AOE 3134
Stability and Control
Final Exam

This is an open-book/open-note exam. Do not discuss this exam with anyone except the instructor.

Honor Pledge: I have neither given nor received unauthorized assistance on this assignment.

______________________________ Printed Name   ______________________________ Signature

Problem 1. (35 points) Figure 1 shows lift force and pitch moment data for an airplane with a T-tail. In a T-tail configuration, the horizontal stabilizer is mounted at the top of the vertical stabilizer, well above the fuselage. At large angles of attack, the horizontal stabilizer passes through the wing wake and loses some effectiveness. At even larger angles of attack, the tail is again exposed to the free stream flow and regains its effectiveness. This phenomenon accounts for the "kinks" in the lift and moment data shown in Figure 1.

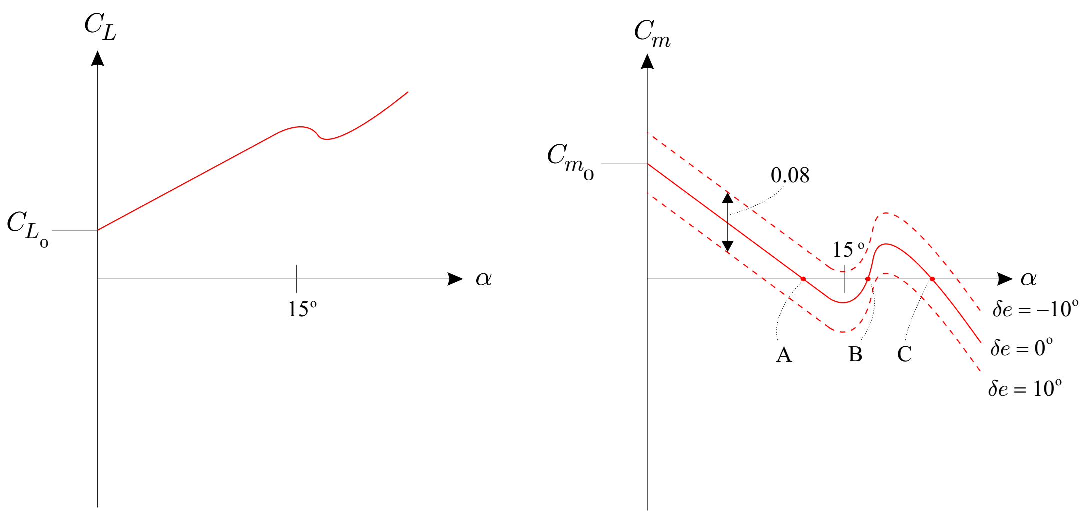

Figure 1: Lift and pitch coefficients for a T-tail airplane.

For this airplane,

$$C_{L_0} = 0.10, \quad C_{L_\alpha} = 0.07 \text{ deg}^{-1}, \quad C_{m_0} = 0.10, \quad \text{and} \quad C_{m_\alpha} = -0.01 \text{ deg}^{-1}.$$

The angle of attack does not exceed 15° degrees in normal operation. Elevator deflections are limited to ±10°. For the data shown in Figure 1, the center of gravity (CG) is located at $h = x_{\text{cg}}/\bar{c} = 0.25$.

Part A. (10 points) Determine the (stick-fixed) neutral point, which serves as the aft limit for the CG location.

Part B. (10 points) Determine a forward limit for the CG location. Describe your method and state any assumptions.

Part C. (10 points) There are three equilibrium angles of attack corresponding to $\delta e = 0°$; they are labeled A, B, and C in Figure 1. For each equilibrium, state whether it is statically stable or unstable.

Part D. (5 points) Using the data shown in Figure 1, explain how it is possible to bring the airplane to equilibrium state C. Next, explain why recovery to equilibrium state A from equilibrium state C may be difficult.

Problem 2. (40 points) Consider an airplane in steady, wings level, equilibrium flight at constant altitude. The following parameter values describe the airplane's inertia properties, its stability properties, and the nominal flight condition.

$$W = 40,000 \text{ lb} \qquad S = 550 \text{ ft}^2$$
$$I_x = 120,000 \text{ slug-ft}^2 \qquad b = 55 \text{ ft}$$
$$I_z = 240,000 \text{ slug-ft}^2 \qquad V = 250 \text{ ft/s}$$
$$I_{xz} = 5000 \text{ slug-ft}^2 \qquad \rho = 0.0024 \text{ slug/ft}^3$$

The table below shows the lateral-directional stability coefficients, where all angles are expressed in *radians*.

	$C_{Y_{(\cdot)}}$	$C_{l_{(\cdot)}}$	$C_{n_{(\cdot)}}$
β	-0.72	-0.10	0.14
p	0	-0.37	-0.14
r	0	0.11	-0.16
δa	0	0.05	-0.01
δr	0.18	0.03	-0.06

You may use a computer to solve the following problems, but write the procedure out explicitly. Also write the values of any parameters (e.g., dimensional stability derivatives) which appear in the expressions. Finally, include the computer script and output with your solutions.

Part A. (20 points) Determine the maximum cross-wind in which the aircraft can land at the given speed, with the following actuator limits:

$$|\delta a| \leq 20^\circ \qquad \text{and} \qquad |\delta r| \leq 15^\circ.$$

Part B. (20 points) Estimate the time to half or double amplitude for the roll and spiral modes. Also estimate the time and number of cycles to half or double amplitude for the Dutch roll mode.

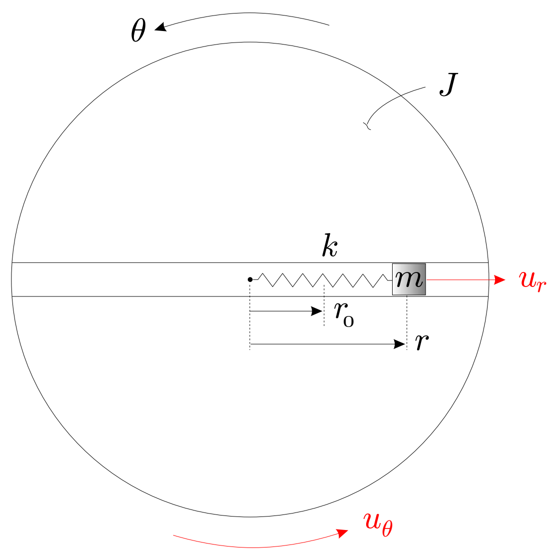

Figure 2: Sketch for Problem 3

Problem 3. (25 points) In Figure 2, a rigid wheel with moment of inertia J spins in the θ direction under the influence of a control torque u_θ. The mass particle m moves radially along the track under the influence of a control force u_r. The spring provides a restoring force in proportion to its displacement from the nominal length r_0. There is no damping in this system. Defining the state vector $\boldsymbol{x} = \left[\theta,\ r,\ \dot{\theta},\ \dot{r}\right]^T$ and the input vector $\boldsymbol{u} = [u_\theta,\ u_r]^T$, the equations of motion are

$$\dot{\boldsymbol{x}} = \begin{pmatrix} x_3 \\ x_4 \\ \frac{1}{J+mx_2^2}\left(-2mx_2x_3x_4 + u_\theta\right) \\ \frac{1}{m}\left(mx_2x_3^2 - k\left(x_2 - r_0\right) + u_r\right) \end{pmatrix}.$$

Part A. (10 points) Find all equilibria for which $\boldsymbol{u} = \boldsymbol{0}$.

Part B. (15 points) One equilibrium is

$$\boldsymbol{x}_{\text{eq}} = \begin{pmatrix} 0 \\ r_0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{u}_{\text{eq}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Linearize the dynamic equations about this equilibrium.