Lecture 16: Longitudinal Stability Derivatives

The term *stability derivative* arises from the linearization of the aerodynamic terms in the nonlinear dynamic equations. The connection to "stability" comes from the role these terms play in the stability of wings-level equilibrium flight. As an example, recall that the linearized aerodynamic force in the X-direction is

$$X = X_0 + \Delta X = X_0 + (X_u \Delta u + X_w \Delta w + X_{\delta e} \Delta \delta e + X_{\delta T} \Delta \delta T)$$

where

$$X_u = \left. \frac{\partial X}{\partial u} \right|_0$$

and so on. The subscript "0" indicates that the partial derivative is evaluated at the nominal, equilibrium flight condition. The term X_u, and similar terms, are sometimes referred to as the "dimensional stability derivatives" because these terms have physical dimensions. The shorter phrase "stability derivative" is typically reserved for the nondimensional form of these terms. We now turn to the problem of relating stability derivatives to the physical properties of a given aircraft.

The u-derivatives: X_u, Z_u, and M_u. We first compute the longitudinal stability derivatives with respect to u. Aerodynamic forces, such as X, are conventionally represented in coefficient form:

$$X = \left(\frac{1}{2}\rho V^2\right) S C_X$$

where $V^2 = u^2 + v^2 + w^2$. Thus, an aerodynamic force depends on u in two ways: through the dynamic pressure and through the (not necessarily constant) aerodynamic coefficient. In the case of the force X, we have

$$\begin{aligned} X_u &= \left[\rho u S C_X + \left(\frac{1}{2}\rho V^2\right) S \frac{\partial C_X}{\partial u}\right]_0 \\ &= \rho u_0 S C_{X_0} + \left(\frac{1}{2}\rho u_0^2\right) S \left.\frac{\partial C_X}{\partial u}\right|_0 \\ &= \left(\frac{1}{2}\rho u_0^2\right) S \left(\frac{2 C_{X_0}}{u_0} + \left.\frac{\partial C_X}{\partial u}\right|_0\right). \end{aligned}$$

Define the nondimensional term

$$C_{X_u} = \left.\frac{\partial C_X}{\partial (u/u_0)}\right|_0 = u_0 \left.\frac{\partial C_X}{\partial u}\right|_0 .$$

With this definition, we may write

$$\begin{aligned} X_u &= \left(\frac{1}{2}\rho u_0^2\right) S \left(\frac{2 C_{X_0}}{u_0} + \frac{C_{X_u}}{u_0}\right) \\ &= \left(\frac{1}{2}\rho u_0\right) S \left(2 C_{X_0} + C_{X_u}\right). \end{aligned} \quad (1)$$

Similarly, we have

$$Z_u = \left(\frac{1}{2}\rho u_0\right) S \left(2 C_{Z_0} + C_{Z_u}\right) \qquad \text{where} \qquad C_{Z_u} = \left.\frac{\partial C_Z}{\partial (u/u_0)}\right|_0 . \quad (2)$$

Considering the aerodynamic pitching moment

$$M = \left(\frac{1}{2}\rho V^2\right) S \bar{c} C_m,$$

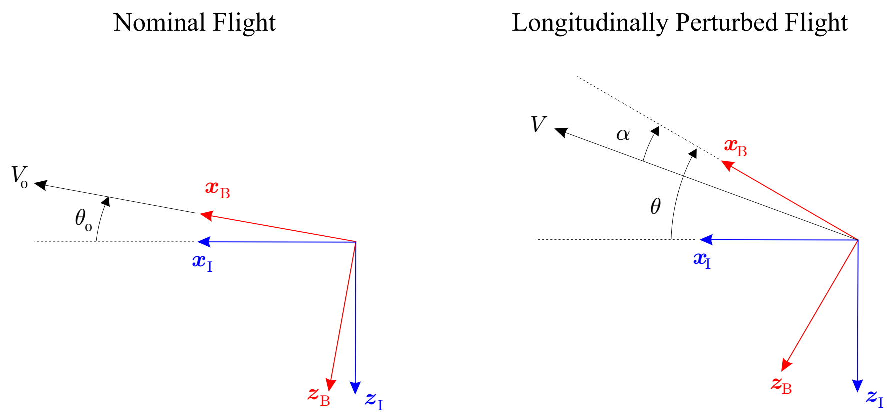

Figure 1: Stability Axes.

we also have

$$M_u = \left(\frac{1}{2}\rho u_0\right) S\bar{c}\,(2C_{m_0} + C_{m_u}) \qquad \text{where} \qquad C_{m_u} = \left.\frac{\partial C_m}{\partial (u/u_0)}\right|_0. \tag{3}$$

We must next relate the body frame forces X and Z to the more familiar aerodynamic forces lift L, drag D, and thrust T. Recall that we are working in stability axes, a body-fixed coordinate frame defined such that the longitudinal ($\boldsymbol{x}_B$) axis is aligned with the velocity vector *when the airplane is in its nominal, wings-level equilibrium condition*. Of course, this frame moves with the aircraft when the flight condition is perturbed.

Consider first the case of nominal flight. For simplicity, we also assume that thrust and drag are aligned with the $\boldsymbol{x}_B$ axis and that lift acts in the negative $\boldsymbol{z}_B$ direction.[1] In nominal flight, we have

$$\begin{aligned} X_0 &= T_0 - D_0 && = mg\sin\theta_0 \\ Z_0 &= -L_0 && = -mg\cos\theta_0 \\ M_0 &= 0 \end{aligned}$$

Nondimensionalizing gives

$$\begin{aligned} C_{X_0} &= C_{T_0} - C_{D_0} && = C_{W_0}\sin\theta_0 \\ C_{Z_0} &= -C_{L_0} && = -C_{W_0}\cos\theta_0 \\ C_{m_0} &= 0 \end{aligned}$$

(For convenience, we have assumed that the reference area for the thrust coefficient is the wing planform area $S_p = S$.)

Now suppose we allow small perturbations in speed and angle of attack; see Figure 1. In the general case, we have

$$\begin{aligned} X &= -D\cos\alpha + L\sin\alpha + T && \approx -D + L\alpha + T \\ Z &= -D\sin\alpha - L\cos\alpha && \approx -D\alpha - L. \end{aligned}$$

Dividing through by dynamic pressure and area gives

$$\begin{aligned} C_X &= -C_D + C_L\alpha + C_T \\ C_Z &= -C_D\alpha - C_L. \end{aligned}$$

[1] This is not an especially good assumption, but it simplifies the discussion. The more general case simply involves some geometry.

The (nondimensional) stability derivative C_{X_u} is obtained by computing

$$\begin{aligned} C_{X_u} &= \left[u_0 \frac{\partial}{\partial u} \left(-C_D + C_L \alpha + C_T \right) \right]_0 \\ &= \left[-u_0 \frac{\partial C_D}{\partial u} + u_0 \frac{\partial C_L}{\partial u} \alpha + u_0 C_L \frac{\partial \alpha}{\partial u} + u_0 \frac{\partial C_T}{\partial u} \right]_0 . \end{aligned}$$

Recall that

$$\tan \alpha = \frac{w}{u},$$

so

$$\sec^2 \alpha \frac{\partial \alpha}{\partial u} = -\frac{w}{u^2} = -\frac{1}{u} \tan \alpha.$$

Solving for $\frac{\partial \alpha}{\partial u}$ gives

$$\frac{\partial \alpha}{\partial u} = -\frac{1}{u} \sin \alpha \cos \alpha = -\frac{1}{2u} \sin 2\alpha.$$

We therefore have

$$\begin{aligned} C_{X_u} &= \left[-u_0 \frac{\partial C_D}{\partial u} + u_0 \frac{\partial C_L}{\partial u} \alpha + C_L \left(-\frac{u_0}{2u} \sin 2\alpha \right) + u_0 \frac{\partial C_T}{\partial u} \right]_0 \\ &= -u_0 \left. \frac{\partial C_D}{\partial u} \right|_0 + u_0 \left. \frac{\partial C_T}{\partial u} \right|_0 . \end{aligned}$$

Similarly, we find that

$$\begin{aligned} C_{Z_u} &= \left[u_0 \frac{\partial}{\partial u} \left(-C_D \alpha - C_L \right) \right]_0 \\ &= \left[-u_0 \frac{\partial C_D}{\partial u} \alpha - u_0 C_D \left(-\frac{u_0}{2u} \sin 2\alpha \right) - u_0 \frac{\partial C_L}{\partial u} \right]_0 \\ &= -u_0 \left. \frac{\partial C_L}{\partial u} \right|_0 . \end{aligned}$$

Defining

$$\begin{aligned} C_{D_u} &= \left. \frac{\partial C_D}{\partial (u/u_0)} \right|_0 &= u_0 \left. \frac{\partial C_D}{\partial u} \right|_0 \\ C_{L_u} &= \left. \frac{\partial C_L}{\partial (u/u_0)} \right|_0 &= u_0 \left. \frac{\partial C_L}{\partial u} \right|_0 \\ C_{T_u} &= \left. \frac{\partial C_T}{\partial (u/u_0)} \right|_0 &= u_0 \left. \frac{\partial C_T}{\partial u} \right|_0 , \end{aligned}$$

we have

$$\begin{aligned} C_{X_u} &= -C_{D_u} + C_{T_u} \\ C_{Z_u} &= -C_{L_u} . \end{aligned}$$

Summarizing the development to this point:

$$
\begin{aligned}
X_u &= \left(\frac{1}{2}\rho u_0\right) S \left[2\left(-C_{D_0} + C_{T_0}\right) + \left(-C_{D_u} + C_{T_u}\right)\right] \\
&= \left(\frac{1}{2}\rho u_0\right) S \left[2C_{W_0}\sin\theta_0 + \left(-C_{D_u} + C_{T_u}\right)\right] \\
Z_u &= \left(\frac{1}{2}\rho u_0\right) S \left(-2C_{L_0} - C_{L_u}\right) \\
&= \left(\frac{1}{2}\rho u_0\right) S \left(-2C_{W_0}\cos\theta_0 - C_{L_u}\right) \\
M_u &= \left(\frac{1}{2}\rho u_0\right) S\bar{c}\left(C_{m_u}\right)
\end{aligned}
$$

It still remains for us to determine how C_D, C_T, and C_L depend on u. In [1], the authors claim that the dependence arises primarily through three mechanisms: the type of propulsion, compressibility effects, and aeroelastic effects. We will consider only the first two.

First, consider the thrust coefficient

$$
C_T = \frac{T}{\left(\frac{1}{2}\rho V^2\right) S}.
$$

We compute

$$
\begin{aligned}
C_{T_u} &= u_0 \left[\frac{1}{\left(\frac{1}{2}\rho V^2\right) S}\frac{\partial T}{\partial u} - \frac{2T}{\left(\frac{1}{2}\rho V^3\right) S}\frac{\partial V}{\partial u}\right]_0 \\
&= \frac{1}{\left(\frac{1}{2}\rho u_0\right) S}\left.\frac{\partial T}{\partial u}\right|_0 - 2C_{T_0}.
\end{aligned}
$$

The variation of thrust with u depends on the type of engine and/or mode of flight. For unpowered flight (e.g., for a sailplane), thrust is identically zero so that $C_{T_u} = 0$. For *constant thrust* propulsion (e.g., for a jet aircraft in cruising flight), thrust does not vary with speed so that $C_{T_u} = -2C_{T_0}$. For *constant power* propulsion (e.g., for propeller driven aircraft), the propulsive power Tu does not vary with speed. In this case, we compute

$$
\frac{\partial}{\partial u}(Tu) = 0 = \frac{\partial T}{\partial u}u + T \qquad \Rightarrow \qquad \left[\frac{\partial T}{\partial u}\right]_0 = -\frac{1}{u_0}T_0
$$

so that $C_{T_u} = -3C_{T_0}$.

Propulsion Type	C_{T_u}
Zero Thrust	0
Constant Thrust	$-2C_{T_0}$
Constant Power	$-3C_{T_0}$

The variation of C_D, C_L, and C_m with u is primarily due to compressibility effects which are characterized by the Mach number Ma. Consider a general nondimensional coefficient C_A which depends on u through its dependence on Mach number. Since

$$
\mathrm{Ma} = \frac{V}{a},
$$

where a is the speed of sound (assumed to remain constant), we may compute

$$
\left.\frac{\partial C_A}{\partial u}\right|_0 = \left[\frac{\partial C_A}{\partial \mathrm{Ma}}\frac{\partial \mathrm{Ma}}{\partial u}\right]_0 = \frac{1}{a}\left.\frac{\partial C_A}{\partial \mathrm{Ma}}\right|_0.
$$

Multiplying through by u_0 to nondimensionalize, we obtain

$$C_{A_u} = \left.\frac{\partial C_A}{\partial(u/u_0)}\right|_0 = u_0 \left.\frac{\partial C_A}{\partial u}\right|_0 = \frac{u_0}{a} \left.\frac{\partial C_A}{\partial \mathrm{Ma}}\right|_0 = \mathrm{Ma}_0 \left.\frac{\partial C_A}{\partial \mathrm{Ma}}\right|_0 .$$

In the case of the longitudinal stability derivatives, and omitting the subscript '0', we have

$$\boxed{C_{D_u} = \frac{\partial C_D}{\partial \mathrm{Ma}}\,\mathrm{Ma} \qquad C_{L_u} = \frac{\partial C_L}{\partial \mathrm{Ma}}\,\mathrm{Ma} \qquad \text{and} \qquad C_{m_u} = \frac{\partial C_m}{\partial \mathrm{Ma}}\,\mathrm{Ma}.}$$

Explicit formulas for the partial derivatives on the right can be obtained from inviscid flow theory; see [1], for example.

It should be pointed out, once again, that aeroelasticity can have an important effect on stability derivatives. The deformations in the fuselage and lifting surfaces which occur as a consequence of changes in speed, can dramatically affect the aircraft's dynamics and stability. These considerations are, however, beyond the scope of this course.

Aside: The phugoid approximation revisited. Recall that in the previous lecture we used Lanchester's assumptions to obtain the following approximate expressions for the natural frequency and damping ratio of the phugoid mode:

$$\omega_{n_\mathrm{P}} \approx \sqrt{-\frac{Z_u g}{m u_0}} \qquad \text{and} \qquad \zeta_\mathrm{P} \approx -\frac{X_u}{2m\omega_{n_\mathrm{P}}}.$$

It was claimed (without proof) that under certain assumptions, these expressions simplify to

$$\omega_{n_\mathrm{P}} \approx \sqrt{2}\frac{g}{u_0} \qquad \text{and} \qquad \zeta_\mathrm{P} \approx \frac{1}{\sqrt{2}}\frac{1}{L/D}.$$

The assumptions are that

- the aircraft is rigid (i.e., no aeroelastic effects),
- the flow is incompressible (i.e., no Mach number effects),
- the propulsion system is a constant-thrust system (i.e., $C_{T_u} = -2C_{T_0}$), and
- the nominal equilibrium flight condition is constant-altitude flight (i.e., $\theta_0 = 0$).

Under these assumptions, we have

$$X_u = -\rho u_0 S C_{T_0} \qquad \text{and} \qquad Z_u = -\rho u_0 S C_{W_0}.$$

We therefore compute

$$\omega_{n_\mathrm{P}} \approx \sqrt{-\frac{Z_u g}{m u_0}} = \sqrt{-\frac{(-\rho u_0 S C_{W_0})\, g}{m u_0}} = \sqrt{\frac{\rho u_0 S \left(\frac{mg}{(\frac{1}{2}\rho u_0^2)S}\right) g}{m u_0}} = \sqrt{2}\frac{g}{u_0}$$

Turning now to the damping ratio, we compute

$$\zeta_\mathrm{P} \approx -\frac{X_u}{2m\omega_{n_\mathrm{P}}} = -\frac{(-\rho u_0 S C_{T_0})}{2m\left(\sqrt{2}\frac{g}{u_0}\right)} = \frac{1}{\sqrt{2}}\frac{C_{T_0}}{\left(\frac{mg}{(\frac{1}{2}\rho u_0^2)S}\right)} = \frac{1}{\sqrt{2}}\frac{C_{T_0}}{C_{W_0}}.$$

In nominal, constant-altitude flight, $C_{T_0} = C_{D_0}$ and $C_{W_0} = C_{L_0}$ so we find that

$$\zeta_{\rm P} \approx \frac{1}{\sqrt{2}} \frac{1}{C_{L_0}/C_{D_0}} = \frac{1}{\sqrt{2}} \frac{1}{L_0/D_0},$$

as claimed. Once we've completed our discussion of longitudinal dimensional stability derivatives, we will compute the *actual* phugoid natural frequency and damping ratio and see how well this approximation compares. We will also develop an approximate expression for the short period mode and see how well it compares with reality.

The w-derivatives. Equivalently, the α-derivatives: X_α, Z_α, and M_α. Recall that the state variables of interest in studying longitudinal dynamics are u, w, q, and θ. (We ignore x and z, which are irrelevant to the question of dynamic stability.) Logically, it would seem that we should next study the dependence of X, Z, and M on the plunge rate w. Equivalently, we may study the dependence of X, Z, and M on the angle of attack α. Recall once again that

$$\tan\alpha = \frac{w}{u}.$$

Differentiating with respect to w gives

$$\sec^2\alpha \frac{\partial\alpha}{\partial w} = \frac{1}{u}$$

or

$$\frac{\partial\alpha}{\partial w} = \frac{1}{u}\cos^2\alpha.$$

For small perturbations from nominal flight, we have

$$\frac{\partial\alpha}{\partial w} \approx \frac{1}{u_0}.$$

Thus, for the linearized equations at least, we may write

$$\frac{\partial}{\partial\alpha} = u_0 \frac{\partial}{\partial w}.$$

Stability derivatives may appear either with respect to α or with respect to w; for small perturbations, the two are directly proportional. In particular, we have

$$X_\alpha = u_0 X_w, \qquad Z_\alpha = u_0 Z_w, \qquad \text{and} \qquad M_\alpha = u_0 M_w.$$

Define the nondimensional coefficients

$$C_{X_\alpha} = \left.\frac{\partial C_X}{\partial\alpha}\right|_0, \qquad C_{Z_\alpha} = \left.\frac{\partial C_Z}{\partial\alpha}\right|_0, \qquad \text{and} \qquad C_{m_\alpha} = \left.\frac{\partial C_m}{\partial\alpha}\right|_0$$

and recall the force coefficient approximations

$$\begin{aligned} C_X &= -C_D + C_L\alpha + C_T \\ C_Z &= -C_D\alpha - C_L. \end{aligned}$$

One may compute

$$\begin{aligned} C_{X_\alpha} &= \frac{\partial}{\partial\alpha}\left[-C_D + C_L\alpha + C_T\right]_0 \\ &= \left[-C_{D_\alpha} + C_L + C_{L_\alpha}\alpha + \frac{\partial C_T}{\partial\alpha}\right]_0 \\ &= -C_{D_\alpha} + C_{L_0}, \end{aligned}$$

where we assume that thrust does not vary with angle of attack.

Turning to the Z force coefficient, we find that

$$\begin{aligned} C_{Z_\alpha} &= \left[-\left(C_D + \frac{\partial C_D}{\partial \alpha}\alpha + C_{L_\alpha} \right) \right]_0 \\ &= -\left(C_{D_0} + C_{L_\alpha} \right). \end{aligned}$$

Summarizing the force coefficients:

$$\boxed{\begin{aligned} C_{X_\alpha} &= -C_{D_\alpha} + C_{L0} \\ C_{Z_\alpha} &= -C_{D_0} - C_{L_\alpha} \end{aligned}}$$

Of course, we have already spent a great deal of time discussing the term C_{m_α}.

$$\boxed{\begin{aligned} C_{m_\alpha} &= C_{m_{\alpha w}} + C_{m_{\alpha t}} + C_{m_{\alpha f}} + C_{m_{\alpha p}} \\ &= C_{L_{\alpha w}}\left(h - h_{\mathrm{ac}_w}\right) - V_{\mathrm{H}} C_{L_{\alpha t}}\left(1 - \frac{d\epsilon}{d\alpha}\right) + C_{m_{\alpha f}} + C_{m_{\alpha p}} \\ &= C_{L_\alpha}\left(h - h_n\right). \end{aligned}}$$

Re-dimensionalizing, we have

$$\boxed{\begin{aligned} X_\alpha &= C_{X_\alpha}\left(\frac{1}{2}\rho u_0^2\right) S \\ &= \left(-C_{D_\alpha} + C_{L0}\right)\left(\frac{1}{2}\rho u_0^2\right) S \\ Z_\alpha &= C_{Z_\alpha}\left(\frac{1}{2}\rho u_0^2\right) S \\ &= \left(-C_{D_0} - C_{L_\alpha}\right)\left(\frac{1}{2}\rho u_0^2\right) S \\ M_\alpha &= C_{m_\alpha}\left(\frac{1}{2}\rho u_0^2\right) S\bar{c} \\ &= \left(C_{L_\alpha}\left(h - h_n\right)\right)\left(\frac{1}{2}\rho u_0^2\right) S\bar{c} \end{aligned}}$$

The force and moment derivatives X_w, Z_w, and M_w can be computed using the approximate relation $w = u_0\alpha$:

$$X_w = \frac{1}{u_0}X_\alpha, \qquad Z_w = \frac{1}{u_0}Z_\alpha, \qquad \text{and} \qquad M_w = \frac{1}{u_0}M_\alpha.$$

The q-derivatives: Z_q and M_q. We have already considered stability derivatives related to pitch rate in a previous lecture. Those results are restated here. Define the nondimensional terms

$$\begin{aligned} C_{L_q} = \frac{\partial C_L}{\partial \hat{q}} &= \frac{2u_0}{\bar{c}}\frac{\partial C_L}{\partial q} \\ C_{m_q} = \frac{\partial C_m}{\partial \hat{q}} &= \frac{2u_0}{\bar{c}}\frac{\partial C_m}{\partial q}. \end{aligned}$$

Considering the increment in lift generated by the tail due to a nonzero pitch rate, one finds that

$$\boxed{\begin{aligned} C_{Z_q} &= -C_{L_q} = -2C_{L_{\alpha_t}} \mathcal{V}_{\mathrm{H}} \\ C_{m_q} &= \qquad\quad = -2kC_{L_{\alpha_t}} \mathcal{V}_{\mathrm{H}} \frac{l_t}{\bar{c}}, \end{aligned}}$$

where k corrects for the contribution to pitch damping from the wing. (Recall that one may generally choose $k \approx 1.1$.) Re-dimensionalizing, we have

$$\boxed{\begin{aligned} Z_q &= \frac{\bar{c}}{2u_0} C_{Z_q} \left(\frac{1}{2}\rho u_0^2\right) S \\ M_q &= \frac{\bar{c}}{2u_0} C_{m_q} \left(\frac{1}{2}\rho u_0^2\right) S\bar{c}. \end{aligned}}$$

The $\dot{w}$-derivatives. Equivalently, the $\dot{\alpha}$-derivatives: $Z_{\dot{\alpha}}$ and $M_{\dot{\alpha}}$. Recall that, for small angles of attack, we may change variables by replacing w everywhere with $u_0\alpha$. Similarly, we may replace $\dot{w}$ everywhere with $u_0\dot{\alpha}$. Stability derivatives with respect to $\dot{w}$ appeared only in the Z-force and M-moment equations. These terms account for the fact that there is a time delay before any change in downwash generated by the wing is felt at the tail. That delay is approximately $\Delta t = \frac{l_t}{u_0}$. The downwash at the tail at an instant t corresponds to the wing angle of attack at a previous instant $t - \Delta t$. Therefore the deviation of the downwash from its nominal value at an instant t corresponds to the deviation of the wing angle of attack from its nominal value at $t - \Delta t$. Treating ϵ as an explicit function of time, we have

$$\Delta\epsilon = \epsilon(t) - \epsilon(t - \Delta t).$$

Of course, ϵ is actually a function of α (which is, itself, a function of time) so that $\epsilon = \epsilon(\alpha(t))$. We therefore have

$$\Delta\epsilon = \frac{\partial\epsilon}{\partial\alpha}\Delta\alpha = \frac{\partial\epsilon}{\partial\alpha}(\alpha(t) - \alpha(t - \Delta t)).$$

Multiplying the right-hand side by $\frac{\Delta t}{\Delta t}$ and assuming that $\frac{l_t}{u_0}$ is small, we may write

$$\begin{aligned} \Delta\epsilon &= \frac{\partial\epsilon}{\partial\alpha}\Delta\alpha \\ &= \frac{\partial\epsilon}{\partial\alpha}\dot{\alpha}\Delta t \\ &= \frac{\partial\epsilon}{\partial\alpha}\dot{\alpha}\frac{l_t}{u_0}. \end{aligned}$$

This is precisely the change in the tail angle of attack due to $\dot{\alpha}$:

$$\Delta\alpha_t = \Delta\epsilon = \frac{\partial\epsilon}{\partial\alpha}\dot{\alpha}\frac{l_t}{u_0}.$$

$$\boxed{\begin{aligned} C_{Z_{\dot{\alpha}}} &= -C_{L_{\dot{\alpha}}} = -2C_{L_{\alpha_t}} \mathcal{V}_{\mathrm{H}} \frac{\partial\epsilon}{\partial\alpha} \\ C_{m_{\dot{\alpha}}} &= \qquad\quad = -2C_{L_{\alpha_t}} \mathcal{V}_{\mathrm{H}} \frac{l_t}{\bar{c}} \frac{\partial\epsilon}{\partial\alpha} \end{aligned}}$$

Re-dimensionalizing, we have:

$$Z_{\dot{\alpha}} = -\frac{\bar{c}}{2u_0} C_{L_{\dot{\alpha}}} \left(\frac{1}{2}\rho u_0^2\right) S$$
$$M_{\dot{\alpha}} = \frac{\bar{c}}{2u_0} C_{m_{\dot{\alpha}}} \left(\frac{1}{2}\rho u_0^2\right) S\bar{c}$$

The force and moment derivatives $Z_{\dot{w}}$ and $M_{\dot{w}}$ can be computed using the approximate relation $\dot{w} = u_0\dot{\alpha}$:

$$\frac{\partial}{\partial\dot{\alpha}} = \frac{\partial}{\partial(\dot{w}/u_0)} = u_0\frac{\partial}{\partial\dot{w}}.$$

Thus,

$$Z_{\dot{w}} = \frac{1}{u_0} Z_{\dot{\alpha}} \qquad \text{and} \qquad M_{\dot{w}} = \frac{1}{u_0} M_{\dot{\alpha}}.$$

References

[1] B. Etkin and L. D. Reid. *Dynamics of Flight: Stability and Control*. John Wiley and Sons, New York, NY, third edition, 1996.