Lecture 1: Introductory Remarks

Aerospace engineering encompass a broad range of challenging topics which must be mastered in order to design atmospheric and space flight vehicles. Topics of fundamental importance include

- aerodynamics
- propulsion
- structures and materials, and
- vehicle dynamics and control.

A fifth topic, vehicle design, envelops all of these more basic areas. It involves integrating knowledge in each of the four subject areas in order to synthesize a complete vehicle which satisfies prescribed performance requirements.

This course focuses on *dynamics and control* of atmospheric flight vehicles, particularly fixed-wing aircraft. This topic is also referred to as *flight mechanics*. Flight mechanics comprises three major subtopics:

- performance,
- stability and control, and
- aeroelasticity.

Conventionally, each of these subtopics is studied individually although the three are very much related. In studying aircraft performance, one considers issues such as range, take-off and landing distance, and trajectory planning for a given aircraft. This involves determining the forces necessary to achieve a given path of motion, assuming that these desired forces can be generated. Thus, one typically models the aircraft as a point mass subject to three "control" forces: lift, side force, and thrust. Performance is concerned with the large-scale aircraft motions associated with takeoff, landing, turning, etc.

In studying stability and control, one takes a closer look at the aircraft and recognizes that lift and side force are not *true* control forces. Rather, these forces are a consequence of the aircraft's orientation with respect to the local air flow. To generate a desired lift force, for example, the vehicle must effect a particular angle of attack. Thus, in stability and control, one is concerned with how the vehicle's orientation, or *attitude*, changes under the influence of *moments* generated by the actuators. These moments are typically generated by a pilot through a suitably designed interface (such as a stick and pedals).

In studying performance, one assumes that the aircraft is a point mass. In studying stability and control, one typically assumes that the aircraft is a rigid body. In studying aeroelasticity, one recognizes that no aircraft is truly rigid and, moreover, that changes in the vehicle shape due to varying load conditions can have dramatic effects on the vehicle's motion. Aeroelastic phenomena that can arise for real aircraft include wing or control surface flutter, roll control reversal, and other effects.

In this course, we will consider only the second sub-topic: stability and control. As the course title suggests, there are two issues of primary importance. *Stability* relates to the intrinsic flying qualities of the aircraft. Stability is a characteristic of the vehicle dynamics which is independent of the pilot's actions. *Control* concerns the interaction between a pilot (human or otherwise) and the aircraft.

We will begin by considering stability. To discuss stability of a steady motion, we must first introduce some terminology to describe the motion. Suppose we fix a reference frame to some point in the aircraft, as shown in Figure 1. We denote by $\boldsymbol{x}_{\mathrm{B}}$ the unit vector pointing through the nose of the aircraft. This axis is often referred to as the *longitudinal axis*. We let $\boldsymbol{z}_{\mathrm{B}}$ represent the unit vector pointing through the belly of the aircraft; this is often called the *directional axis*. Finally, we define the *lateral axis* in terms of the unit vector $\boldsymbol{y}_{\mathrm{B}} = \boldsymbol{z}_{\mathrm{B}} \times \boldsymbol{x}_{\mathrm{B}}$. Viewing the aircraft from behind, $\boldsymbol{y}_{\mathrm{B}}$ points to the right.

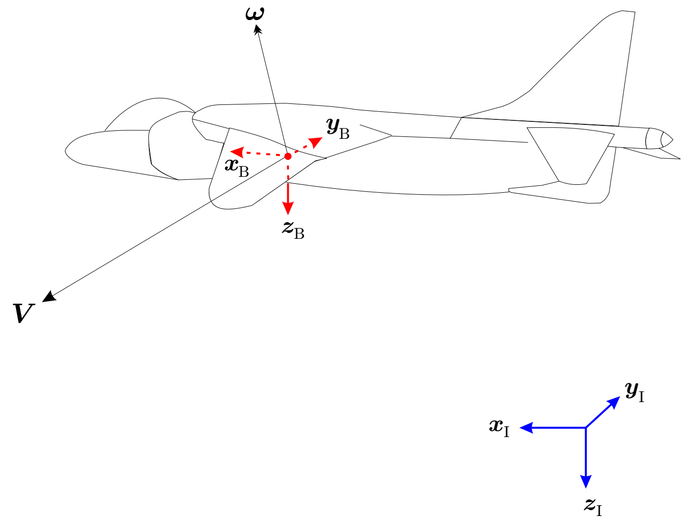

Figure 1: Inertial and body-fixed reference frames.

To describe the orientation of the aircraft, we define an inertial reference frame, which is denoted by the *fixed* unit vectors $\boldsymbol{x}_{\mathrm{I}}$, $\boldsymbol{y}_{\mathrm{I}}$, and $\boldsymbol{z}_{\mathrm{I}}$. The reason we choose to describe the aircraft's orientation with respect to an inertial frame is that Newton's laws of motion only hold in an inertially fixed frame. In this course, we will typically consider an earth-fixed frame to be an "inertial" frame. Although the resulting equations of motion will technically be incorrect, the error due to the earth's rotation, its revolution about the sun, etc. will be small over the time periods of interest in studying stability and control.

As the aircraft is assumed to be rigid, the location of any point in the airplane is uniquely determined by the position and orientation of the body-fixed reference frame. Therefore, we will often represent the aircraft simply by its body-fixed reference frame. Suppose that the aircraft (i.e., the body frame) translates at some velocity with respect to the inertial frame. We let

$$\boldsymbol{V} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

denote the translational velocity of the body with respect to the inertial frame, but *expressed in the body frame*.[1] Also, suppose that the aircraft rotates at some angular velocity with respect to the the inertial frame. We let

$$\boldsymbol{\omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

denote the angular velocity of the body with respect to the inertial frame, but expressed in the body frame.

[1]Note the distinction, here! While any given vector can be expressed in any given reference frame, derivatives are always taken *with respect to a specific frame*.

Axis	Linear Velocity	Aerodynamic Force	Angular Displacement	Angular Velocity	Aerodynamic Moment
Longitudinal ($\boldsymbol{x}_\mathrm{B}$)	u	X	ϕ	p	L
Lateral ($\boldsymbol{y}_\mathrm{B}$)	v	Y	θ	q	M
Directional ($\boldsymbol{z}_\mathrm{B}$)	w	Z	ψ	r	N

The angular displacement variables ϕ, θ, and ψ do not generally represent angles about the body-fixed axes. These angles, referred to as the *Euler angles*, define a series of three rotations which transform vectors from the inertial frame to the body frame, and vice versa. We will discuss the parameterization of vehicle attitude in more detail later in the course. Until then, we will only consider simple motions in which, for example, the pitch angle θ is truly a rotation about the lateral ($\boldsymbol{y}_\mathrm{B}$) axis.

The aerodynamic forces and moments are conventionally denoted in terms of dimensionless coefficients. Let $V = \|\boldsymbol{V}\|$ be the airspeed, let S denote a reference area, and let l denote a reference length. Then one writes

$$
\begin{aligned}
X &= C_X \left(\frac{1}{2}\rho V^2\right) S \\
Y &= C_Y \left(\frac{1}{2}\rho V^2\right) S \\
Z &= C_Z \left(\frac{1}{2}\rho V^2\right) S \\
L &= C_l \left(\frac{1}{2}\rho V^2\right) Sl \\
M &= C_m \left(\frac{1}{2}\rho V^2\right) Sl \\
N &= C_n \left(\frac{1}{2}\rho V^2\right) Sl
\end{aligned}
$$

Note: The reference area is typically chosen to be the wing planform area. Reference lengths may differ depending on the context. For the pitch moment coefficient, for example, one typically takes $l = \bar{c}$, the mean aerodynamic chord. For the roll and yaw moment coefficients, one takes $l = b$, the wing span.

The use of upper-case subscripts in the force coefficients is consistent with the notation for aerodynamic forces. The apparently inconsistent use of lower-case subscripts in the moment coefficients avoids a potential ambiguity between roll moment coefficient and lift coefficient.

The dimensionless coefficients C_X, C_Y, C_Z, C_l, C_m, and C_n, are primarily functions of the Mach number $M = V/a$ (where a is the speed of sound), the Reynolds number $\mathrm{Re} = (\rho V l)/\mu$ (where ρ is the fluid density and μ is the dynamic viscosity), and the aerodynamic angles α and β. Recall that the aerodynamic angles are defined solely in terms of the body translational velocity:

$$\alpha = \arctan\frac{w}{u} \qquad \text{and} \qquad \beta = \arcsin\frac{v}{V}.$$

These angles are shown in Figure 2. In normal flight conditions, the dimensionless coefficients depend primarily on the variables and parameters mentioned above. They also depend on the body angular rate $\boldsymbol{\omega}$ as well as the aerodynamic angle rates $\dot{\alpha}$ and $\dot{\beta}$.

An often-used simplifying assumption is that an aircraft is symmetric about the $\boldsymbol{x}_\mathrm{B}$-$\boldsymbol{z}_\mathrm{B}$ plane. Motions which are restricted to this plane of symmetry, such as wings-level climbs and loops, are called *symmetric* or

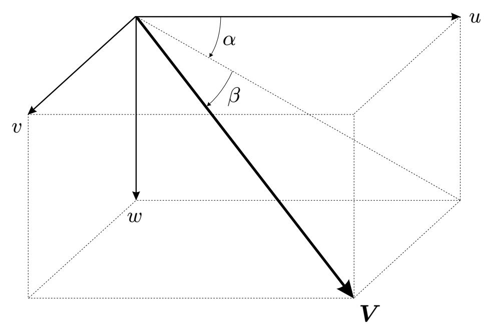

Figure 2: Aerodynamic angles.

longitudinal motions. Motions out of the plane of symmetry, such as banked turns, are called *asymmetric* or *lateral-directional* motions. Accordingly, the components of velocity and aerodynamic force and moment are often decomposed into these two groups:

- **Longitudinal (or symmetric) quantities:** u, w, q, X, Z, M
- **Lateral-directional (or asymmetric) quantities:** v, p, r, Y, L, N

Static longitudinal stability. When discussing flight of atmospheric vehicles, the term "stability" refers to a property of a special class of motion known as *steady motion*. For a vehicle in steady motion, all components of body translational velocity $\boldsymbol{V}$ and body angular velocity $\boldsymbol{\omega}$ are constant. A special case of steady motion is *equilibrium flight*, in which the vehicle acceleration is zero. Note that these two definitions are distinct. Steady, wings-level flight at constant altitude is equilibrium flight. A horizontal turn at constant radius and velocity is not equilibrium flight; the constant yaw rate turn requires a constant centripetal acceleration. Equilibrium flight is a steady motion for which $\boldsymbol{\omega} = \mathbf{0}$.

Stability (or instability) is a property corresponding to a steady motion. Loosely speaking, if a vehicle which is slightly perturbed from a steady motion returns to that steady motion, the motion is *stable*. If the vehicle motion diverges in response to a small perturbation, the motion is *unstable*.

The flight mechanics literature distinguishes between two finer notions of stability: static and dynamic stability. The term static stability is somewhat of a misnomer because, by definition, stability (or instability) refers to a system's *motion* in response to a disturbance. *Static stability* refers to the *initial* tendency of a vehicle, if displaced from a given steady motion, to return to that motion. No information about the vehicle's subsequent motion is required, only its initial tendency. Thus, one may determine static stability without solving the differential equations that describe the airplane's motion. Moreover, one may typically determine static stability without even knowing those equations, that is, without knowing the full mathematical model for the aircraft. The notion of static stability is therefore very important in aircraft design, where the effect of preliminary sizing and configuration decisions on stability must be determined immediately.

Even though a given steady motion may be statically stable, the vehicle may diverge from the given motion with time. To characterize this latter phenomenon, one must consider *dynamic stability* in which the complete vehicle motion, not just its initial motion, is important. A given steady motion is dynamically stable if, after a small displacement, the aircraft returns to the steady motion asymptotically in time. Dynamic stability is stronger than static stability:

$$\text{Dynamic stability} \quad \Rightarrow \quad \text{Static stability} \qquad \text{but} \qquad \text{Static stability} \quad \not\Rightarrow \quad \text{Dynamic stability}$$

For example, a vehicle's state may undergo diverging oscillations about a statically stable steady motion. See Figure 3.

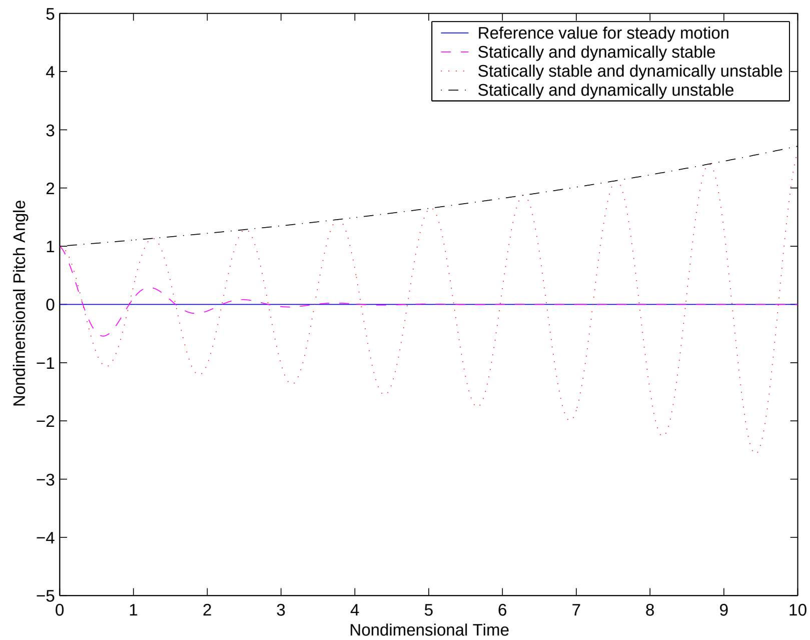

Figure 3: Sketches depicting static and dynamic pitch stability.

We will begin by investigating the conditions under which steady wings level flight is statically stable in *pitch*. Consider a rigid aircraft with a reference frame fixed in the body at its center of gravity (CG) as shown in Figure 1. If the $\boldsymbol{x}_{\mathrm{B}}$-$\boldsymbol{z}_{\mathrm{B}}$ plane is a plane of symmetry, then the pitch rate equation is

$$\dot{q} = \frac{1}{I_y}\left((I_z - I_x)pr + I_{xz}(r^2 - p^2) + M\right), \tag{1}$$

where I_i is the moment of inertia about the i^{th} coordinate axis and I_{xz} is a product of inertia. This is one of three first order ODEs for the body angular rate; there are also equations for p and r. In addition, there are three first order ODEs for the components u, v, and w of translational velocity. These six equations describe the aircraft dynamics. Six more first order ODEs describe the variation of position and attitude due to changes in velocity, that is, the aircraft kinematics.

Equation (1) is a *nonlinear* ordinary differential equation; the dependent variables p and r appear *quadratically*. We consider the case of steady, wings-level flight. In this case, $p = r = 0$. Also, $v = 0$. In the absence of asymmetric disturbances, the lateral-directional variables remain zero; the motion is purely longitudinal. The pitch rate equation becomes simply

$$\dot{q} = \frac{1}{I_y}M = \frac{1}{I_y}\left(\frac{1}{2}\rho V^2 S\bar{c}\right)C_m.$$

The dimensionless coefficient C_m depends on a number of variables and parameters. For pure longitudinal flight, the primary influences are angle of attack α, Reynolds number Re, and Mach number M. To a lesser extent, C_m also depends on q and $\dot{\alpha}$. For now, we will ignore all dependencies save α and write $C_m = C_m(\alpha)$. Formally expanding this expression in a Taylor series about zero angle of attack gives

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + \text{h.o.t.} \tag{2}$$

If we assume that α (given in radians) remains fairly small, then we may neglect the higher order terms in (2). The term C_{m_0} is the pitch moment coefficient at zero angle of attack and

$$C_{m_\alpha} = \left.\frac{\partial C_m}{\partial \alpha}\right|_{\alpha=0}$$

is the slope of the pitch moment coefficient curve. Because of its critical role in determining both static and dynamic stability, C_{m_α} is referred to as a *stability derivative*.

One should keep in mind that (2) is only an approximation. It ignores the dependency of C_m on Re and M, as well as higher order terms in α. Moreover, for fast, asymmetric maneuvers, C_m also depends on $\dot{\alpha}$, β, $\dot{\beta}$, p, q, and r and possibly other variables and parameters. For now, we consider only the case of symmetric (wings-level) equilibrium flight

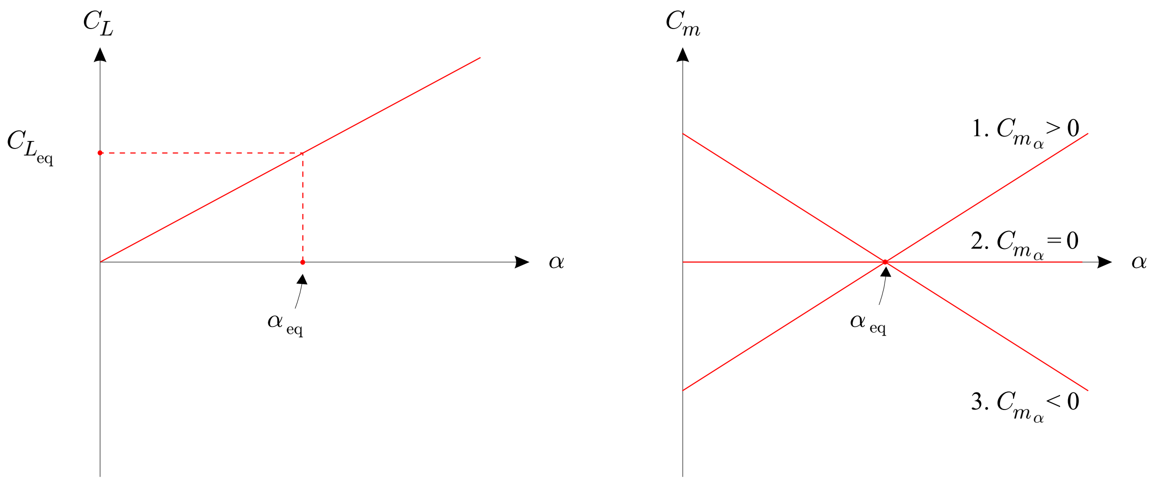

Figure 4: Pitch coefficient possibilities.

From a force balance in the $\boldsymbol{z}_I$ direction, we see that the lift which the aircraft generates must perfectly balance its weight for equilibrium flight. Suppose that we measure the angle of attack α from the airplane's zero-lift line, so that

$$C_L = C_{L_\alpha}\alpha.$$

Note that, in this case, the constant component C_{m_0} in the pitch moment coefficient is the zero-lift pitch moment coefficient: $C_{m_{0L}}$.

ASIDE: If the angle of attack (call it $\bar{\alpha}$, for now) is given in reference to some other line in the body so that

$$C_L = C_{L_0} + C_{L_\alpha}\bar{\alpha},$$

we may shift the origin by defining

$$\alpha = \bar{\alpha} - \bar{\alpha}_{0L}$$

where

$$\bar{\alpha}_{0L} = -\frac{C_{L_0}}{C_{L_\alpha}}.$$

Doing so gives

$$C_L = C_{L_\alpha}\alpha,$$

as we have assumed. □

Given that $C_{L_\alpha} > 0$ and that lift must act in the upward direction to balance the airplane's weight, a constant, positive angle of attack, say $\alpha = \alpha_{\rm eq} > 0$, is required for equilibrium flight:

$$\sum F_{z_{\rm I}} = 0 = W - \left(\frac{1}{2}\rho V^2 S\right) C_{L_\alpha}\alpha_{\rm eq}.$$

Moreover, equilibrium flight requires that the angular rate and all its derivatives be zero. Thus,

$$\dot{q} = 0 = \frac{1}{I_y}\left(\frac{1}{2}\rho V^2 S\bar{c}\right) C_m(\alpha_{\rm eq}) \qquad \Rightarrow \qquad C_m(\alpha_{\rm eq}) = C_{m_{0L}} + C_{m_\alpha}\alpha_{\rm eq} = 0.$$

Referring to Figure 4, there are three possibilities to consider:

1. $C_{m_{0L}} < 0$ and $C_{m_\alpha} > 0$
2. $C_{m_{0L}} = 0$ and $C_{m_\alpha} = 0$
3. $C_{m_{0L}} > 0$ and $C_{m_\alpha} < 0$

In each case, the pitch coefficient is zero when $\alpha = \alpha_{\text{eq}}$. What distinguishes the three cases is what happens when the equilibrium is disturbed.

Case 1. If an impulsive pitch disturbance causes the angle of attack to decrease ($\alpha < \alpha_{\text{eq}}$), then the pitch moment coefficient becomes negative. This results in a nose-down pitching moment which drives the angle of attack even lower. Alternatively, if a pitch disturbance causes the angle of attack to increase ($\alpha > \alpha_{\text{eq}}$), then the pitch moment coefficient becomes positive. This results in a nose-up pitching moment which drives the angle of attack even higher. Thus, if $C_{m_{\text{cg}_\alpha}} > 0$, steady wings-level flight is *statically unstable.*

Case 2. In this case, variations in the angle of attack have no effect on the pitch moment coefficient. Thus no additional moment is developed which would either drive the aircraft away from the equilibrium motion or cause equilibrium to be restored. If $C_{m_{\text{cg}_\alpha}} = 0$ (with $C_{m_{\text{cg}_0}} = 0$), steady wings-level flight is called *neutrally stable.*

Case 3. If an impulsive pitch disturbance causes the angle of attack to decrease, then the pitch moment coefficient becomes positive, resulting in a nose-up pitching moment which drives the angle of attack back up toward α_{eq}. Alternatively, if a pitch disturbance causes the angle of attack to become positive, then the pitch moment coefficient becomes negative, resulting in a nose-down pitching moment which drives the angle of attack back down toward α_{eq}. Thus, if $C_{m_{\text{cg}_\alpha}} < 0$ (with $C_{m_{\text{cg}_0}} > 0$), steady wings-level flight is *statically stable.*

Static pitch stability clearly requires that $C_m(\alpha)$ have a negative slope when it crosses the α-axis at the equilibrium angle of attack α_{eq}. Moreover:

Static longitudinal stability requires $C_{m_\alpha} < 0$ and $C_{m_{0L}} > 0$.