AOE 3134 Homework #2 Solutions

Assigned: January 18, 2007
Due: January 30, 2007 (Place your homework in the box outside my office by 5 PM.)

Problem 1. Wind tunnel tests yield the following data for lift and pitch moment about the quarter chord line of a given wing:

C_L	$C_{m_{1/4}}$
0.3	-0.039
0.6	-0.036

1. Find h_{n_w}.
2. Find $C_{m_{\text{ac}}}$.
3. Find h_{cp} when $C_L = 0.4$.

Solution. The equation for finding the pitch moment about a point h is

$$C_m = C_{m_{ac_w}} + C_{L_w}\left(h - h_{n_w}\right).$$

We have two data points for pitch moment about $h = \frac{1}{4}$:

$$\begin{aligned} C_{m_{1/4}} = -0.039 &= C_{m_{ac_w}} + 0.3\left(\frac{1}{4} - h_{n_w}\right) \\ -0.036 &= C_{m_{ac_w}} + 0.6\left(\frac{1}{4} - h_{n_w}\right). \end{aligned}$$

Solving these two equations in two unknowns gives

$$C_{m_{ac_w}} = -0.042 \qquad \text{and} \qquad h_{n_w} = 0.24.$$

Remark: Differentiating the expression

$$C_m = C_{m_{ac_w}} + C_{L_w}\left(h - h_{n_w}\right)$$

with respect to α gives

$$C_{m_\alpha} = C_{L_{\alpha_w}}\left(h - h_{n_w}\right),$$

assuming that C_m and C_L depend only on α. Thus, the neutral point is also given by the expression

$$h_{n_w} = h - \frac{C_{m_\alpha}}{C_{L_{\alpha_w}}}.$$

If C_m and C_{L_w} truly depend only on α, then we may write $C_m(C_{L_w}(\alpha))$ and compute

$$\frac{dC_m}{d\alpha} = \frac{dC_m}{dC_{L_w}}\frac{dC_{L_w}}{d\alpha} \qquad \Rightarrow \qquad \frac{dC_m}{dC_{L_w}} = \frac{C_{m_\alpha}}{C_{L_{\alpha_w}}}.$$

We could thus obtain the neutral point directly from moment versus lift data, as given here. While this approach is acceptable, it is not the *preferred* method of determining h_{n_w}; see Etkin and Reid's discussion early in Section 2.3.

To determine the location of the center of pressure, note that

$$C_{m_{cp}} = 0 = C_{m_{ac_w}} + C_{L_w}\left(h_{cp} - h_{n_w}\right).$$

Substituting $C_{L_w} = 0.4$ and the other values from above gives

$$h_{cp} = 0.345$$

Problem 2. Consider an airplane with the following characteristics

$$\begin{aligned} C_L &= 0.07\,\alpha + 0.002\,\delta e \\ C_m &= 0.10 - 0.02\,\alpha - 0.03\,\delta e \\ C_{L_t} &= 0.06\,\alpha_t + 0.004\,\delta e \end{aligned}$$

where all angles are measured in degrees. These equations are valid below stall. The lift coefficient at stall is $C_{L_{\max}} = 1.4$. The center of gravity is located at the point $x_{\mathrm{cg}} = 1.5$ meters, where x is measured positive aft of the wing apex. Other relevant parameters are

$$b = 15\text{ m}, \qquad S = 45\text{ m}^2, \qquad \bar{c} = 3\text{ m}, \qquad V_{\mathrm{H}} = 1, \qquad W = 120\text{ kN}.$$

1. Determine the equilibrium angle of attack and elevator deflection corresponding to steady, wings-level flight at speed $V = 70$ m/s in air of density $\rho = 1$ kg/m^3.
2. Compute the stick-fixed static margin.
3. Suppose that elevator deflections are limited to the range $-15^\circ \leq \delta e \leq 15^\circ$. Find the forward limit on the CG location defined by requiring that the minimum equilibrium elevator deflection $\delta e_{\min}$ correspond to the stall lift coefficient $C_{L_{\max}}$.

Solution. The relevant equilibrium conditions are that weight balance lift and pitch moment about the center of gravity be zero. The trim lift coefficient necessary for lift to balance drag is

$$C_{L_{\mathrm{trim}}} = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{120,000}{\frac{1}{2}(1)(4900)(45)} \approx 1.09.$$

The corresponding trim angle of attack is

$$\begin{aligned} \alpha_{\mathrm{trim}} &= \frac{C_{m_{\delta e}} C_{L_{\mathrm{trim}}} + C_{L_{\delta e}} C_{m_0}}{C_{L_\alpha} C_{m_{\delta e}} - C_{L_{\delta e}} C_{m_\alpha}} \\ &= \frac{(-0.03)(1.09) + (0.002)(0.10)}{(0.07)(-0.03) - (0.002)(-0.02)} \\ &= 15.8^\circ \end{aligned}$$

and

$$\begin{aligned} \delta e_{\mathrm{trim}} &= -\frac{C_{m_\alpha} C_{L_{\mathrm{trim}}} + C_{L_\alpha} C_{m_0}}{C_{L_\alpha} C_{m_{\delta e}} - C_{L_{\delta e}} C_{m_\alpha}} \\ &= -\frac{(-0.02)(1.09) + (0.08)(0.10)}{(0.07)(-0.03) - (0.002)(-0.02)} \\ &= -6.7^\circ. \end{aligned}$$

Differentiating

$$C_m = C_{m_{0L}} + C_L(h - h_n)$$

with respect to α gives

$$C_{m_\alpha} = 0 + C_{L_\alpha}(h - h_n).$$

Rearranging terms, we find that

$$(h - h_n) = \frac{C_{m_\alpha}}{C_{L_\alpha}} = \frac{-0.02}{0.07} = -0.29,$$

so

$$K_n = h_n - h = 0.29.$$

Note that, since we were given

$$h = \frac{1.5}{3} = \frac{1}{2},$$

we may determine h_n explicitly:

$$h_n = 0.79.$$

The maximum forward CG location can be determined as

$$\begin{aligned} h_{\min} &= h_n - \frac{C_{m_0} + C_{m_{\delta e}} \delta e_{\min}}{C_{L_{\max}} - C_{L_{\delta e}} \delta e_{\min}} \\ &= (0.79) - \frac{0.10 + (-0.03)(-15)}{1.4 - (0.002)(-15)} \\ &= 0.41. \end{aligned}$$

Thus, $x_{\text{cg}_{\min}}$ is $(0.41)(3) = 1.2$ meters aft of the wing apex.

Problem 3. Problem #2.8 in Etkin and Reid. Although we did not cover this topic in lecture, the problem is quite straight forward after a bit of reading and independent thought.

Solution. Download, open, and run the Matlab script `HW2_prob3.m`.

Problem 4. Problem #2.11 in Etkin and Reid. The discussion must be typed and should not exceed one page.

Solution. There is no single "correct" answer to this question. Grades were assigned based on the effective use of non-technical language and the degree of clarity.