AOE 3134 Homework #1 Solutions

Problem 1. Wind tunnel tests yield the following data for lift coefficient and pitch moment coefficient about the half chord line of a given wing:

$\bar{\alpha}$ (deg)	C_L	$C_{m_{1/2}}$
-4.0	-0.257	0.149
-2.0	0.030	0.084
0.0	0.194	0.062
2.0	0.210	0.020
4.0	0.540	-0.033
6.0	0.752	-0.066

Using the software of your choice (e.g., Matlab or Excel), determine $C_L(\bar{\alpha})$ and $C_{m_{1/2}}(\bar{\alpha})$ using a linear least squares fit. Plot the data and the curve fits. Determine h_{ac} and $C_{m_{\text{ac}}}$.

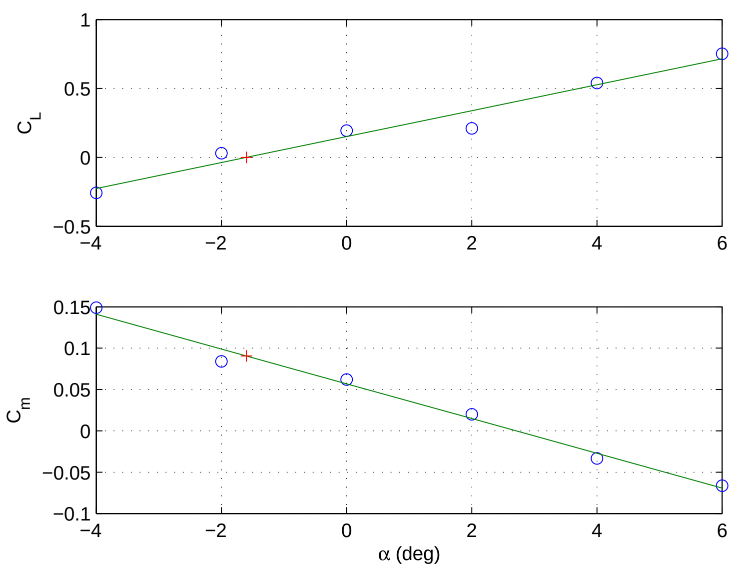

Figure 1: Least squares fits for $C_L(\bar{\alpha})$ and $C_m(\bar{\alpha})$ versus $\bar{\alpha}$.

Solution. Using Matlab's `polyfit` and `polyval` functions, one may obtain the plots shown in Figure 1. The data fits suggest that

$$C_L(\bar{\alpha}) = 0.1507 + 0.0942\,\bar{\alpha} \tag{1}$$

$$C_{m_{1/2}}(\bar{\alpha}) = 0.0569 - 0.021\,\bar{\alpha}. \tag{2}$$

The neutral point can be found by differentiating the identity

$$C_{m_{1/2}} = C_{m_{\text{ac}}} + C_L\left(h_{1/2} - h_{\text{ac}}\right)$$

with respect to $\bar{\alpha}$ and solving for h_{ac}:

$$h_{\text{ac}} = h_{1/2} - \frac{1}{C_{L_\alpha}}\left(\frac{\partial C_{m_{1/2}}}{\partial \bar{\alpha}}\right)$$

$$
\begin{aligned}
&= \frac{1}{2} - \frac{-0.0197}{0.093} \\
&\approx 0.72
\end{aligned}
$$

The easiest way to find $C_{m_{\text{ac}}}$ is to solve (1) for $\bar{\alpha}_{0L}$ and substitute into (2). Doing so gives

$$C_{m_{\text{ac}}} = 0.0905.$$

Problem 2. Consider a 30 pound, rectangular flying wing with a span $b = 10$ feet and a chord length $c = 2$ feet. For this aircraft, $C_{m_{0L}} = 0.018$ and $C_{m_\alpha} = -0.003$ per degree, where C_m is computed about the center of gravity. The slope of the lift curve is $C_{L_\alpha} = 0.1$ per degree and $C_{L_0} = 0$. Determine the speed required for equilibrium flight at sea level.

Solution. We know that

$$C_m = C_{m_{0L}} + C_{m_\alpha}\alpha$$

where α is measured from the zero-lift line. In equilibrium flight, we must have

$$C_{m_{\text{eq}}} = 0 = C_{m_{0L}} + C_{m_\alpha}\alpha_{\text{eq}} \qquad \Rightarrow \qquad \alpha_{\text{eq}} = 6^\circ.$$

In this problem, zero lift corresponds to $\alpha = 0^\circ$, since $C_{L_0} = 0$. Having satisfied the condition for pitch equilibrium, the remaining conditions for equilibrium flight are that drag balances thrust and lift balances weight. We assume that the thrust is adjusted as necessary. The equilibrium lift coefficient is

$$
\begin{aligned}
C_{L_{\text{eq}}} &= C_{L_0} + C_{L_\alpha}\alpha_{\text{eq}} \\
&= 0 + 0.1\alpha_{\text{eq}} \\
&= 0.60
\end{aligned}
$$

For equilibrium flight, we require

$$L_{\text{eq}} = W = C_{L_{\text{eq}}}\left(\frac{1}{2}\rho V^2\right) S$$

where $S = bc$. Solving for the speed, we find

$$
\begin{aligned}
V &= \sqrt{\frac{2W}{C_{L_{\text{eq}}}\rho b c}} \\
&= \sqrt{\frac{2(30 \text{ lb}_\text{f})}{(0.60)(0.00238 \text{ slugs/ft}^3)(10 \text{ ft})(2 \text{ ft})}} \\
&= 45.8 \text{ ft/s}.
\end{aligned}
$$

Problem 3. Consider a lighter-than-air vehicle modeled as a prolate spheroid with a horizontal stabilizer. (See Figure 2 on the following page.) For longitudinal, equilibrium flight, the pitch moment about the center of buoyancy (CB) is

$$M_{\text{CB}} = (m_w - m_u)\, V^2\alpha - mg\Delta\theta - \left(\frac{1}{2}\rho V^2\right) \text{Vol}_t C_{L_{\alpha_t}}\alpha.$$

The first term above is the so-called "Munk moment," a potential flow effect which tends to destabilize longitudinal translation. In this expression, m_w is the *added mass* along the $\boldsymbol{z}_\text{B}$ axis and m_u is the added mass along the $\boldsymbol{x}_\text{B}$ axis. The second term is a moment due to the vertical separation between the center of gravity (CG) and the CB – the "bottom-heaviness" parameter Δ has units of length and is positive when the CG is *below* the CB. The third term accounts for the moment due to the horizontal stabilizer, where the "horizontal tail volume" Vol_t is the product of the horizontal stabilizer area (S_t) and the moment arm (l_t) from the CB to the tail.

Consider a 9 meter long airship with a 3 meter diameter that has the following properties:

$$I_y = 350 \text{ kg m}^2, \quad m = 52 \text{ kg}, \quad m_u = 6 \text{ kg}, \quad m_w = 42 \text{ kg}, \quad \text{and} \quad C_{L_{\alpha_t}} = 1 \text{ rad}^{-1}$$

Assume that the vehicle's flight path is somehow constrained to be horizontal so that $\theta = \alpha$ in the pitch moment expression given above. In analogy to aircraft, static pitch stability requires that $\partial M_\text{CB}/\partial\alpha < 0$. On a plot for which Δ is the abscissa (x-axis) and Vol_t is the ordinate (y-axis), indicate the parameter regions in which flight at $V = 1$ m/s and $V = 5$ m/s are statically stable, respectively. Discuss your results.

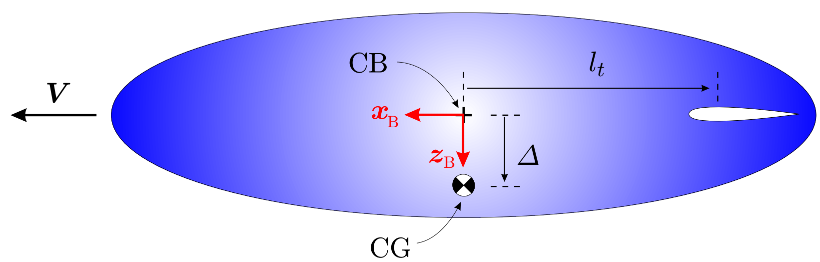

Figure 2: An airship.

Solution. For static stability, we need the moment due to small changes in α (equivalently θ) to tend to restore the equilibrium condition. Thus, at sea level conditions, we need

$$(m_w - m_u)\, V^2 - mg\Delta - \left(\frac{1}{2}\rho V^2\right) \text{Vol}_t C_{L_{\alpha_t}} < 0.$$

Substituting values, we obtain

$$(42 - 6)\, V^2 - (52)(9.81)\Delta - \left(\frac{1}{2}(1.225)V^2\right) \text{Vol}_t(1) < 0$$

or

$$\text{Vol}_t > 59 - \frac{836}{V^2}\Delta$$

Letting $V = 1$ m/s and $V = 5$ m/s, respectively, gives the lines shown in Figure 3. In either case, the parameter region *above* the line corresponds to a statically stable flight condition. Notice that, as the airship flies faster, the tail moment is more effective at providing stability than is the bottom-heaviness. This is indicated by the relatively small variation in minimum tail volume required versus bottom-heaviness. At lower speeds, the horizontal tail provides a much smaller restoring moment, making the bottom-heaviness a more important factor in determining static stability. In fact, at speed $V = 1$ m/s with the CG about 7 cm below the CB, no horizontal tail is even required for longitudinal static stability. Also note that the airship remains statically stable in longitudinal flight even if it is *top-heavy*, provided the horizontal tail volume ratio is sufficiently large positive. Keep in mind, however, that our simple analysis does not consider static *roll* stability.

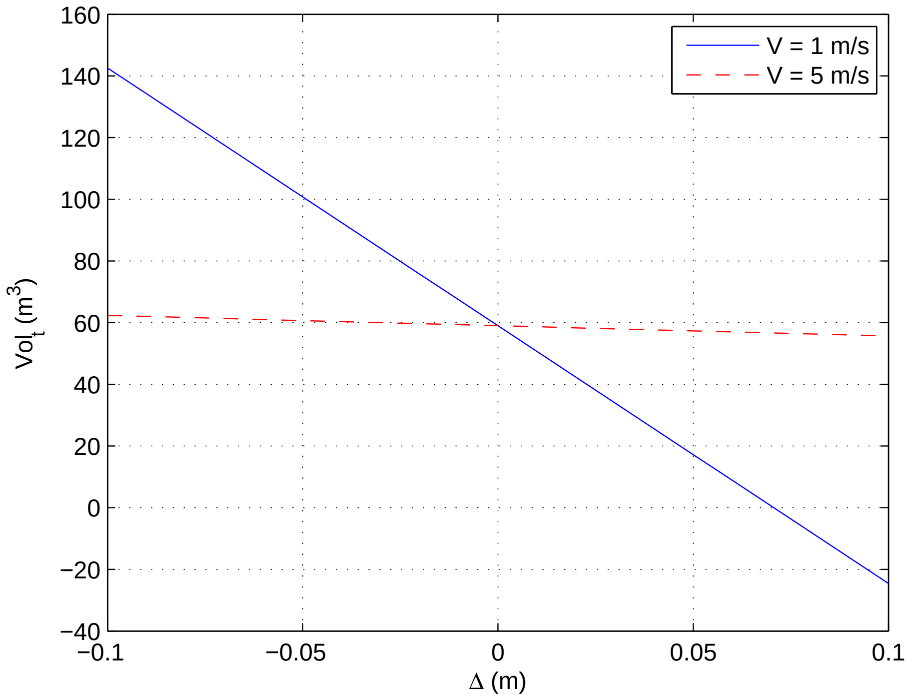

Figure 3: Tail volume versus bottom-heaviness required for static stability.