AOE 3134 Homework #5

Assigned: March 22, 2007
Due: March 29, 2007 (Place your homework in the box outside my office by 5 PM.)

Problem 1. The dynamic equations for a freely rotating rigid body are

$$\dot{\boldsymbol{\omega}} = \boldsymbol{I}^{-1}\left((\boldsymbol{I}\boldsymbol{\omega}) \times \boldsymbol{\omega}\right).$$

Assume that the body-fixed reference frame is fixed in the principal axes of inertia so that $\boldsymbol{I} = \mathrm{diag}(I_x, I_y, I_z)$. Linearize the equations about the equilibrium $\boldsymbol{\omega}_{\mathrm{eq}} = [\omega_0,\ 0,\ 0]^T$. (Note that there will be no input matrix $\boldsymbol{B}$ for this problem.)

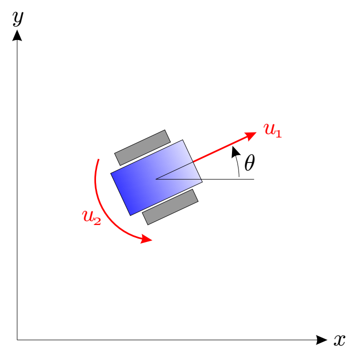

Figure 1: Kinematic model of a mobile robot.

Problem 2. A simple, kinematic model for a mobile robot is

$$\underbrace{\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix}}_{\dot{\boldsymbol{x}}} = \begin{pmatrix} \cos x_3 & 0 \\ \sin x_3 & 0 \\ 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}}_{\boldsymbol{u}}$$

where $x_1 = x$ and $x_2 = y$ denote the position of the robot and $x_3 = \theta$ denotes its orientation. The inputs are forward speed u_1 and turn rate u_2. Linearize these equations about the following two trajectories:

$$(a)\quad \boldsymbol{x}_{\mathrm{e}} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{u}_{\mathrm{e}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{and} \qquad (b)\quad \boldsymbol{x}_{\mathrm{e}} = \begin{pmatrix} \frac{1}{\omega}\sin\omega t \\ -\frac{1}{\omega}\cos\omega t \\ \omega t \end{pmatrix}, \quad \boldsymbol{u}_{\mathrm{e}} = \begin{pmatrix} 1 \\ \omega \end{pmatrix}$$

Problem 3. Using the Taylor series approach, verify the six linearized kinematic equations given in Lecture 14:

$$\begin{aligned}
\Delta\dot{x} &= \cos\theta_0\Delta u - (u_0\sin\theta_0)\Delta\theta + \sin\theta_0\Delta w \\
\Delta\dot{y} &= (u_0\cos\theta_0)\Delta\psi + \Delta v \\
\Delta\dot{z} &= -\sin\theta_0\Delta u - (u_0\cos\theta_0)\Delta\theta + \cos\theta_0\Delta w \\
\Delta\dot{\phi} &= \Delta p + \tan\theta_0\Delta r \\
\Delta\dot{\theta} &= \Delta q \\
\Delta\dot{\psi} &= \sec\theta_0\Delta r
\end{aligned}$$