# AOE 3134 Homework #3

Assigned: February 8, 2007

Due: February 20, 2007 (Place your homework in the box outside my office by 5 PM.)

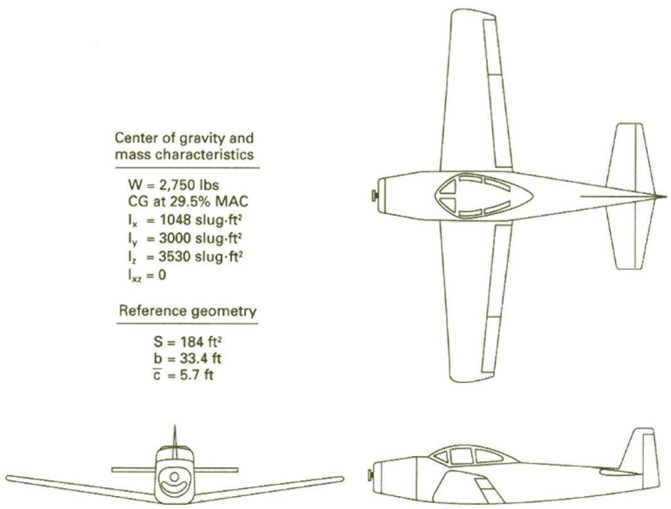


Figure 1: The Navion

Table 1: Data for the Navion

| $C_{L_{\text{trim}}}$ | $C_{L_\alpha}$ | $C_{m_\alpha}$ | $C_{L_q}$ | $C_{m_q}$ | $C_{L_{\delta e}}$ | $C_{m_{\delta e}}$ |
|---|---|---|---|---|---|---|
| 0.41 | 4.44 | -0.683 | 3.8 | -9.96 | 0.355 | -0.923 |

Problems 1 and 2 concern a Navion general aviation airplane. Geometry and data for this aircraft are provided in Figure 1 and in Table 1 for *sea level, equilibrium flight.* All coefficients and stability derivatives in Table 1 are nondimensional. (For example, the value given for $C_{m_\alpha}$ is *per radian*). Data in Figure 1 are given in English units, however *your calculations should be carried out in SI units.*

**Problem 1.** For simplicity, define the body reference frame such that $\theta = 0$ in equilibrium flight. (Such a body-fixed reference frame is sometimes referred to as the *stability frame.*) Also, suppose that the angle of attack $\bar{\alpha}$ is defined to be zero in the given equilibrium flight condition. Thus,

$$C_L = C_{L_0} + C_{L_\alpha}\bar{\alpha} + C_{L_{\delta e}}\delta e$$

where $C_{L_{\text{trim}}} = C_{L_0} + C_{L_{\delta e}}\delta e_{\text{trim}}$.

1. In equilibrium flight, the engine generates 1.84 kN of thrust. Using the expression

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e AR},$$

compute the cruise speed of the aircraft, assuming that thrust directly balances drag. In the above expression, $C_{D_0} = 0.05$ is the parasite drag coefficient, $AR = \frac{b^2}{S}$ is the wing aspect ratio, and $e = 0.8$ is the Oswald efficiency factor.

2. Suppose that the engine suddenly fails and that the pilot adjusts the elevator to maintain the pitch angle at zero as the airplane glides steadily downward. Draw the free body diagram for the new steady flight condition. What are the new trim values of $\bar{\alpha}$ and $\delta e$ and what is the new airspeed in descending flight? (*Note:* You will have to solve this problem numerically. Assume that $\delta e = 0$ in solving the force balance equations for $\bar{\alpha}$ and speed. Then solve the moment balance equation for $\delta e$ – the answer will be approximate, but fairly accurate.)

3. *Extra Credit:* Assume that the airplane is flying over level ground at a ground-relative altitude $h = 2000m$ when the engine fails. In the given flight condition, and ignoring the variation in density, how much time will elapse before the aircraft hits the ground? If the original destination is 1000 m ahead when the engine fails, but there is a 5 m/s headwind, will the pilot make it to the airport?

**Problem 2.** A crude model for the longitudinal dynamics (valid if the airplane were pinned in a wind tunnel through its center of gravity) is

$$I_y \dot{q} = \left(\frac{1}{2}\rho V^2 S \bar{c}\right) \left(C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e + C_{m_q}\hat{q}\right)$$

where $\alpha = \theta$ and $q = \dot{\theta}$. (Recall that $\hat{q} = \frac{\bar{c}}{2V}q$.) For the given problem, by our definition of $\bar{\alpha}$, $C_{m_0} = 0$. (Note, however, that $C_{m_{0L}} > 0$ as it must be.)

This equation can be rewritten in the form

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = bu \qquad (1)$$

where $u = \delta e$.

1. Compute explicit expressions for the constants $\omega_n$, $\zeta$, and $b$ and then evaluate those expressions using the given parameter values. (Assume that the Mach number is 0.16 so that $V \approx 54\frac{\text{m}}{\text{s}}$.)

2. Using software of your choice (or pencil and paper, if you prefer), investigate the time response $\theta(t)$ to the initial state $\theta(0) = \frac{\pi}{12}$ and $\dot{\theta}(0) = 0$ for two cases:

   - *Stick-fixed* response: $u = 0$
   - *Stability-augmented* response:

     $$u = -\frac{1}{b}\left[\left(2\zeta_d\omega_d\dot{\theta} + \omega_d^2\theta\right) - \left(2\zeta\omega_n\dot{\theta} + \omega_n^2\theta\right)\right]$$

     where $\omega_d = 5$ rad/s and $\zeta_d = 0.7$ are desired values for natural frequency and damping ratio.

**Problem 3.** Consider an aircraft in trimmed, wings-level flight at speed $V_0$ and lift coefficient $C_{L_{\max}}$.

1. Suppose that the pilot wishes to execute a constant radius, coordinated turn at a roll angle $\phi = 60°$. What should be the new airspeed $V$ (relative to $V_0$) if the coordinated turn also occurs at $C_{L_{\max}}$?

2. Suppose the aircraft is designed to withstand a load factor of 3.8 g's. That is, the flight condition should always satisfy $n \leq 3.8$. Is it possible to make a coordinated turn with $\phi = 75°$ without exceeding the structural limit?

**Problem 4.** The static yaw moment derivatives for the four-engine Boeing 747-100 in sea level flight at $M = 0.2$ are

$$C_{n_\beta} = 0.150, \quad \text{and} \quad C_{n_{\delta r}} = -0.109.$$

Using the information in Appendix E of the book, determine the rudder deflection necessary to counter the yaw moment induced by the loss of the outer starboard engine. Assume that the thrust of the remaining three engines is adjusted evenly in order to maintain the given flight speed. (Use the approximate technique discussed in Lecture 7.)