AOE 3134 Homework #6

Assigned: April 3, 2007
Due: April 12, 2007 (Place your homework in the box outside my office by 5 PM.)

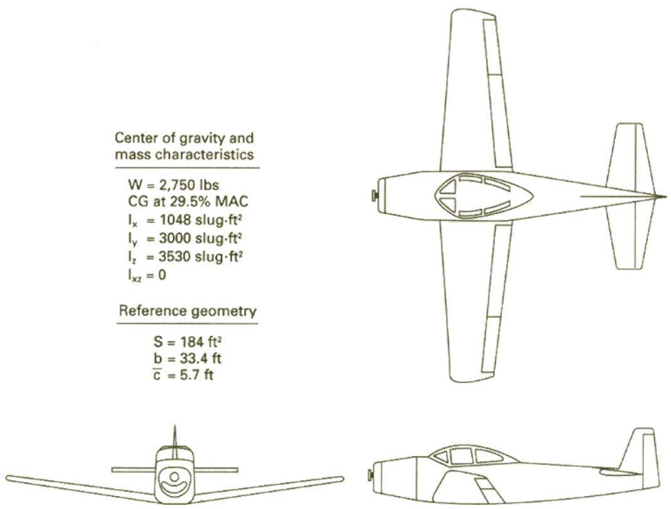

Figure 1: The Navion

$C_{L_{\text{trim}}}$	$C_{D_{\text{trim}}}$	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}
0.41	0.05	4.44	0.33	-0.683	0.0	-4.36	3.8	-9.96
C_{Y_β}	C_{l_β}	C_{n_β}	C_{Y_p}	C_{l_p}	C_{n_p}	C_{Y_r}	C_{l_r}	C_{n_r}
-0.564	-0.074	0.071	0	-0.410	-0.0575	0	0.107	-0.125

Table 1: Data for the Navion

Problems 1 and 2 concern a Navion general aviation airplane. Geometry and data for this aircraft are provided in Figure 1 and in Table 1 for sea level, equilibrium flight at Mach number 0.158. (Assume that compressibility effects are negligible at this speed.) All coefficients and stability derivatives in Table 1 are nondimensional; angles are expressed in radians.

Problem 1. The stick-fixed longitudinal dynamic equations, linearized about wings-level, constant-altitude flight, are

$$\underbrace{\frac{d}{dt}\begin{pmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{pmatrix}}_{\dot{\boldsymbol{x}}_{\mathrm{L}}} = \underbrace{\begin{pmatrix} \frac{1}{m}X_u & \frac{1}{m}X_w & 0 & -g \\ \frac{Z_u}{m-Z_{\dot w}} & \frac{Z_w}{m-Z_{\dot w}} & \frac{(Z_q+mu_0)}{m-Z_{\dot w}} & 0 \\ \frac{1}{I_{\mathrm{y}}}\left(M_u+\frac{M_{\dot w}Z_u}{m-Z_{\dot w}}\right) & \frac{1}{I_{\mathrm{y}}}\left(M_w+\frac{M_{\dot w}Z_w}{m-Z_{\dot w}}\right) & \frac{1}{I_{\mathrm{y}}}\left(M_q+\frac{M_{\dot w}(Z_q+mu_0)}{m-Z_{\dot w}}\right) & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\bar{\boldsymbol{A}}_{\mathrm{L}}} \underbrace{\begin{pmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{pmatrix}}_{\bar{\boldsymbol{x}}_{\mathrm{L}}}.$$

Part 1) Develop a Matlab script to construct $\bar{\boldsymbol{A}}_{\mathrm{L}}$ using the given parameter values for the Navion. Name the script `LongitudinalDynamics.YourLastName.m`. Print and submit the script as well as the resulting matrix $\bar{\boldsymbol{A}}_{\mathrm{L}}$.

Part 2) Compute the eigenvalues of $\bar{\boldsymbol{A}}_{\mathrm{L}}$, as developed in Part 1. Using these eigenvalues, determine the phugoid and short period natural frequency and damping ratio.

Part 3) Compute the approximate phugoid natural frequency and damping ratio using the approximation given in Lecture 15. Compare with the exact phugoid parameter values computed in Part 2.

Problem 2. The stick-fixed lateral-directional dynamic equations, linearized about wings-level, constant-altitude flight, are

$$\underbrace{\frac{d}{dt}\begin{pmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix}}_{\dot{\boldsymbol{x}}_{\mathrm{LD}}} = \underbrace{\begin{pmatrix} \frac{1}{m}Y_v & \frac{1}{m}Y_p & \frac{1}{m}Y_r - u_0 & g \\ \frac{1}{\xi}\left(I_{\mathrm{z}}L_v + I_{\mathrm{xz}}N_v\right) & \frac{1}{\xi}\left(I_{\mathrm{z}}L_p + I_{\mathrm{xz}}N_p\right) & \frac{1}{\xi}\left(I_{\mathrm{z}}L_r + I_{\mathrm{xz}}N_r\right) & 0 \\ \frac{1}{\xi}\left(I_{\mathrm{xz}}L_v + I_{\mathrm{x}}N_v\right) & \frac{1}{\xi}\left(I_{\mathrm{xz}}L_p + I_{\mathrm{x}}N_p\right) & \frac{1}{\xi}\left(I_{\mathrm{xz}}L_r + I_{\mathrm{x}}N_r\right) & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}_{\bar{\boldsymbol{A}}_{\mathrm{LD}}} \underbrace{\begin{pmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix}}_{\bar{\boldsymbol{x}}_{\mathrm{LD}}}$$

where $\xi = I_{\mathrm{x}}I_{\mathrm{z}} - I_{\mathrm{xz}}^2$.

Part 1) Develop a Matlab script to construct $\bar{\boldsymbol{A}}_{\mathrm{LD}}$ using the given parameter values for the Navion. Name the script `LateralDirectionalDynamics.YourLastName.m`. Print and submit the script as well as the resulting matrix $\bar{\boldsymbol{A}}_{\mathrm{LD}}$.

Part 2) Compute the eigenvalues of $\bar{\boldsymbol{A}}_{\mathrm{LD}}$, as developed in Part 1.

Problem 3. Using the eigenvector approach described in Lecture 18, solve the planar system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}, \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

for general initial conditions $\boldsymbol{x}_0$ where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -3 \\ 0 & -2 \end{pmatrix}.$$

Problem 4. Compute the natural frequency and damping ratio associated with the underdamped planar system

$$\dot{\boldsymbol{x}} = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}\boldsymbol{x}.$$

Also compute the time t_{half} and the number of cycles N_{half} to half-amplitude for the initial condition response. Recall that

$$t_{\mathrm{half}} = \frac{0.69}{\zeta\omega_n} \qquad \text{and} \qquad N_{\mathrm{half}} = \frac{t_{\mathrm{half}}}{T_d} \quad \text{where} \quad T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{1-\zeta^2}\,\omega_n}$$

(See supplemental notes on LTI ODEs.)