Lecture 7: Directional Stability & Control

With the exception of steady turning flight at constant radius, we have only considered wings level flight. In this setting, the three lateral-directional velocities (v, p, and r) are initially zero and they remain zero under the assumption that there are no lateral-directional disturbance forces or torques to perturb their values. In reality, there are always disturbances. A major design objective, therefore, is to provide *lateral-directional stability* through careful choice of available parameters (vertical tail size and location, wing dihedral and sweep angle, etc.). Following the textbook, we will divide our discussion of lateral-directional stability into two topics: directional (or "weathervane") stability and roll stability. We first discuss directional stability.

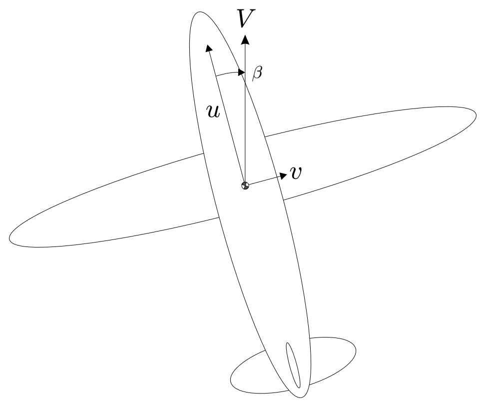

Figure 1: Top view of an aircraft in sideslip.

Directional stability. Note that the assumption that v is zero implies that the sideslip angle $\beta = \sin^{-1}\frac{v}{V}$ is zero. If a yaw disturbance occurs which changes the value of β, then we would prefer that the airplane naturally "reject" this disturbance by driving β back toward zero. This is the purpose of the vertical stabilizer.

A conventional vertical stabilizer is a vertical lifting surface which produces a lateral force in response to its "angle of attack," which is a function of the sideslip angle. This force acts through a moment arm to produce a yaw moment about the airplane's center of gravity in response to sideslip. Let l_{vt} denote the longitudinal distance from the center of gravity to the aerodynamic center of the vertical tail. The yaw moment due to sideslip is

$$N = N_{\text{wb}} - L_{\text{vt}} l_{\text{vt}} \cos\alpha_{\text{vt}} - D_{\text{vt}} l_{\text{vt}} \sin\alpha_{\text{vt}}$$

where α_{vt} is the angle from the longitudinal axis to the relative air velocity at the vertical tail. In [1], tail lift is defined to be positive when the resulting side force on the aircraft is positive. Thus, the sign of α_{vt} is opposite that of β. The contribution N_{wb} of the wing-body to the yaw moment vanishes at zero sideslip angle and typically makes a destabilizing contribution when $\beta \neq 0$.

Making the usual assumptions that α_{vt} is small and that $D_{\text{vt}} \ll L_{\text{vt}}$, and normalizing by $\left(\frac{1}{2}\rho V^2\right) Sb$, we obtain

$$\begin{aligned} C_n &= C_{n_{\text{wb}}} + C_{n_{\text{vt}}} && (1) \\ &\approx C_{n_{\text{wb}}} - \frac{1}{\left(\frac{1}{2}\rho V^2\right) Sb}\left(C_{L_{\text{vt}}}\left(\frac{1}{2}\rho V_{\text{vt}}^2\right) S_{\text{vt}}\right) l_{\text{vt}} \\ &= C_{n_{\text{wb}}} - \mathcal{V}_{\text{V}}\left(\frac{V_{\text{vt}}}{V}\right)^2 C_{L_{\text{vt}}} && (2) \end{aligned}$$

where

$$\mathcal{V}_{\text{V}} = \frac{S_{\text{vt}} l_{\text{vt}}}{Sb}.$$

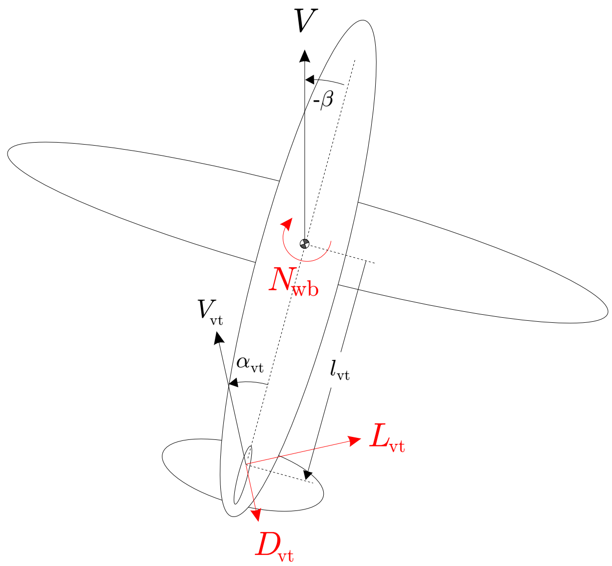

Figure 2: Tail forces and yaw moment on an aircraft in sideslip.

The parameter $\forall_{\mathrm{V}}$ is referred to as the *vertical tail volume ratio*. In analogy with the horizontal tail volume ratio V_{H}, the vertical tail volume ratio governs the effectiveness of the vertical tail at providing directional stability.

A symmetric vertical tail generates no lift when α_{vt} is zero, so

$$C_{L_{\mathrm{vt}}} = C_{L_{\alpha_{\mathrm{vt}}}} \alpha_{\mathrm{vt}}.$$

The angle of attack α_{vt} is

$$\alpha_{\mathrm{vt}} = -\beta + \sigma,$$

where the *sidewash* angle σ results from the asymmetry in the distribution of vorticity generated by the wing. In [1], the sidewash angle is defined to be positive when the induced flow is in the positive y direction. In simple terms, one may observe that, of the two trailing vortices for an airplane in *negative* sideslip (as shown in Figure 2), the trailing vortex from the *left* wing tip will dominate the local flow at the tail. If the vertical tail is mounted level with or above the wings, the induced flow at the vertical tail will have a substantial component in the positive y direction. Thus, a negative sideslip angle will cause a positive sidewash angle. Conversely, a positive sideslip condition ($\beta > 0$) will cause a negative sidewash angle. In both cases, the effect of sidewash on a top-mounted vertical tail is to *increase* the magnitude of the angle of attack α_{vt}, as indicated by the definition above. Note that the sidewash angle σ is a function of the sideslip angle β just as the downwash angle was a function of the wing angle of attack.

For an airplane for which the xz-plane is a plane of symmetry, $C_n = 0$ when $\beta = 0$. Thus, for small β,

$$C_n = C_{n_\beta} \beta$$

where

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}.$$

Referring to (2), we have

$$\begin{aligned} C_n &= C_{n_{\mathrm{wb}}} + C_{n_{\mathrm{vt}}} \\ &= C_{n_{\mathrm{wb}}} - \forall_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V} \right)^2 C_{L_{\alpha_{\mathrm{vt}}}} \left(-\beta + \sigma \right). \end{aligned}$$

Thus, we find

$$C_{n_\beta} = C_{n_{\beta_{\mathrm{wb}}}} + C_{n_{\beta_{\mathrm{vt}}}}$$

where

$$C_{n_{\beta_{\mathrm{vt}}}} = \mathcal{V}_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V}\right)^2 C_{L_{\alpha_{\mathrm{vt}}}} \left(1 - \frac{d\sigma}{d\beta}\right).$$

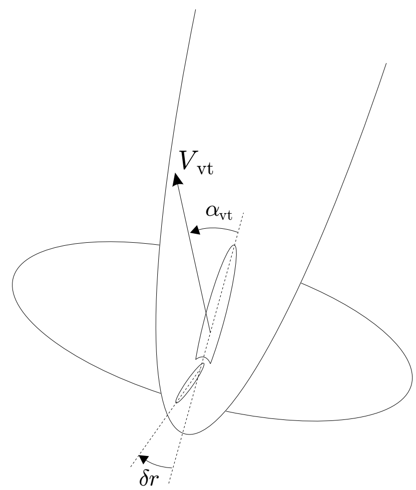

Figure 3: Sketch of a (positively) deflected rudder.

Directional control. Now, suppose that a rudder flap is included on the vertical tail. Rudder deflections δr are defined to be positive when the trailing edge of the flap moves to the left. Recalling that tail lift is defined to be positive when the resulting side force on the aircraft is positive, it is easy to see that a positive rudder deflection will cause a positive increment in tail lift. Thus $C_{L_{\delta r_{\mathrm{vt}}}} > 0$.

The total yaw moment is

$$C_n = C_{n_\beta}\beta + C_{n_{\delta r}}\delta r,$$

where $C_{n_{\delta r}}$ is the *rudder power*. To determine the rudder power, we refer once again to (2)

$$C_n = C_{n_{\mathrm{wb}}} + \mathcal{V}_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V}\right)^2 C_{L_{\mathrm{vt}}}.$$

With the rudder included, we now have

$$C_{L_{\mathrm{vt}}} = C_{L_{\alpha_{\mathrm{vt}}}}\alpha_{\mathrm{vt}} + C_{L_{\delta r_{\mathrm{vt}}}}\delta r$$

It follows by the previous argument that

$$C_n = C_{n_{\mathrm{wb}}} - \mathcal{V}_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V}\right)^2 \left(C_{L_{\alpha_{\mathrm{vt}}}}\alpha_{\mathrm{vt}} + C_{L_{\delta r_{\mathrm{vt}}}}\delta r\right),$$

from which we can easily see that

$$C_{n_{\delta r}} = -\mathcal{V}_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V}\right)^2 C_{L_{\delta r_{\mathrm{vt}}}}.$$

Note that the rudder power is negative which is consistent with intuition given the sign convention for rudder deflections. (By convention, a positive rudder deflection results in a negative increment in yaw moment, causing the airplane to nose left.)

To summarize, the total yaw moment due to steady sideslip and rudder deflections is

$$C_n = C_{n_\beta}\beta + C_{n_{\delta r}}\delta r,$$

where

$$C_{n_\beta} = C_{n_{\beta_{\mathrm{wb}}}} + \mathcal{V}_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V}\right)^2 C_{L_{\alpha_{\mathrm{vt}}}} \left(1 - \frac{d\sigma}{d\beta}\right)$$

and

$$C_{n_{\delta r}} = -\mathcal{V}_{\mathrm{V}} \left(\frac{V_{\mathrm{vt}}}{V}\right)^2 C_{L_{\delta r_{\mathrm{vt}}}}.$$

Rudder considerations. The vertical tail alone tends to direct the nose of the aircraft into the wind, thus driving the sideslip angle to zero. So why does one need a rudder and how does one decide how large to make it? There are several considerations that require the use of a rudder and that dictate its size.

1. *Adverse yaw.* The outer (upper) wing of an airplane which is performing a banked turn generates more lift and therefore more drag than the inner wing. This results in a yaw moment which is counter to the desired turn direction. The rudder must counter this adverse yaw moment in order to execute a coordinated turn.

2. *Asymmetric power effects.*

   (a) *Propeller effects.*

   - For an airplane with one or more propellers rotating in one direction, there will be a steady yaw torque generated which must be countered by the rudder. In particular, for an airplane flying at low speed (and thus at high angle of attack), each propeller will generate a yaw moment due to the differential angle of attack seen by the downward-moving and upward-moving propeller blades. The downward moving blade sees a higher angle of attack and therefore generates more thrust which creates a yaw moment.
   - The reaction torque from the propeller shaft onto the airplane causes a negative roll moment which must be countered by the ailerons (still to be discussed). A side effect of the compensatory roll torque is a yaw moment due to the drag differential between the wings; the phenomenon is essentially identical to the case of "adverse yaw" described above.
   - The swirling flow from the propeller causes additional sidewash at the tail which generates a yaw moment.

   (b) *Engine out.* A multi-engine aircraft flying at low speed must be able to maintain equilibrium flight with a single engine failure. The rudder must be sizable enough to counter the yaw moment resulting from this asymmetric flight condition.

3. *Cross wind landings.* In order to align an aircraft's longitudinal axis with the runway on final approach when landing in a cross wind, it is necessary to maintain a steady sideslip angle for some period of time. A nonzero rudder deflection is necessary to balance the resulting yaw moment due to sideslip.

Crude Rudder Sizing for Asymmetric Thrust. Suppose a two engine airplane has lost its left engine. For the airplane to maintain equilibrium flight, it is necessary for the rudder to counter the yaw moment generated by the thrust of the good engine. Assuming that thrust is parallel to longitudinal axis, the moment about the airplane center of gravity is

$$N_T = -T y_p,$$

where y_p is the lateral distance to the line of action of thrust. Thrust is often given in terms of a dimensionless coefficient C_T:

$$T = C_T \left(\frac{1}{2} \rho V^2 \right) S_p,$$

where S_p is an appropriate reference area for the engine (as specified by the manufacturer). The contribution of thrust to the yaw moment coefficient is

$$\begin{aligned} C_{n_T} &= \frac{1}{\left(\frac{1}{2}\rho V^2\right) S b} \left(-T y_p\right) \\ &= -C_T \frac{S_p y_p}{S b}. \end{aligned}$$

The total yaw moment coefficient is then

$$C_n = C_{n_\beta} \beta + C_{n_{\delta r}} \delta r + C_{n_T}.$$

For equilibrium flight, we require that $C_n = 0$. Moreover, to minimize drag the airplane should fly at zero sideslip angle: $\beta = 0$. Suppose that the rudder deflection satisfies $-(\delta_r)_{\max} \leq \delta_r \leq (\delta_r)_{\max}$ for some maximum rudder deflection $(\delta_r)_{\max}$. Then, designing for the critical case of an engine failure, the rudder should be sized such that

$$C_{n_{\delta r}} \leq -\frac{C_{n_T}}{(\delta_r)_{\max}}.$$

This is a relatively crude analysis. We will refine the condition once we have discussed roll stability and control.

References

[1] B. Etkin and L. D. Reid. *Dynamics of Flight: Stability and Control*. John Wiley and Sons, New York, NY, third edition, 1996.