Modifying Hamiltonian Structure to Stabilize an Underwater Vehicle

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Abstract

This paper presents new results on stabilization of an underwater vehicle using internal rotors. In previous work, a stabilizing control law was derived which preserves the open-loop Hamiltonian structure in the closed-loop system, but with a modified Hamiltonian. The results presented here illustrate the utility of feedback control that not only shapes the energy but also modifies the Hamiltonian structure.

1 Introduction

Internal rotors can provide energy shaping that stabilizes steady forward motion of an underwater vehicle with dynamics described by Kirchhoff's equations [6]. Kirchhoff's equations are Hamiltonian (Lie-Poisson) and a feedback law may be chosen to preserve the Lie-Poisson structure in the closed loop, with a modified Hamiltonian. The control law can be derived by the *method of controlled Lagrangians* specialized to Euler-Poincaré systems [1]. Stability requires choosing control gains so that a Lyapunov function constructed from the modified Hamiltonian is negative definite, i.e., the equilibrium becomes an "energy maximum". When physical dissipation (in this case, viscous drag due to the body's motion through the fluid) is added to the system dynamics, it has the effect of decreasing energy and therefore destabilizing the equilibrium. The control law proposed in this paper provides stabilization in such a way that physical dissipation enhances stability. The control law provides a Hamiltonian closed-loop system with a modified Hamiltonian structure, as well as a modified Hamiltonian.

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2 Feedback Stabilization

Consider an ellipsoidal vehicle with three internal rotors and let the ellipsoid principal axes define a body-fixed coordinate frame. Each of the three rotors is symmetric and spins about its axis of symmetry under the influence of a control torque. The rotors are mounted orthogonally within the vehicle so that each rotor's spin axis is collinear with a body coordinate axis. Assume that the vehicle center of gravity coincides with its center of buoyancy.

Let the diagonal matrix $\boldsymbol{I} = \mathrm{diag}(I_1, I_2, I_3)$ represent the inertia of the vehicle without rotors plus the added inertia of the fluid. Similarly, let the diagonal matrix $\boldsymbol{M} = \mathrm{diag}(m_1, m_2, m_3)$ represent the mass of the vehicle multiplied by the identity matrix plus the added mass matrix of the fluid. We assume that the vehicle's 1-axis is longest and its 3-axis is shortest. Then $m_1 < m_2 < m_3$.

Let the diagonal matrix with diagonal elements (J_1^i, J_2^i, J_3^i) denote the inertia of the rotor which spins about the ith body coordinate axis ($i = 1, 2,$ or 3). The total vehicle inertia, with the rotors locked in place, is $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ where

$$\lambda_j = I_j + J_j^1 + J_j^2 + J_j^3, \quad j = 1, 2, 3.$$

For convenience, The matrix $\boldsymbol{J_r} = \mathrm{diag}(J_1^1, J_2^2, J_3^3)$ is composed of the moments of inertia of the rotors about their spin axes.

Assume that the ith rotor spins at a rate Ω_{r_i} relative to the vehicle and define $\boldsymbol{\Omega_r} = (\Omega_{r_1}, \Omega_{r_2}, \Omega_{r_3})^T$. Define the body coordinate momenta

$$\begin{pmatrix} \boldsymbol{\Pi} \\ \boldsymbol{P} \\ \boldsymbol{l} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Lambda} & \boldsymbol{0} & \boldsymbol{J_r} \\ \boldsymbol{0} & \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{J_r} & \boldsymbol{0} & \boldsymbol{J_r} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Omega} \\ \boldsymbol{v} \\ \boldsymbol{\Omega_r} \end{pmatrix}$$

where $\boldsymbol{\Omega}$ and $\boldsymbol{v}$ are angular and linear velocity, respectively, in body coordinates. The Hamiltonian for the vehicle/fluid system is the total kinetic energy

$$H = \frac{1}{2} \begin{pmatrix} \boldsymbol{\Pi} \\ \boldsymbol{P} \\ \boldsymbol{l} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{\Lambda} & \mathbf{0} & \boldsymbol{J_r} \\ \mathbf{0} & \boldsymbol{M} & \mathbf{0} \\ \boldsymbol{J_r} & \mathbf{0} & \boldsymbol{J_r} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{\Pi} \\ \boldsymbol{P} \\ \boldsymbol{l} \end{pmatrix}.$$

Then the equations of motion are

$$\begin{pmatrix} \dot{\boldsymbol{\Pi}} \\ \dot{\boldsymbol{P}} \\ \dot{\boldsymbol{l}} \end{pmatrix} = \begin{pmatrix} \hat{\boldsymbol{\Pi}} & \hat{\boldsymbol{P}} & \mathbf{0} \\ \hat{\boldsymbol{P}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \nabla H + \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{u} \end{pmatrix},$$

where $\boldsymbol{u} = (u_1, u_2, u_3)^T$ is the control; u_i is the torque applied to the ith internal rotor about its spin axis. With $\boldsymbol{u} = \mathbf{0}$, it can be verified that steady translation of the vehicle along its long axis is an unstable relative equilibrium [2].

The control law developed in our earlier work, and that prescribed by the method of controlled Lagrangians, is

$$\boldsymbol{u} = k\dot{\boldsymbol{\Pi}} = k(\boldsymbol{\Pi} \times \boldsymbol{\Omega} + \boldsymbol{P} \times \boldsymbol{v}), \qquad (2.1)$$

where k is a control gain. Consider the change of coordinates $(\boldsymbol{\Pi}, \boldsymbol{P}, \boldsymbol{l}) \to (\boldsymbol{\Pi}, \boldsymbol{P}, \boldsymbol{\zeta})$ where

$$\boldsymbol{\zeta} = \frac{1}{1-k}(\boldsymbol{l} - k\boldsymbol{\Pi}).$$

By construction, $\boldsymbol{\zeta}$ is conserved. The closed-loop equations are Hamiltonian (Lie-Poisson) with respect to the energy $H_R =$

$$\frac{1}{2}(\boldsymbol{\Pi} - \boldsymbol{\zeta}) \cdot \boldsymbol{I_C}^{-1}(\boldsymbol{\Pi} - \boldsymbol{\zeta}) + \frac{1}{2}\boldsymbol{P} \cdot \boldsymbol{M}^{-1}\boldsymbol{P}$$

where

$$\boldsymbol{I_C} = \frac{1}{1-k}\bar{\boldsymbol{I}}.$$

The equations of motion are

$$\begin{pmatrix} \dot{\boldsymbol{\Pi}} \\ \dot{\boldsymbol{P}} \\ \dot{\boldsymbol{\zeta}} \end{pmatrix} = \begin{pmatrix} \hat{\boldsymbol{\Pi}} & \hat{\boldsymbol{P}} & \mathbf{0} \\ \hat{\boldsymbol{P}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \nabla H_R,$$

Conditions for stability may be determined by applying the energy-Casimir method. The method indicates that choosing $k > 1$ ($\boldsymbol{I_C} < \mathbf{0}$) stabilizes steady long-axis translation. The method also provides a Lyapunov function, H_Φ, constructed from the energy H_R and other conserved quantities. The desired equilibrium is a maximum of H_Φ. Fluid drag, which is not included in this model, tends to destabilize the stabilized equilibrium by decreasing H_Φ [5].

The proposed control law is

$$\boldsymbol{u} = k(\boldsymbol{P} \times \boldsymbol{v}).$$

This control law is a modification of the original control law (2.1) which was formulated from physical intuition partly motivated by Leonard (1996) . The closed-loop dynamics are

$$\begin{pmatrix} \dot{\boldsymbol{\Pi}} \\ \dot{\boldsymbol{P}} \\ \dot{\boldsymbol{\zeta}} \end{pmatrix} = \begin{pmatrix} \hat{\boldsymbol{\Pi}} & \hat{\boldsymbol{P}} & \mathbf{0} \\ \hat{\boldsymbol{P}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{k}{1-k}\hat{\boldsymbol{\Pi}} \end{pmatrix} \nabla H_R.$$

This system is not Lie-Poisson but is an implicit generalized Hamiltonian system in the sense of van der Schaft (1998); that is, the corresponding Poisson bracket does not satisfy the Jacobi identity. Stability of steady long-axis translation can be proven by choosing $k > 1$ and using H_R to construct a negative semidefinite Lyapunov function. In this case, however, fluid drag tends to increase the Lyapunov function, enhancing stability of the desired equilibrium.

References

[1] A. M. Bloch, N. E. Leonard, and J. E. Marsden. Matching and stabilization by the method of controlled Lagrangians. In *Proc. IEEE Conf. Decision and Control*, pages 1446–1451, Tampa, FL, December 1998.

[2] H. Lamb. *Hydrodynamics.* Dover, New York, 6th edition, 1932.

[3] N. E. Leonard. Stability and stabilization of underwater vehicle dynamics. In *Proc. Conf. Information Sciences and Systems*, pages 771–775, Princeton, NJ, 1996.

[4] A.J. van der Schaft. Implicit hamiltonian systems with symmetry. *Reports on Mathematical Physics*, 41(2):203–221, 1998.

[5] C. Woolsey and N. E. Leonard. Global asymptotic stabilization of an underwater vehicle using internal rotors. In *Proc. IEEE Conf. Decision and Control*, Phoenix, AZ, December 1999. To appear.

[6] C. Woolsey and N. E. Leonard. Underwater vehicle stabilization by internal rotors. In *Proc. American Control Conf.*, pages 3417–3421, San Diego, CA, June 1999.