AOE 3134 Homework #3 Solutions

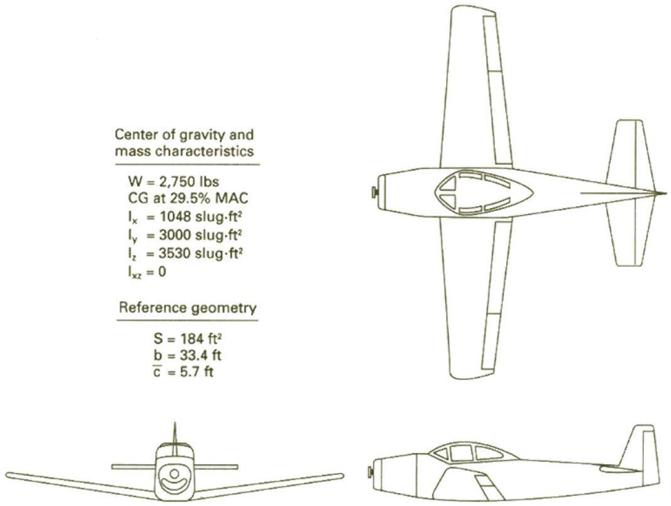


Figure 1: The Navion

Table 1: Data for the Navion

$C_{L_{\text{trim}}}$	C_{L_α}	C_{m_α}	C_{L_q}	C_{m_q}	$C_{L_{\delta e}}$	$C_{m_{\delta e}}$
0.41	4.44	-0.683	3.8	-9.96	0.355	-0.923

Problems 1 and 2 concern a Navion general aviation airplane. Geometry and data for this aircraft are provided in Figure 1 and in Table 1 for *sea level, equilibrium flight.* All coefficients and stability derivatives in Table 1 are nondimensional. (For example, the value given for C_{m_α} is *per radian*). Data in Figure 1 are given in English units, however *your calculations should be carried out in SI units.*

Problem 1. For simplicity, define the body reference frame such that $\theta = 0$ in equilibrium flight. (Such a body-fixed reference frame is sometimes referred to as the *stability frame.*) Also, suppose that the angle of attack $\bar{\alpha}$ is defined to be zero in the given equilibrium flight condition. Thus,

$$C_L = C_{L_0} + C_{L_\alpha}\bar{\alpha} + C_{L_{\delta e}}\delta e$$

where $C_{L_{\text{trim}}} = C_{L_0} + C_{L_{\delta e}}\delta e_{\text{trim}}$.

1. In equilibrium flight, the engine generates 1.84 kN of thrust. Using the expression

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e AR},$$

compute the cruise speed of the aircraft, assuming that thrust directly balances drag. In the above expression, $C_{D_0} = 0.05$ is the parasite drag coefficient, $AR = \frac{b^2}{S}$ is the wing aspect ratio, and $e = 0.8$ is the Oswald efficiency factor.

2. Suppose that the engine suddenly fails and that the pilot adjusts the elevator to maintain the pitch angle at zero as the airplane glides steadily downward. Draw the free body diagram for the new steady flight condition. What are the new trim values of $\bar{\alpha}$ and δe and what is the new airspeed in descending flight? (*Note:* You will have to solve this problem numerically.)

3. *Extra Credit:* Assume that the airplane is flying over level ground at a ground-relative altitude $h = 2000m$ when the engine fails. In the given flight condition, and ignoring the variation in density, how much time will elapse before the aircraft hits the ground? If the original destination is 1000 m ahead when the engine fails, but there is a 5 m/s headwind, will the pilot make it to the airport?

Solution. Consider the equations

$$\begin{aligned} W &= L \\ T &= D. \end{aligned}$$

Substituting

$$\begin{aligned} L &= P_{\text{dyn}} S C_{L_{\text{trim}}} \\ D &= P_{\text{dyn}} S C_D = P_{\text{dyn}} S \left(C_{D_0} + \frac{C_{L_{\text{trim}}}^2}{\pi e AR} \right), \end{aligned}$$

we can solve the second equation for P_{dyn}

$$T = P_{\text{dyn}} S \left(C_{D_0} + \frac{C_{L_{\text{trim}}}^2}{\pi e AR} \right)$$

The solution is $P_{\text{dyn}_2} = 1773$ Pa. This corresponds to a cruise speed of $V = 54\frac{\text{m}}{\text{s}}$, or equivalently $M = 0.16$.

Now consider the new equilibrium flight. The pitch angle is still zero, but the aircraft is flying at a nonzero angle of attack. Since the accelerations are zero, we can write the static equations

$$W = L \cos\bar{\alpha} + D \sin\bar{\alpha} \tag{1}$$
$$0 = L \sin\bar{\alpha} - D \cos\bar{\alpha} \tag{2}$$

These equations are transcendental in α. We could use a numeric solver (Matlab or Mathematica, for example) to obtain a solution for the two unknowns $\bar{\alpha}$ and P_{dyn}. Alternatively, we can assume that $\bar{\alpha} \ll 1$, solve the simultaneous equations, and then check whether, indeed, $\bar{\alpha} \ll 1$. Taking this latter approach, consider the second equation above. Dividing by $P_{\text{dyn}} S$ gives

$$0 \approx C_L \bar{\alpha} - C_D.$$

If we ignore, for the moment, the small change in C_L due to the change in δe, then we obtain

$$0 \approx \left(C_{L_{\text{trim}}} + C_{L_\alpha} \bar{\alpha} \right) \bar{\alpha} - \left(C_{D_0} + \frac{1}{\pi e AR} \left(C_{L_{\text{trim}}} + C_{L_\alpha} \bar{\alpha} \right)^2 \right).$$

We thus obtain a quadratic equation for $\bar{\alpha}$:

$$\left(\frac{C_{L_\alpha}^2}{\pi e AR} - C_{L_\alpha} \right) \bar{\alpha}^2 + \left(\frac{2 C_{L_\alpha}}{\pi e AR} - 1 \right) C_{L_{\text{trim}}} \bar{\alpha} + \left(C_{D_0} + \frac{C_{L_{\text{trim}}}^2}{\pi e AR} \right) = 0.$$

The two solutions are -0.169 rad and 0.115 rad. The former value makes no physical sense. The latter value, approximately $6.6°$, is the (approximate) solution. Note that $\bar{\alpha} = 0.115 \ll 1$, so the small angle assumption was well-justified. (We have *not* assumed that $\bar{\alpha} \ll 1$ implies that terms quadratic in $\bar{\alpha}$ vanish in the expression above. If we were to ignore the terms quadratic in $\bar{\alpha}$, we would find an angle of attack around $20.4°$, which stretches the limits of the small angle assumption. That might imply that we should return to the original equation and solve numerically.) We may now substitute $\bar{\alpha} = 0.115$ into equation (1) to compute P_{dyn}, and thus the descent speed. Doing so, we find that $V = 35.4$ m/s.

Now consider the moment equation

$$0 = C_{m_\alpha}\bar{\alpha} + C_{m_{\delta_e}}\delta_e.$$

Solving gives $\delta_e = -4.9°$. Note that the error in our approximation $C_L = C_{L_{\text{trim}}} + C_{L_\alpha}\bar{\alpha}$ (in which we neglected the term $C_{L_{\delta e}}\delta e$) is about 3%.

Extra Credit: The vertical component of the velocity is $\dot{z} = V\sin\alpha = 4.1$ m/s. From $h = 2000$ m it takes about 8 minutes (493 s) to touch the ground. The longitudinal component of the velocity, decreased by the wind speed (i.e. the ground speed) is $u_g = 11.9\frac{\text{m}}{\text{s}}$. With this speed the airplane can travel 14.9 km, before hitting the ground, so it can certainly make it to the airport.

Problem 2. A crude model for the longitudinal dynamics (valid if the airplane were pinned in a wind tunnel through its center of gravity) is

$$I_y\dot{q} = \left(\frac{1}{2}\rho V^2 S\bar{c}\right)\left(C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e + C_{m_q}\hat{q}\right)$$

where $\alpha = \theta$ and $q = \dot{\theta}$. (Recall that $\hat{q} = \frac{\bar{c}}{2V}q$.) For the given problem, by our definition of $\bar{\alpha}$, $C_{m_0} = 0$. (Note, however, that $C_{m_{0L}} > 0$ as it must be.)

This equation can be rewritten in the form

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = bu \tag{3}$$

where $u = \delta e$.

1. Compute explicit expressions for the constants ω_n, ζ, and b and then evaluate those expressions using the given parameter values. (Assume that the Mach number is 0.16 so that $V \approx 54\frac{\text{m}}{\text{s}}$.)
2. Using software of your choice (or pencil and paper, if you prefer), investigate the time response $\theta(t)$ to the initial state $\theta(0) = \frac{\pi}{12}$ and $\dot{\theta} = 0$ for two cases:
   - *Stick-fixed* response: $u = 0$
   - *Stability-augmented* response:

     $$u = -\frac{1}{b}\left[\left(2\zeta_d\omega_d\dot{\theta} + \omega_d^2\theta\right) - \left(2\zeta\omega_n\dot{\theta} + \omega_n^2\theta\right)\right]$$

     where $\omega_d = 5$ rad/s and $\zeta_d = 0.7$ are desired values for natural frequency and damping ratio.

Solution. Letting $\kappa = \frac{\rho V^2 S\bar{c}}{2I_y}$, we have

$$\ddot{\theta} = \kappa\left(C_{m_\alpha}\theta + C_{m_{\delta e}}u + C_{m_q}\left(\frac{\bar{c}}{2V}\right)\dot{\theta}\right).$$

or

$$\ddot{\theta} - \kappa C_{m_q}\left(\frac{\bar{c}}{2V}\right)\dot{\theta} - \kappa C_{m_\alpha}\theta = \kappa C_{m_{\delta e}}u.$$

Comparing with the required form gives

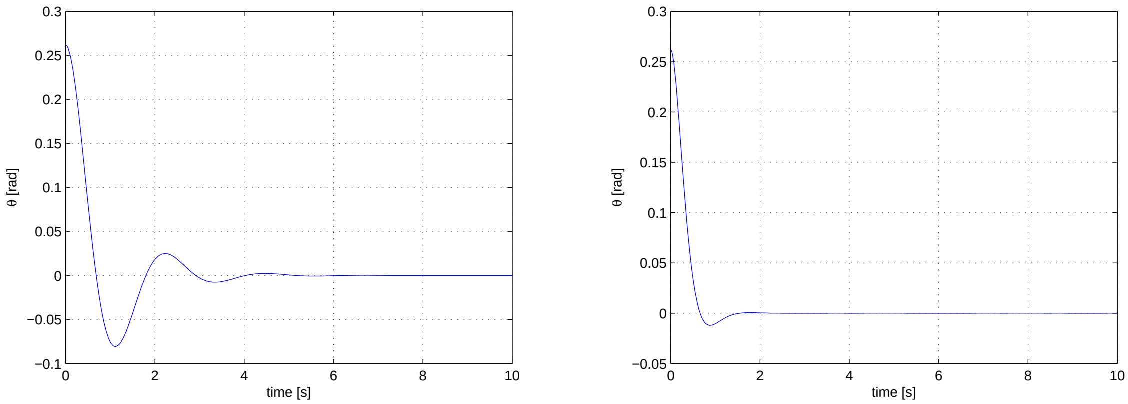
$$\omega_n = \sqrt{-\kappa C_{m_\alpha}}, \quad \zeta = \frac{-\kappa\bar{c}C_{m_q}}{4V\omega_n} = \frac{-\sqrt{\kappa}\bar{c}C_{m_q}}{4V\sqrt{-C_{m_\alpha}}}, \quad b = \kappa C_{m_{\delta e}}.$$

For the given parameter values

$$\omega_n = 3.0 \text{ rad/s}, \quad \zeta = 0.35, \quad \text{and} \quad b = -12.2 \text{ rad/s}^2$$

Figures 2(a) and 2(b) show the initial condition and feedback-controlled response, respectively. Note that convergence is faster and better damped in Figure 2(b).

Problem 3. Consider an aircraft in trimmed, wings-level flight at speed V_0 and lift coefficient $C_{L_{\max}}$.



(a) $u = 0$. (b) $u(\theta, \dot{\theta})$

Figure 2: Pitch angle histories for Problem 3.

1. Suppose that the pilot wishes to execute a constant radius, coordinated turn at a roll angle $\phi = 60°$. What should be the new airspeed V (relative to V_0) if the coordinated turn also occurs at $C_{L_{\max}}$?

2. Suppose the aircraft is designed to withstand a load factor of 3.8 g's. That is, the flight condition should always satisfy $n \leq 3.8$. Is it possible to make a coordinated turn with $\phi = 75°$ without exceeding the structural limit?

Solution.

$$\begin{aligned} \cos(\phi) &= \frac{1}{n} = \frac{1}{2} \rightarrow \frac{L}{W} = 2 \\ V &= \sqrt{\frac{2L}{C_{L_{\max}} \rho S}} = \sqrt{2} V_0 \end{aligned} \tag{4}$$

The pilot should increase the speed to $\sqrt{2}V_0$ to maintain the level turn.

When the bank angle is equal to 75°, it is impossible to maintain a level turn without violating the structural limit because $\frac{L}{W} = 3.8637 > 3.8$.

Problem 4. The static yaw moment derivatives for the four-engine Boeing 747-100 in sea level flight at $M = 0.2$ are

$$C_{n_\beta} = 0.150, \quad \text{and} \quad C_{n_{\delta r}} = -0.109.$$

Using the information in Appendix E of the book, determine the rudder deflection necessary to counter the yaw moment induced by the loss of the outer starboard engine. Assume that the thrust of the remaining three engines is adjusted evenly in order to maintain the given flight speed. (Use the approximate technique discussed in Lecture 7.)

Solution. The total yaw moment coefficient, with an unbalanced moment due to thrust, is

$$C_n = C_{n_\beta} \beta + C_{n_{\delta r}} \delta r + C_{n_T}.$$

For equilibrium flight, we require that $C_n = 0$. Moreover, to minimize drag the airplane should fly at zero sideslip angle: $\beta = 0$. We must therefore solve

$$0 = C_{n_{\delta r}} \delta r + C_{n_T}$$

for δr. First, we must compute C_{n_T}. The total thrust must balance drag which, at $M = 0.2$ is

$$\begin{aligned} D &= C_D \left(\frac{1}{2}\rho V^2\right) S \\ &= (0.263)\left(\frac{1}{2}(0.00238 \text{ slugs/ft}^3)(223.3 \text{ ft/s})^2\right)(5500 \text{ ft}^2) \\ &\approx 85,800 \text{ lb.} \end{aligned}$$

Split evenly among the three remaining engines, the outer port engine generates a yaw moment

$$\begin{aligned} N_T &= \left(\frac{1}{3}85,800 \text{ lb}\right)(69 \text{ ft}) \\ &= 1.97e6 \text{ ft}-\text{lb.} \end{aligned}$$

Normalizing gives

$$\begin{aligned} C_{n_T} &= \frac{N_T}{\left(\frac{1}{2}\rho V^2\right) Sb} \\ &= 0.031. \end{aligned}$$

Given

$$C_{n_{\delta r}} = -0.109,$$

we find that the rudder angle to trim at zero sideslip is

$$\delta r = 0.284 \text{ rad} = 16.2^\circ.$$

This is quite a large rudder angle and, in fact, the result may be quite different than the true rudder angle required. In reality, rudder deflections induce roll moments which must be countered by the ailerons which, in turn, contribute to the yaw moment. Generally, one should consider the full lateral-directional equations and compute the rudder and aileron angles simultaneously.