ratios and efficiencies, for the real turbojet case, of the unique components within a typical turbojet engine (Fig. 5). Much like the "Ramjet Engine" section, the specific heat ratios will fill in automatically with whatever value is input into γ_a and the efficiencies across each engine component will default to 1 for the ideal case. Below are the equations used to calculate the conditions at each station for typical turbojet engine cycle analysis. These equations are derived from those expressed in Hill and Peterson (1992).

1. Ambient

Equation used to calculate the flight velocity, u_a , which will be used for performance value calculations:

$$u_a = M_a \sqrt{\gamma_a R_{air} T_a} \tag{29}$$

2. Diffuser Outlet/Compressor Inlet

Equations used to calculate the stagnation conditions at the exit of the diffuser/entrance to the compressor:

$$T_{02} = T_a \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)$$
(30)

$$p_{02} = p_a \left[1 + \eta_d \left(\frac{T_{02}}{T_a} - 1 \right) \right]^{\frac{\gamma_d}{\gamma_d - 1}}$$
(31)

3. Compressor Outlet/Burner Inlet

Equations used to calculate the stagnation conditions at the exit of the compressor/entrance to the burner:

$$T_{03} = T_{02} \left[1 + \frac{1}{\eta_c} \left(p_{rc}^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right) \right]$$
(32)

$$p_{03} = p_{rc} p_{02} \tag{33}$$

4. Burner Outlet/Turbine Inlet

Equations used to calculate the stagnation conditions at the exit of the burner/entrance to the turbine, c_{pb} , and f:

$$p_{04} = p_{03} \tag{34}$$

$$c_{pb} = \frac{R_{prod}\gamma_b}{\gamma_b - 1} \tag{35}$$

$$f = \frac{\frac{T_{04}}{T_{03}} - 1}{\frac{T_{04}}{c_{pb}T_{03}} - \frac{T_{04}}{T_{03}}}$$
(36)

5. Turbine Outlet/Nozzle Inlet

Equations used to calculate the stagnation conditions at the exit of the turbine/entrance to the nozzle, specific heat ratio of the compressor, c_{pc} , and specific heat ratio of the turbine, c_{pt} :

$$c_{pc} = \frac{R_{air}\gamma_c}{\gamma_c - 1} \tag{37}$$

$$c_{pt} = \frac{R_{prod}\gamma_t}{\gamma_t - 1} \tag{38}$$

$$T_{05} = \frac{(1+f)T_{04}c_{pt} - c_{pc}(T_{03} - T_{02})}{(1+f)c_{pt}}$$
(39)

$$p_{05} = p_{04} \left[1 - \frac{1}{\eta_t} \left(1 - \frac{T_{05}}{T_{04}} \right) \right]^{\frac{\gamma_t}{\gamma_t - 1}}$$
(40)

$$T_{06} = T_{05} \tag{41}$$

$$p_{06} = p_{05} \tag{42}$$

6. Nozzle Exit

Equations used to calculate the engine exit conditions necessary to perform desired performance calculations:

$$p_{0e} = p_{06} \tag{43}$$

$$T_{0e} = T_{06}$$
 (44)

$$u_e = \sqrt{2\eta_n \frac{\gamma_n}{\gamma_n - 1} R_{prod} T_{0e} \left[1 - \left(\frac{p_e}{p_{0e}}\right)^{\frac{\gamma_n - 1}{\gamma_n}} \right]}$$
(45)

Note that Eq. (39) was derived from balancing the power between the turbine and compressor. This derivation is expressed in greater detail in Section 5.4 of Hill and Peterson (1992). Power balance is a key step in the engine cycle analysis of turbojets, as per definition, the power from the turbine is what drives the compressor. Much like the "Ramjet Engine" section of the calculator, the results of this engine cycle analysis are plugged into Eqs. (2-6) to obtain the performance value output.

C. Turbofan Equations

Below is a screenshot of the "Turbofan Engine" section of this online propulsion calculator:



Fig. 12 Screenshot of "Turbofan Engine" Section of Calculator.

Similar to the "Turbojet Engine" section, the input design parameters include T_{04} , M_a , p_{rc} , and Q_R . The "Turbofan Engine" section, however, contains additional design parameters to be input - fan pressure ratio, p_{rf} , and the bypass ratio, \mathcal{B} . p_{rf} is the ratio of stagnation pressure at the fan outlet (station (2.5) in Fig. 8) to the stagnation pressure at the fan inlet (station (2) in Fig. 8). Note that there are additional input engine parameters corresponding to specific heat ratios and efficiencies for the fan (γ_f and η_f) and the fan nozzle (γ_{nf} and η_{nf}) needed for the real turbofan case. For the ideal case, the default values for all of the engine parameters are filled in when "CALCULATE" is clicked, in a similar manner as the "Turbojet Engine" section. Below are the equations used to calculate the conditions at each station for typical turbofan engine cycle analysis. These equations are derived from those expressed in Hill and Peterson (1992).

1. Ambient

Equation used to calculate the flight velocity, u_a , which will be used for performance value calculations:

$$u_a = M_a \sqrt{\gamma_a R_{air} T_a} \tag{46}$$

2. Diffuser Outlet/Fan Inlet/Compressor Inlet

Equations used to calculate the stagnation conditions at the exit of the diffuser/entrance to the fan/entrance to the compressor:

$$T_{02} = T_a \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)$$
(47)

$$p_{02} = p_a \left[1 + \eta_d \left(\frac{T_{02}}{T_a} - 1 \right) \right]^{\frac{\gamma_d}{\gamma_d - 1}}$$
(48)

3. Fan Outlet/Fan Nozzle Inlet

Equations used to calculate the stagnation conditions at the exit of the fan/entrance to the fan nozzle:

$$T_{02.5} = T_{02} \left[1 + \frac{1}{\eta_f} \left(p_{rf}^{\frac{\gamma_f - 1}{\gamma_f}} - 1 \right) \right]$$
(49)

$$p_{02.5} = p_{rf} \, p_{02} \tag{50}$$

4. Fan Nozzle Exit

Equations used to calculate the fan nozzle exit conditions necessary to perform desired performance calculations:

$$u_{ef} = \sqrt{2\eta_{nf} \frac{\gamma_{nf}}{\gamma_{nf} - 1} R_{air} T_{02.5} \left[1 - \left(\frac{p_a}{p_{02.5}}\right)^{\frac{\gamma_{nf} - 1}{\gamma_{nf}}} \right]}$$
(51)

5. Compressor Outlet/Burner Inlet

Equations used to calculate the stagnation conditions at the exit of the compressor/entrance to the burner:

$$T_{03} = T_{02} \left[1 + \frac{1}{\eta_c} \left(p_{r_c}^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right) \right]$$
(52)

$$p_{03} = p_{rc} p_{02} \tag{53}$$

6. Burner Outlet/Turbine Inlet

Equations used to calculate the stagnation conditions at the exit of the burner/entrance to the turbine, c_{pb} , and f:

$$p_{04} = p_{03} \tag{54}$$

$$c_{pb} = \frac{R_{prod}\gamma_b}{\gamma_b - 1} \tag{55}$$

$$f = \frac{\frac{T_{04}}{T_{03}} - 1}{\frac{\eta_b Q_R}{c_{pb} T_{03}} - \frac{T_{04}}{T_{03}}}$$
(56)

7. Turbine Outlet/Nozzle Inlet

Equations used to calculate the stagnation conditions at the exit of the turbine/entrance to the nozzle, c_{pc} , c_{pt} , and specific heat ratio of the fan, c_{pf} :

$$c_{pc} = \frac{R_{air}\gamma_c}{\gamma_c - 1} \tag{57}$$

$$c_{pt} = \frac{R_{prod}\gamma_t}{\gamma_t - 1} \tag{58}$$

$$c_{pf} = \frac{R_{air}\gamma_f}{\gamma_f - 1} \tag{59}$$

$$T_{05} = \frac{(1+f)T_{04}c_{pt} - c_{pc}(T_{03} - T_{02}) - \mathscr{B}c_{pf}(T_{2.5} - T_{02})}{(1+f)c_{pt}}$$
(60)

$$p_{05} = p_{04} \left[1 - \frac{1}{\eta_t} \left(1 - \frac{T_{05}}{T_{04}} \right) \right]^{\frac{\gamma_t}{\gamma_t - 1}}$$
(61)

$$T_{06} = T_{05} \tag{62}$$

$$p_{06} = p_{05} \tag{63}$$

8. Nozzle Exit

Equations used to calculate the engine exit conditions necessary to perform desired performance calculations:

$$p_{0e} = p_{06} \tag{64}$$

$$T_{0e} = T_{06}$$
 (65)

$$u_e = \sqrt{2\eta_n \frac{\gamma_n}{\gamma_n - 1} R_{prod} T_{0e} \left[1 - \left(\frac{p_e}{p_{0e}}\right)^{\frac{\gamma_n - 1}{\gamma_n}} \right]}$$
(66)

9. Modifications to Performance Equations

These slight modifications to Eqs. (2-6) account for the effect of the fan bypass on the performance of this turbofan engine:

$$ST = (1+f)u_e + \mathcal{B}u_{ef} - (1+\mathcal{B})u_a$$
(67)

$$TSFC = \frac{\dot{m}_f}{\mathcal{T}} = \frac{f}{\mathcal{T}/\dot{m}_a} = \frac{f}{ST} = \frac{f}{(1+f)u_e + \mathcal{B}u_{ef} - (1+\mathcal{B})u_a}$$
(68)

$$\eta_p = \frac{\mathcal{T}u}{\dot{m}_a[(1+f)(u_e^2/2) + \mathcal{B}(u_{ef}^2/2) - (1+\mathcal{B})(u_a^2/2)]} = \frac{STu_a}{(1+f)(u_e^2/2) + \mathcal{B}(u_{ef}^2/2) - (1+\mathcal{B})(u_a^2/2)} \tag{69}$$

$$\eta_{th} = \frac{(1+f)(u_e^2/2) + \mathcal{B}(u_{ef}^2/2) - (1+\mathcal{B})(u_a^2/2)}{fQ_R}$$
(70)

$$\eta_0 = \eta_p \eta_{th} \tag{71}$$

Now, when the user clicks "CALCULATE", the output performance values will be calculated using Eqs. (67-71) instead of Eqs. (2-6).

D. Turboprop Equations

Below is a screenshot of the "Turboprop Engine" section of this online propulsion calculator.

Turboprop Engine Assumptions: No Afterburner, R _{prod} = R _{ain} , pe = pa
DESIGN PARAMETERS:
$T_{O4}(K) = \square M_{0} = \square P_{rc} = \square Q_{R}(J/kg) = \square$
FLIGHT CONDITIONS:
$T_a(K) =$ $p_a(Pa) =$ $R_{air}(J/kg/K) =$ $Y_a =$
IDEAL or REAL: IDEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
$Y_d =$ $\eta_d =$ $\gamma_c =$ $\eta_c =$ $\gamma_b =$ $\eta_b =$ $\gamma_t =$ $\eta_r =$ $\eta_n =$
$V_{f} = \qquad $
TURBOPROP CHARACTERISTICS (if IDEAL, no need to fill in)
$n_{\text{prop}} =$ $n_{\text{pt}} =$ $n_{\text{g}} =$
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = TSFC ((kg/s)/N) = \eta_p = \eta_p = \eta_{th} = \eta_0 = \eta_0 = 0$
OPTIMAL THRUST SPLIT
Thrust Contribution from Propeller (%) = Thrust Contribution from Core (%) =

Fig. 13 Screenshot of "Turboprop Engine" Section of Calculator.

The input design parameters and flight conditions are the same as the "Turbojet Engine" section. However, the input engine parameters are the same as the "Turboprop" section, as fan specific heat ratios and efficiencies are now representative of propeller conditions. Additionally, efficiencies for the propeller (η_{prop}), power turbine (η_{pt}), and gearbox (η_g) are needed to be input to the "Turboprop Characteristics" (for the real turboprop case). Much like the previous calculator sections discussed, once the "CALCULATE" button is clicked for an ideal case, the default values for the engine parameters and turboprop characteristics shall automatically fill in. Below are the equations used to calculate the performance values, as well as optimal thrust split between the propeller and engine core, of a typical turboprop engine. Note that the equations used throughout the cycle analysis will look quite similar to those used for the "Turbofan Engine" section. It is such a similar analysis because of the incorporation of the propeller "fan", but since the bypass flow does not contribute to the cycle analysis, \mathcal{B} is set to zero. This is why \mathcal{B} is not included as an input parameter. Additionally, p_{rf} is set to one because the flow travelling from the front of the propeller to the back of the propeller is an isentropic process, so the stagnation states from front to back are equal. This is why p_{rf} is not included as an input design parameter for this turboprop analysis.

1. Ambient

Equation used to calculate the flight velocity, u_a , which will be used for performance value calculations:

$$u_a = M_a \sqrt{\gamma_a R_{air} T_a} \tag{72}$$

2. Diffuser Outlet/Compressor Inlet

Equations used to calculate the stagnation conditions at the exit of the diffuser/entrance to the compressor:

$$T_{02} = T_a \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)$$
(73)

$$p_{02} = p_a \left[1 + \eta_d \left(\frac{T_{02}}{T_a} - 1 \right) \right]^{\frac{\gamma_d}{\gamma_d - 1}}$$
(74)

3. Propeller Outlet

Equations used to calculate the stagnation conditions at the exit of the propeller:

$$T_{02.5} = T_{02} \left[1 + \frac{1}{\eta_f} \left(p_{rf}^{\frac{\gamma_f - 1}{\gamma_f}} - 1 \right) \right]$$
(75)

$$p_{02.5} = p_{rf} \, p_{02} \tag{76}$$

$$u_{ef} = \sqrt{2\eta_{nf} \frac{\gamma_{nf}}{\gamma_{nf} - 1} R_{air} T_{02.5} \left[1 - \left(\frac{p_a}{p_{02.5}}\right)^{\frac{\gamma_{nf} - 1}{\gamma_{nf}}} \right]}$$
(77)

4. Compressor Outlet/Burner Inlet

Equations used to calculate the stagnation conditions at the exit of the compressor/entrance to the burner:

$$T_{03} = T_{02} \left[1 + \frac{1}{\eta_c} \left(p_{rc}^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right) \right]$$
(78)

$$p_{03} = p_{rc} p_{02} \tag{79}$$

5. Burner Outlet/Turbine Inlet

Equations used to calculate the stagnation conditions at the exit of the burner/entrance to the turbine, c_{pb} , and f:

$$p_{04} = p_{03} \tag{80}$$

$$c_{pb} = \frac{R_{prod}\gamma_b}{\gamma_b - 1} \tag{81}$$

$$f = \frac{\frac{T_{04}}{T_{03}} - 1}{\frac{\eta_b Q_R}{c_{pb} T_{03}} - \frac{T_{04}}{T_{03}}}$$
(82)

6. Compressor Turbine Outlet/Power Turbine Inlet

Equations used to calculate the stagnation conditions at the exit of the compressor turbine/entrance to the power turbine, c_{pc} , c_{pt} , and c_{pf} :

$$c_{pc} = \frac{R_{air}\gamma_c}{\gamma_c - 1} \tag{83}$$

$$c_{pt} = \frac{R_{prod}\gamma_t}{\gamma_t - 1} \tag{84}$$

$$c_{pf} = \frac{R_{air}\gamma_f}{\gamma_f - 1} \tag{85}$$

$$T_{04.5} = \frac{(1+f)T_{04}c_{pt} - c_{pc}(T_{03} - T_{02})}{(1+f)c_{pt}}$$
(86)

$$p_{04.5} = p_{04} \left[1 - \frac{1}{\eta_t} \left(1 - \frac{T_{04.5}}{T_{04}} \right) \right]^{\frac{\gamma_t}{\gamma_t - 1}}$$
(87)

7. Power Turbine Outlet/Nozzle Inlet

Equations used to calculate the stagnation conditions at the exit of the power turbine:

$$T_{05} = T_{04.5} - \left[\mathscr{B} \frac{c_{pf}}{c_{pt}} (T_{02.5} - T_{02}) \right]$$
(88)

$$p_{05} = p_{04.5} \left[1 - \frac{1}{\eta_t} \left(1 - \frac{T_{05}}{T_{04.5}} \right) \right]^{\frac{\gamma_t}{\gamma_t - 1}}$$
(89)

$$T_{06} = T_{05} \tag{90}$$

$$p_{06} = p_{05} \tag{91}$$

8. Nozzle Exit

Equations used to calculate the engine exit conditions necessary to perform desired performance calculations:

$$p_{0e} = p_{06} \tag{92}$$

$$T_{0e} = T_{06}$$
 (93)

$$u_e = \sqrt{2\eta_n \frac{\gamma_n}{\gamma_n - 1} R_{prod} T_{0e}} \left[1 - \left(\frac{p_e}{p_{0e}}\right)^{\frac{\gamma_n - 1}{\gamma_n}} \right]$$
(94)

The results of these equations are input into Eqs. (67-71) to obtain the performance values for this turboprop analysis. These numbers are output in the "Performance Values" portion of the "Turboprop Engine" section when the user clicks the "CALCULATE" button.

9. Optimal Thrust Split

Equations used to calculate the optimal thrust split (as a percentage) of the contribution of thrust from the propeller and the core;

$$\Delta h = \frac{\gamma_t}{\gamma_t - 1} R_{air} T_{04.5} \left[1 - \left(\frac{p_a}{p_{04.5}} \right)^{\frac{\gamma_t - 1}{\gamma_t}} \right]$$
(95)

$$\alpha = 1 - \frac{u_a^2}{2\Delta h} \left(\frac{\eta_n}{\eta_{prop}^2 \eta_g^2 \eta_{pt}^2} \right)$$
(96)

$$\dot{m} = \frac{Thrust}{\frac{\eta_{prop}\,\eta_g\,\eta_{pt}\,\alpha\Delta h}{u_a} + \sqrt{2(1-\alpha)\eta_n\Delta h} - u_a} \tag{97}$$

$$\%Thrust_{prop} = \frac{\dot{m}\frac{\eta_{prop}\eta_g\eta_{pr}\alpha\Delta h}{u_a}}{Thrust} * 100$$
(98)

$$\% Thrust_{core} = 100 - \% Thrust_{prop}$$
⁽⁹⁹⁾

The results of these equations are output in the "Optimal Thrust Split" portion of the "Turboprop Engine" section when the user clicks the "CALCULATE" button.

VI. Sample Cases

To verify the functionality of this air-breathing jet engine calculator, some sample cases shall be analyzed. One case shall be an arbitrarily specified ramjet engine example, as formulated in the AOE 3164 course. Similarly, a case of be an arbitrarily specified turbojet engine design shall be analyzed. Three turbofan engine cases associated with Virginia Tech shall be Pratt & Whitney's JT15D-1, Honeywell's TFE731-2B, and the Rolls Royce Trent 1000. Lastly, one turboprop engine case associated with Virginia Tech shall be Pratt & Whitney's PT6A-20. These cases serve as sample inputs/outputs that can be checked by users of the calculator to test functionality. For each sample case, the following input flight conditions and engine parameters will be used for consistency.

Flight Conditions/Engine Parameters			
Ma	0.85	γ_{nf}	1.4
Q_R	45E6 J/kg	η_d	0.94
T_a	298 K	η_c	0.83
p_a	101.3E3 Pa	η_b	1
Rair	287 J/kg/K	η_t	0.89
γ_a	1.4	η_n	0.98
γ_d	1.4	η_f	0.85
γ_c	1.4	η_{nf}	0.98
γ _b	1.3	η_{prop}	0.85
γ_t	1.32	η_{pt}	0.89
γ_n	1.34	η_g	0.97
γ_f	1.4		

Table 1 Flight Conditions and Engine Parameters Used for All Sample Cases.

A. Ramjet Case

The following table shows the input design parameters for a random ramjet engine design.

Table 2Design Parameters	for Random	Ramjet Engine.
--------------------------	------------	----------------

Ramjet Design Parameters		
<i>T</i> ₀₄	1500 K	
r_d	0.85	
r_b	0.99	
r_n	0.95	

After inputting all the necessary design parameters and flight conditions, the performance values output, when choosing an ideal case, are displayed in Fig. 14 below.

Ramjet Engine Assumptions: No Afterburner, R _{prod} = R _{air} , P _e = P _a
DESIGN PARAMETERS:
$T_{04}(K) = 1500$ $M_a = 0.85$ $Q_R(J/kg) = 45000000$
FLIGHT CONDITIONS:
$T_{a}(K) = 298 \qquad p_{a}(Pa) = 101300 \qquad R_{air}(J/kg/K) = 287 \qquad Y_{a} = 1.4$
IDEAL or REAL: IDEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
$Y_{d} = \begin{bmatrix} 1.4 \\ 1.4 \end{bmatrix} \begin{bmatrix} r_{d}(p_{02}/p_{0a}) = \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ Y_{b} = \\ 1.4 \end{bmatrix} \begin{bmatrix} 1.4 \\ r_{b}(p_{04}/p_{03}) = \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ Y_{n} = \\ 1.4 \end{bmatrix} \begin{bmatrix} 1.4 \\ r_{n}(p_{0e}/p_{06}) = \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1$
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = 339.21 TSFC ((kg/s)/N) = 0.0000789 \eta_p = 0.656 \eta_{th} = 0.126 \eta_0 = 0.083$

Fig. 14 Screenshot of Results for Ideal Ramjet Sample Case.

Note how since the ideal case was chosen, the default values for the engine parameters were automatically filled in. The real case, where the stagnation pressure ratios from Table 2 would need to be input to their respective places in the "Engine Parameters" section, is displayed in Fig. 15 below.

Ramjet Engine Assumptions: No Afterburner, R _{prod} = R _{air} , P _e = P _a			
DESIGN PARAMETERS:			
$T_{0.4}(K) = 1500$ $M_a = 0.85$ $Q_R(J/kg) = 45000000$			
FLIGHT CONDITIONS:			
$T_a(K) =$ 298 $p_a(Pa) =$ 101300 $R_{air}(J/kg/K) =$ 287 $Y_a =$ 1.4			
IDEAL or REAL: REAL			
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):			
$Y_d =$ 1.4 $r_d (p_{02}/p_{0a}) =$ 0.85 $Y_b =$ 1.3 $r_b (p_{04}/p_{03}) =$ 0.99 $Y_n =$ 1.34 $r_n (p_{0e}/p_{06}) =$ 0.95			
PERFORMANCE VALUES CALCULATE CLEAR			
$ST (N/(kg/s)) = 176.474 TSFC ((kg/s)/N) = 0.0001893 \eta_p = 0.812 \eta_{th} = 0.042 \eta_0 = 0.034$			

Fig. 15 Screenshot of Results for Real Ramjet Sample Case.

B. Turbojet Case

The following table shows the input design parameters for a random turbojet engine design.

Table 3 Design Parameters for Random Turbojet Engine.

Turbojet Design Parameters		
7 ₀₄ 1500 K		
prc	50	

After inputting all the necessary design parameters and flight conditions, the performance values output, when choosing an ideal case, are displayed in Fig. 16 below.

Turbojet Engine Assumptions: No Afterburner, R _{prod} = R _{air} , Pe = Pa
DESIGN PARAMETERS:
$T_{04} (K) = \begin{bmatrix} 1500 \\ 0.85 \end{bmatrix} M_{0} = \begin{bmatrix} 0.85 \\ 0.85 \end{bmatrix} P_{TC} = \begin{bmatrix} 50 \\ 0_{R} (J/kg) = \end{bmatrix} \begin{bmatrix} 4500000 \\ 4500000 \end{bmatrix}$
FLIGHT CONDITIONS:
$T_{a}(K) = \boxed{298} p_{a}(Pa) = \boxed{101300} R_{air}(J/kg/K) = \boxed{287} Y_{a} = \boxed{1.4}$
IDEAL or REAL: IDEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Y_d I_{cd}
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = 585.19 TSFC ((kg/s)/N) = 0.000018 \eta_p = 0.507 \eta_{th} = 0.714 \eta_0 = 0.362$

Fig. 16 Screenshot of Results for Ideal Turbojet Sample Case.

The real case, where the specific heat ratios and efficiencies from Table 1 would need to be input to their respective places in the "Engine Parameters" section, is displayed in Fig. 17 below.

Turbojet Engine Assumptions: No Afterburner, R _{prod} = R _{air} , pe = pa
DESIGN PARAMETERS:
$T_{04} (K) = \begin{bmatrix} 1500 \\ 0.85 \end{bmatrix} M_{R} = \begin{bmatrix} 0.85 \\ 0.85 \end{bmatrix} p_{rc} = \begin{bmatrix} 50 \\ 0_{R} (J/kg) = \end{bmatrix} 4500000$
FLIGHT CONDITIONS:
$T_{a}(K) = \boxed{298} p_{a}(Pa) = \boxed{101300} R_{air}(J/kg/K) = \boxed{287} Y_{a} = \boxed{1.4}$
IDEAL or REAL: REAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Yd = 1.4 Nc = 0.83 Yb = 1.3 Nb = 1 Yt = 1.32 Nc = 0.89 Yn = 1.34 Nn = 0.98
PERFORMANCE VALUES CALCULATE CLEAR
ST (N/(kg/s)) = 394.449 TSFC ((kg/s)/N) = 0.000229 $\eta_p = 0.605$ $\eta_{th} = 0.472$ $\eta_0 = 0.286$

Fig. 17 Screenshot of Results for Real Turbojet Sample Case.

C. Turbofan Cases

1. JT15D-1

The following table shows the design parameters for Pratt & Whitney's JT15D-1 turbofan engine.

JT15D-1 Design Parameters		
<i>T</i> ₀₄ 1233.15 K		
p_{rc}	10	
p_{rf}	1.5	
\mathscr{B}	3.3	

Table 4Design Parameters for JT15D-1 Turbofan Engine [7].

After inputting these necessary design parameters and flight conditions, the performance values output, when choosing an ideal case, are displayed in Fig. 18 below.

Turbofan Engine Assumptions: No Afterburner, R _{prod} = R _{air} , Pe = Pa
DESIGN PARAMETERS:
$T_{04} (K) = 1233.15 \qquad M_{6} = 0.85 \qquad P_{rc} = 10 \qquad P_{rf} = 1.5 \qquad \mathcal{B} = 3.3 \qquad Q_{R} (J/kg) = 4500000$
FLIGHT CONDITIONS:
$T_{a}(K) = \begin{array}{ c c } \hline P_{a}(Pa) = \begin{array}{ c } \hline 101300 \end{array} \\ \hline R_{air}(J/kg/K) = \begin{array}{ c } \hline 287 \end{array} \\ \hline Y_{a} = \begin{array}{ c } \hline 1.4 \end{array} \\ \hline \end{array}$
IDEAL or REAL: DEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
$Y_d =$ $I.4$ $\eta_c =$ $I.4$ $\eta_c =$ $I.4$ $\eta_b =$ $I.4$ $\eta_t =$ $I.4$ $\eta_n =$ $I.4$
$Yr =$ 1.4 $\eta_{r} =$ 1 $\eta_{nf} =$ 1
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = \boxed{780.286} TSFC ((kg/s)/N) = \boxed{0.000169} \eta_p = \boxed{0.706} \eta_{th} = \boxed{0.547} \eta_0 = \boxed{0.386}$

Fig. 18 Screenshot of Results for Ideal JT15D-1 Sample Case.

The real case, where the specific heat ratios and efficiencies from Table 2 would need to be input to their respective places in the "Engine Parameters" section, is displayed in Fig. 19 below.

Turbofan Engine Assumptions: No Afterburner, R _{prod} = R _{air} , P _e = P _a
DESIGN PARAMETERS:
$T_{04} (K) = 1233.15 \qquad M_0 = 0.85 \qquad P_{rc} = 10 \qquad P_{rf} = 1.5 \qquad \mathcal{A} = 3.3 \qquad Q_R (J/kg) = 45000000$
FLIGHT CONDITIONS:
$T_{a}(K) = 298 \qquad p_{a}(Pa) = 101300 \qquad R_{air}(J/kg/K) = 287 \qquad Y_{a} = 1.4$
IDEAL or REAL: REAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Yd = 1.4 nc = 0.83 Yb = 1.3 nb = 1 Yt = 0.89 Yn = 1.34 nn = 0.98
Yf = I.4 Nr = I.4 Nnr = 0.98
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = 553.71 TSFC ((kg/s)/N) = 0.0000263 \eta_p = 0.82 \eta_{th} = 0.303 \eta_0 = 0.248$

Fig. 19 Screenshot of Results for Real JT15D-1 Sample Case.

2. TFE731-2B

The following table shows the design parameters for Honeywell's TFE731-2B turbofan engine.

TFE731-2B Design Parameters	
<i>T</i> ₀₄	1283.15 K
p_{rc}	13
p_{rf}	1.5
B	2.8

Table 5Design Parameters for TFE731-2B Turbofan Engine [7].

After inputting these necessary design parameters and flight conditions, the performance values output, when choosing an ideal case, are displayed in Fig. 20 below.

Turbofan Engine Assumptions: No Afterburner, R _{prod} = R _{air} , Pe = Pa
DESIGN PARAMETERS:
$T_{04} (K) = 1283.15 \qquad M_a = 0.85 \qquad p_{rc} = 13 \qquad p_{rf} = 1.5 \qquad \mathcal{B} = 2.8 \qquad Q_R (J/kg) = 45000000$
FLIGHT CONDITIONS:
$T_{a}(K) = \begin{bmatrix} 298 \\ p_{a}(Pa) = \end{bmatrix} \begin{bmatrix} 101300 \\ R_{air}(J/kg/K) = \end{bmatrix} \begin{bmatrix} 287 \\ R_{air}(J/kg/K) = \end{bmatrix} \begin{bmatrix} 1.4 \\ R_{air}(J/kg/K) = R_{air}(J/kg$
IDEAL or REAL: DEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Y_d = $I.4$ η_c = $I.4$ η_c = $I.4$ η_b = $I.4$ η_t = $I.4$ η_r = $I.4$ η_n = $I.4$
$Y_{ff} =$ 1.4 $\eta_{ff} =$ 1
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = 777.956 TSFC ((kg/s)/N) = 0.0000169 \eta_p = 0.665 \eta_{th} = 0.58 \eta_0 = 0.386$

Fig. 20 Screenshot of Results for Ideal TFE731-2B Sample Case.

The real case, where the specific heat ratios and efficiencies from Table 2 would need to be input to their respective places in the "Engine Parameters" section, is displayed in Fig. 21 below.

Turbofan Engine Assumptions: No Afterburner, R _{prod} = R _{air} , p _e = p _a
DESIGN PARAMETERS:
$T_{04} (K) = 1283.15 \qquad M_8 = 0.85 \qquad P_{rc} = 13 \qquad P_{rf} = 1.5 \qquad \mathcal{B} = 2.8 \qquad Q_R (J/kg) = 45000000$
FLIGHT CONDITIONS:
$T_{a}(K) = \begin{bmatrix} 298 \\ p_{a}(Pa) = \end{bmatrix} \begin{bmatrix} 101300 \\ R_{air}(J/kg/K) = \end{bmatrix} \begin{bmatrix} 287 \\ Y_{a} = \end{bmatrix} \begin{bmatrix} 1.4 \\ R_{air}(J/kg/K) = R_{air}(J/kg/K) = \end{bmatrix} \begin{bmatrix} 1.4 \\ R_{air}(J/kg/K) = R_{air}(J/kg/K$
IDEAL or REAL: REAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Y_d = $I_{.4}$ η_c = $I_{.4}$ η_c = $I_{.3}$ η_b = $I_{.32}$ η_t = $I_{.34}$ η_n = $I_{.98}$
$Y_{\rm f}$ = 1.4 $\eta_{\rm f}$ = 0.85 $Y_{\rm nf}$ = 1.4 $\eta_{\rm nf}$ = 0.98
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = 572.569 TSFC ((kg/s)/N) = 0.0000249 \eta_p = 0.776 \eta_{th} = 0.338 \eta_0 = 0.262$

Fig. 21 Screenshot of Results for Real TFE731-2B Sample Case.

3. Trent 1000

The following table shows the design parameters for the Rolls Royce Trent 1000 turbofan engine.

Trent 1000 Design Parameters	
<i>T</i> ₀₄	1750 K
p_{rc}	13
p_{rf}	1.54
\mathscr{B}	10

Table 6Design Parameters for Trent 1000 Turbofan Engine [8].

After inputting these necessary design parameters and flight conditions, the performance values output, when choosing an ideal case, are displayed in Fig. 20 below.

Turbofan Engine Assumptions: No Afterburner, R _{prod} = R _{air} , p _e = p _a
DESIGN PARAMETERS:
T_{04} (K) = M_a = 0.85 p_{rc} = 13 p_{rf} = 1.54 \mathscr{B} = 10 $Q_R(J/kg)$ = 45000000
FLIGHT CONDITIONS:
$T_{a}(K) = 298 \qquad p_{a}(Pa) = 101300 \qquad R_{air}(J/kg/K) = 287 \qquad Y_{a} = 1.4$
IDEAL or REAL: IDEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Yd = 1.4 η_c = 1.4 η_c = 1.4 η_b = 1 Yt = 1.4 η_t = 1.4 η_n = 1
Yr = 1.4 Ynf = 1.4 Nnf = 1
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = 1_{1643.29} TSFC ((kg/s)/N) = 0.0000147 \eta_p = 0.766 \eta_{th} = 0.58 \eta_0 = 0.444$

Fig. 22 Screenshot of Results for Ideal Trent 1000 Sample Case.

The real case, where the specific heat ratios and efficiencies from Table 2 would need to be input to their respective places in the "Engine Parameters" section, is displayed in Fig. 21 below.

Turbofan Engine Assumptions: No Afterburner, R _{prod} = R _{airr} , Pe = Pa
DESIGN PARAMETERS:
$T_{04} (K) = 1750 M_{0} = 0.85 P_{7c} = 13 P_{7f} = 1.54 H = 10 Q_{R} (J/kg) = 4500000$
FLIGHT CONDITIONS:
$T_{a}(K) = 298 \qquad p_{a}(Pa) = 101300 \qquad R_{air}(J/kg/K) = 287 \qquad Y_{a} = 1.4$
IDEAL or REAL: REAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Y_d = $I.4$ η_c = $I.4$ η_c = $I.3$ η_b = I Y_t = $I.32$ η_t = $I.34$ η_n = $I.98$
Yr = I.4 Nr = 0.85 Ynr = I.4 Nnr = 0.98
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = 1039.565 TSFC ((kg/s)/N) = 0.000027 \eta_p = 0.78 \eta_{th} = 0.311 \eta_0 = 0.243$

Fig. 23 Screenshot of Results for Real Trent 1000 Sample Case.

D. Turboprop Case

The following table shows the design parameters for Pratt & Whitney's PT6A-20 turboprop engine.

Table 7 Design Parameters for PT6A-20 Turboprop Engine [1].

Trent 1000 Design Parameters	
<i>T</i> ₀₄	1400 K
p_{rc}	7

After inputting these necessary design parameters and flight conditions, the performance values output, when choosing an ideal case, are displayed in Fig. 24 below.

Turboprop Engine Assumptions: No Afterburner, R _{prod} = R _{airr} , P _e = P _a
DESIGN PARAMETERS:
T_{04} (K) = 1400 M_{0} = 0.85 p_{rc} = 7 Q_{R} (J/kg) = 45000000
FLIGHT CONDITIONS:
$T_{a}(K) = 298 \qquad p_{a}(Pa) = 101300 \qquad R_{air}(J/kg/K) = 287 \qquad Y_{a} = 1.4$
IDEAL or REAL: IDEAL V
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):
Yd = 1.4 Nc = 1.4 Nc = 1.4 Nb = 1.4 Nt = 1.4 Nn
$Y_f =$ 1.4 $\eta_{ff} =$ 1.4
TURBOPROP CHARACTERISTICS (if IDEAL, no need to fill in)
$\eta_{\text{prop}} = 1$ $\eta_{\text{pt}} = 1$ $\eta_{g} = 1$
PERFORMANCE VALUES CALCULATE CLEAR
$ST (N/(kg/s)) = \begin{bmatrix} 673,741 \\ 673,741 \end{bmatrix} TSFC ((kg/s)/N) = \begin{bmatrix} 0.0000275 \\ 0.0000275 \end{bmatrix} \eta_p = \begin{bmatrix} 0.476 \\ 0.476 \end{bmatrix} \eta_{th} = \begin{bmatrix} 0.499 \\ 0.499 \end{bmatrix} \eta_0 = \begin{bmatrix} 0.238 \\ 0.238 \end{bmatrix}$
OPTIMAL THRUST SPLIT
Thrust Contribution from Propeller (%) = 100 Thrust Contribution from Core (%) = 0

Fig. 24 Screenshot of Results for Ideal PT6A-20 Sample Case.

The real case, where the specific heat ratios and efficiencies from Table 2 would need to be input into the "Engine Parameters" and "Turboprop Characteristics" sections, is displayed in Fig. 25 below.

Turboprop Engine Assumptions: No Afterburner, R _{prod} = R _{ain} , Pe = Pa			
DESIGN PARAMETERS:			
$T_{04} (K) = \begin{bmatrix} 1400 \\ M_0 \end{bmatrix} M_0 = \begin{bmatrix} 0.85 \\ 0.85 \end{bmatrix} p_{rc} = \begin{bmatrix} 7 \\ 7 \end{bmatrix} Q_R (J/kg) = \begin{bmatrix} 45000000 \\ 145000000 \end{bmatrix}$			
FLIGHT CONDITIONS:			
$T_{a}(K) = \boxed{298} p_{a}(Pa) = \boxed{101300} R_{air}(J/kg/K) = \boxed{287} Y_{a} = \boxed{1.4}$			
IDEAL or REAL: REAL			
ENGINE PARAMETERS (if IDEAL, no need to fill in engine parameters):			
Yd = 1.4 Nd = 0.94 Yc = 1.4 Nd = 0.83 Yb = 1.3 Nd = 1 Yt = 1.32 Nt = 0.89 Yn = 1.34 Nn =			
$Y_{f'}$ 1.4 $\eta_{f'}$ 0.85 Y_{rf} 1.4 $\eta_{rf'}$			
TURBOPROP CHARACTERISTICS (if IDEAL, no need to fill in)			
$\Pi_{prop} = 0.85$ $\Pi_{pt} = 0.89$ $\Pi_{g} = 0.97$			
PERFORMANCE VALUES CALCULATE CLEAR			
$ST (N/(kg/s)) = 610.038 TSFC ((kg/s)/N) = 0.0000355 \eta_p = 0.503 \eta_{th} = 0.366 \eta_0 = 0.184$			
OPTIMAL THRUST SPLIT			
Thrust Contribution from Propeller (%) = 89.12 Thrust Contribution from Core (%) =			

Fig. 25 Screenshot of Results for Real PT6A-20 Sample Case.

VII. Conclusion

This air-breathing jet engine performance calculator was created to provide a simple way for students to analyze/check their answers to engine cycle analysis problems for ramjet, turbojet, turbofan, and turboprop engines.

Users should note that if an invalid parameter is input into the calculator, an error message will pop up that instructs the user to make sure their input parameters are valid. For example, if the user were to input a value of less than 1 for any of the specific heat ratios (a value that cannot be achieved by this parameter), an error message will remind them that specific heat ratios must be greater than 1. This will only display when the user presses "CALCULATE" and, in turn, the calculator will not finish the calculation and nothing shall be output.

Additionally, if the user wishes to perform back-to-back calculations, they may press the "CLEAR" button, which will reset the calculator to how the user found it when they first visited the page, and allow for a new calculation.

With the verification of the functionality of this HTML-based online propulsion calculator, based on the theory and equations used for different engine cycle analyses, the user may now analyze air-breathing jet engine designs to determine how the engine performs. One must keep in mind the key assumptions made for each engine case considered in this calculator including: non-afterburning engines, $R_{prod} = R_{air}$, $p_e = p_a$, and assumption of calorically perfect gases. In the future, more in-depth versions of this calculator may be created to open up a wider range of real-life engine performance calculations.

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