Lecture 8: Roll Stability & Control

For both pitching motion and yawing motion, the primary effect of the empennage (horizontal and vertical tail assembly) is to provide stabilizing moments which tend to keep the nose of the airplane pointing into the wind. Thus, the notion of *static stability* is well-defined for these motions: the initial tendency of the aircraft, when perturbed from wings level, equilibrium flight, is to return to that equilibrium. More specifically, the horizontal tail provides a counter-pitching moment and the vertical tail provides a counter-yawing moment. Because these restoring moments are proportional to the "deflections" in the aerodynamic angles α and β, the effects are sometimes referred to as "pitch stiffness" and "yaw stiffness." The more effective the tail surfaces are at generating these moments, the more "stiff" the system is in the sense of a mass-spring mechanism.

In the case of rolling motion, there is no feature of an airplane which provides static roll stability *per se*. Etkin and Reid give the example of an airplane model pinned about its longitudinal axis in a wind tunnel [1]. If the airplane is perturbed from wings level, no roll moment will develop to return it to that state. While an aircraft has no first order roll stiffness, some roll stability can be provided by thoughtful design. To see how, first consider an airplane which has suffered a positive roll disturbance so that $\phi > 0$. Because the lift vector does not remain vertical, it no longer balances the weight, which now has a component in the direction of the right wing. The airplane begins to "slide" in the direction of the right wing so that $v > 0$ which means that $\beta > 0$. We will say that wings level, equilibrium flight is "statically stable in roll" if the result of such a perturbation is a negative roll moment, i.e., one which tends to return the airplane to the wings level condition.

Defining the roll moment coefficient

$$C_l = \frac{L}{P_{\text{dyn}} S b},$$

we require that

$$C_{l_\beta} := \frac{\partial C_l}{\partial \beta} < 0$$

for roll stability. There are four primary factors which influence the value of C_{l_β}. The most important of these is the wing dihedral angle. Other contributors are wing-body interaction, wing sweep, and the vertical tail.

Dihedral effect. The principal means of satisfying the condition $C_{l_\beta} < 0$ is to angle the wings upward so that the tip chord is higher than the root chord. The angle Γ between the reference plane of the wing and the aircraft xy-plane is called the *dihedral angle* and the stabilizing effect of this configuration is known as the *dihedral effect*.

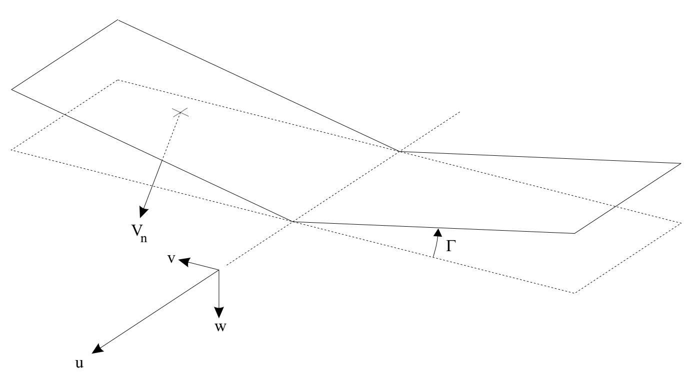

Figure 1: Illustration of wing dihedral.

Let V_n denote the component of velocity normal to the reference plane of the right wing, as shown in

Figure 1, which is adapted from [1]. A bit of thought will verify that

$$V_n = w \cos \Gamma + v \sin \Gamma \approx w + v\Gamma.$$

If $\frac{v}{V}$ and $\frac{w}{V}$ are much less than one, then $u \approx V$ and the local angle of attack of the right wing is

$$\alpha_{\text{right}} \approx \arctan\left(\frac{w + v\Gamma}{u}\right) \approx \alpha + \Gamma\beta.$$

Conversely, the component of velocity normal to the left wing is approximately $w - v\Gamma$ and the local angle of attack is

$$\alpha_{\text{left}} \approx \arctan\left(\frac{w - v\Gamma}{u}\right) \approx \alpha - \Gamma\beta.$$

Thus, if $\Gamma > 0$, then positive sideslip results in an increased angle of attack, and therefore increased lift, on the right wing. Conversely, positive sideslip results in a decreased angle of attack, and therefore decreased lift, on the left wing. The net result is a negative roll moment which tends to restore wings level flight.

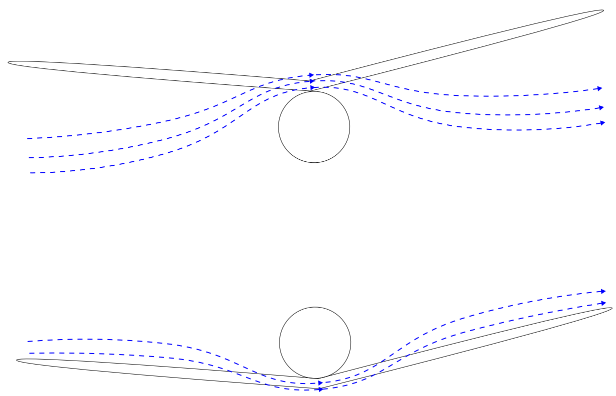

Figure 2: Illustration of fuselage contribution to dihedral effect. (Viewed from the front)

Wing-body interaction. Mounting the wing above the fuselage enhances the dihedral effect while mounting the wing below the fuselage decreases this effect. To understand this, consider Figure 2, which depicts the lateral airflow about a fuselage when the airplane slips sideways. For the high wing configuration, the locally increased angle of attack on the right wing, and the decreased angle of attack on the left wing, augment the dihedral effect. For the low wing configuration, the converse is true. Because the fuselage contribution to roll stability is generally detrimental for a low-wing airplane, the dihedral angle Γ must generally be larger.

Wing sweep. The wing sweep angle Λ affects roll stability, as well. Consider the case of positive sideslip depicted in Figure 3, which might result from a positive roll disturbance. Any given chord line of the right wing (e.g., the leading edge) experiences a relative increase in normal velocity, and thus an increase in the local angle of attack. Conversely, any given chord line of the left wing experiences a relative decrease in normal velocity and a decrease in angle of attack. Consequently, the right wing generates more lift than the left and a negative (restoring) roll moment results.

The contribution $C_{l_{\beta_{\text{wb}}}}$ to C_{l_β} due to the wing and body can be estimated as described in Section B.9 of [1]. The estimate accounts for dihedral angle, sweep angle, taper ratio, and twist, as well as the finite aspect ratio and Mach number effects.

Vertical tail. The contribution to C_{l_β} due to the vertical tail is straight forward to estimate. The key physical observation is that the lift force generated by a vertical tail in sideslip (the same force which exerts the "weathercock" yaw moment that tends to decrease β) acts, in general, at some distance above or below the longitudinal axis of the aircraft. Thus, a roll moment results as well.

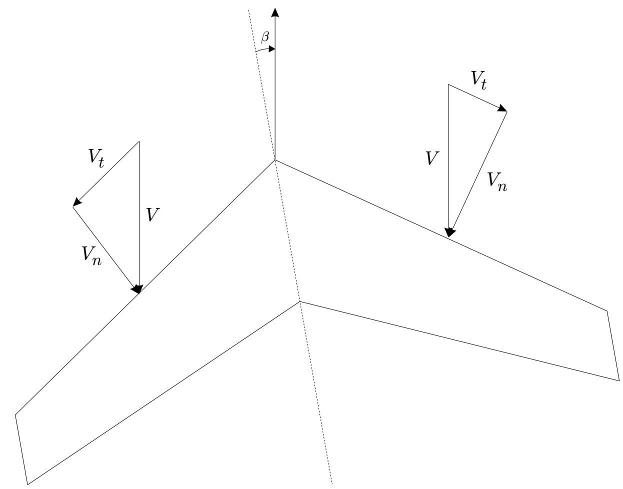

Figure 3: Illustration of wing sweep effect on roll stability.

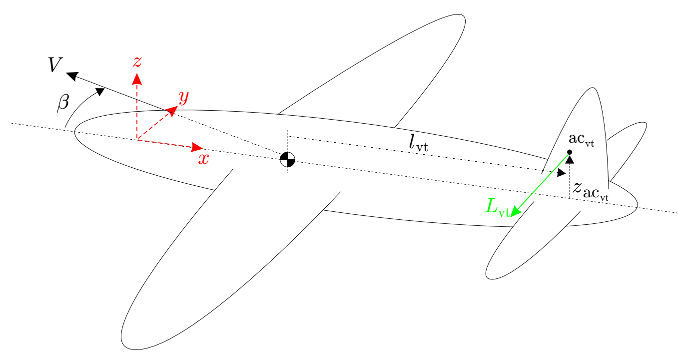

Figure 4: Roll moment due to vertical tail.

Let $z_{\mathrm{ac}_{\mathrm{vt}}}$ denote the vertical distance (measured positive *up*) from the longitudinal axis to the vertical tail aerodynamic center. For positive sideslip angles and for $z_{\mathrm{ac}_{\mathrm{vt}}} > 0$ (i.e., a topside vertical tail), the roll moment is negative, which means that the vertical tail increases roll stability. In fact, it is a simple matter to show that, for an aft, topside vertical tail,

$$C_{l_{\beta_{\mathrm{vt}}}} = -C_{n_{\beta_{\mathrm{vt}}}} \frac{z_{\mathrm{ac}_{\mathrm{vt}}}}{l_{\mathrm{vt}}} < 0,$$

where, from the previous lecture, $l_{\mathrm{vt}} > 0$ and

$$C_{n_{\beta_{\mathrm{vt}}}} = \mathcal{V}_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V}\right)^2 C_{L_{\alpha_{\mathrm{vt}}}} \left(1 - \frac{d\sigma}{d\beta}\right) > 0.$$

In the end, we have

$$C_{l_\beta} = C_{l_{\beta_{\mathrm{wb}}}} + C_{l_{\beta_{\mathrm{vt}}}}.$$

Roll Control. As with the other two control moment devices we have studied (namely the elevator and the rudder), the roll control device acts by exerting a small force over a large moment arm. The ailerons are a pair of flaps located symmetrically about the xz-plane at the trailing edge of the left and right wings. These flaps are slaved to move in opposite directions, so only one parameter δa is necessary to specify the aileron displacement. For $\delta a > 0$, the left aileron is deflected downward to increase the lift generated by the left wing. The right aileron is simultaneously deflected upward to decrease the lift generated by the right wing. The primary result is a positive roll moment.[1]

[1] The convention stated here is *opposite* to that used by Etkin and Reid; compare with Figure 3.20 in the text.

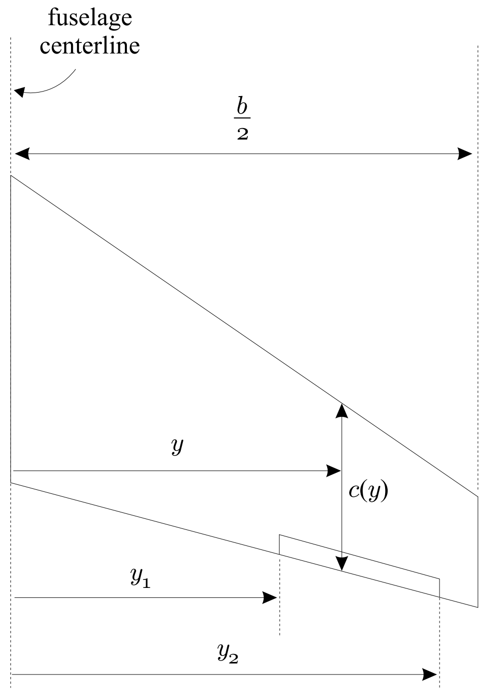

Figure 5: Sketch of the right wing with aileron.

To determine the aileron power, first note that the roll moment increment due to an aileron deflection is

$$\Delta L = -2 \int_0^{b/2} y \Delta \text{Lift}'(y) dy,$$

where y is the spanwise station along the right wing and where $\Delta\text{Lift}'(y)$ is the additional lift *per unit spanwise length* at y due to a positive aileron deflection. The factor of two accounts for the effect of the left aileron. The minus sign accounts for the fact that a positive aileron deflection decreases the lift generated by the right wing. (By convention, $\Delta\text{Lift}'(y)$ for the right wing is negative for positive aileron deflections. However, the resulting roll moment ΔL is positive.)

If we assume that the flow over the wing is only altered at the aileron, then we may write

$$\Delta\text{Lift}'(y) = \begin{cases} 0 & 0 \le y < y_1 \\ -\left(C_{L_{\delta a}} \delta a\right)\left(\frac{1}{2}\rho V^2\right) c(y) & y_1 \le y \le y_2 \\ 0 & y_2 < y < \frac{b}{2} \end{cases} \tag{1}$$

where y_1 is the spanwise location of the aileron's inner edge, y_2 is the location of its outer edge, and $c(y)$ is the wing chord length at y. The aileron effectiveness is

$$C_{L_{\delta a}} = C_{L_{\alpha_w}} \tau,$$

where the parameter τ can be estimated as in Section B.2 of [1], exactly as in the case of an elevator or a rudder. Also see Section B.9. The minus sign in (1) accounts for the fact that a positive aileron deflection *decreases* the lift generated by the right wing.

Substituting $\Delta\text{Lift}'(y)$ into our expression for ΔL, and normalizing by $P_{\text{dyn}} S b$, we obtain the increment in roll moment coefficient due to an aileron deflection:

$$\begin{aligned} C_{l_{\text{aileron}}} &= \frac{1}{P_{\text{dyn}} S b}\left(-2 \int_{y_1}^{y_2} y \left(-\left(C_{L_{\alpha_w}} \tau \delta a\right) P_{\text{dyn}} c(y)\right) dy\right) \\ &= \frac{2 C_{L_{\alpha_w}} \tau \delta a}{S b} \int_{y_1}^{y_2} c(y) y dy. \end{aligned}$$

Therefore, the aileron power is

$$C_{l_{\delta a}} = \frac{\partial C_l}{\partial \delta a} = \frac{2C_{L_{\alpha_w}}\tau}{Sb} \int_{y_1}^{y_2} c(y)ydy.$$

The aileron power can be increased by making the flap chord larger (which increases τ), by placing it farther outboard (which increases $\int_{y_1}^{y_2} c(y)ydy$), or by increasing its span (which also increases $\int_{y_1}^{y_2} c(y)ydy$).

At this point, we have described the primary effects of the lateral directional control surfaces. Namely, we have described how the rudder exerts a yaw moment and how the ailerons exert a roll moment. In addition, these actuators have secondary effects which couple them together. For example, in discussing the necessity of a rudder, we mentioned the *adverse yaw* that results from aileron deflections. The contribution of the ailerons to yaw moment is captured by the term $C_{n_{\delta a}}\delta a$. Conversely, a rudder deflection causes a small roll moment $C_{l_{\delta r}}\delta r$ which must be countered by the ailerons.

References

[1] B. Etkin and L. D. Reid. *Dynamics of Flight: Stability and Control*. John Wiley and Sons, New York, NY, third edition, 1996.