AOE 3134 Homework #6 Solutions

Problem 1 Solution. See Matlab script `NavionLongitudinal.m` posted with these solutions.

Problem 2 Solution. See Matlab script `NavionLateralDirectional.m` posted with these solutions.

Problem 3. Using the eigenvector approach described in Lecture 18, solve the planar system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}, \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0$$

for general initial conditions $\boldsymbol{x}_0$ where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -3 \\ 0 & -2 \end{pmatrix}.$$

Solution. The general solution is

$$\boldsymbol{x}(t) = \boldsymbol{V}(t)\boldsymbol{V}(0)^{-1}\boldsymbol{x}_0$$

where

$$\boldsymbol{V}(t) = \left[\ \boldsymbol{v}_1 e^{\lambda_1 t} \quad \boldsymbol{v}_2 e^{\lambda_2 t}\ \right)$$

with

$$\{\lambda_1, \boldsymbol{v}_1\} = \left\{1, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\} \qquad \text{and} \qquad \{\lambda_2, \boldsymbol{v}_2\} = \left\{-2, \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}.$$

Problem 4. Compute the natural frequency and damping ratio associated with the underdamped planar system

$$\dot{\boldsymbol{x}} = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}\boldsymbol{x}.$$

Also compute the time t_{half} and the number of cycles N_{half} to half-amplitude for the initial condition response. Recall that

$$t_{\text{half}} = \frac{0.69}{\zeta\omega_n} \qquad \text{and} \qquad N_{\text{half}} = \frac{t_{\text{half}}}{T_d} \quad \text{where} \quad T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{1-\zeta^2}\,\omega_n}$$

(See supplemental notes on LTI ODEs.)

Solution. The characteristic polynomial of the state matrix is

$$\det\begin{pmatrix} \lambda+1 & -2 \\ 2 & \lambda+1 \end{pmatrix} = \lambda^2 + 2\lambda + 5.$$

The roots are

$$\lambda_{1,2} = -1 \pm 2j$$

We therefore have

$$-\zeta\omega_n = -1 \qquad \text{and} \qquad \omega_d = \sqrt{1-\zeta^2}\omega_n = 2.$$

Solving for ω_n and ζ, we find

$$\omega_n = \sqrt{5} \approx 2.24 \text{ rad/s} \qquad \text{and} \qquad \zeta = \frac{\sqrt{5}}{5} \approx 0.45.$$

The system is stable, since the real part of each eigenvalue is negative, and the time and number of cycles to half amplitude are

$$t_{\text{half}} \approx \frac{0.69}{\zeta\omega_n} = 0.69 \text{ s} \qquad \text{and} \qquad N_{\text{half}} \approx 0.11\sqrt{\frac{1-\zeta^2}{\zeta^2}} = 0.22 \text{ cycles.}$$