AOE 3134
Stability and Control
Exam #2

Problem 1. **(30 points)** The following second order system describes an oscillator with a stiffening spring and nonlinear damping:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -kx_1(1+x_1^2) - b\sin(x_2) \end{pmatrix}. \quad (1)$$

(a) Find all equilibria.

(b) Linearize equations (1) about the equilibrium $(x_1, x_2)_{\text{eq}} = (0, 0)$.

Bonus (10 points) Define $x = x_1$ and note that $x_2 = \dot{x}$. Rewrite your solution to part (b) as a single, second order ODE in terms of the dependent variable x and its derivatives. Let

$$k = 2 \quad \text{and} \quad b = 3$$

Determine the response $x(t)$ to an arbitrary initial state $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$.

Problem 2. **(40 points)** Table 1 gives the longitudinal stability derivatives for a certain four-engine, transport jet flying horizontally at Mach 0.25 at sea level conditions.[1] Other relevant parameters include:

$$W = 126,000 \text{ lbs} \qquad S = 2,000 \text{ ft}^2 \qquad \bar{c} = 11 \text{ ft} \qquad I_y = 2,450,000 \text{ slug ft}^2.$$

Table 1: Longitudinal Stability Derivatives.

	$C_{D_{(\cdot)}}$	$C_{L_{(\cdot)}}$	$C_{m_{(\cdot)}}$
0 (nominal)	0.08	0.68	0
α	0.3	4.5	-0.9
$\dot{\alpha}$	0	2.7	-4.1
q	0	7.7	-12.1
Ma	0	0	0

(a) Compute approximate values for the natural frequency and damping ratio of the phugoid mode. Use your approximation to estimate the time and number of cycles to half amplitude.

(b) Compute approximate values for the natural frequency and damping ratio of the short period mode. Use your approximation to estimate the time and number of cycles to half amplitude.

[1] For the standard atmosphere at sea level, density is $\rho = 2.3769 \times 10^{-3}$ slugs/ft^3 and the speed of sound is $a = 1116.45$ ft/s.

Problem 3. (30 points) For each of the following questions, circle the letter corresponding to the correct answer. Each question is worth five points. (Don't forget to turn these answers in with your solutions to Problems 1 and 2!)

1. Suppose that, at a given instant, a certain airplane's pitch angle θ passes through 90°. Then, at this same instant,

(a) the body angular velocity $\boldsymbol{\omega}$ becomes undefined.
(b) the matrix $\boldsymbol{R}_{\text{IB}}$ which rotates body frame vectors into the inertial frame becomes singular.
(c) the matrix $\boldsymbol{L}_{\text{BI}}$ which relates $\boldsymbol{\omega}$ to the Euler angle rates becomes singular.
(d) the airplane explodes.

2. The stability axes are a body-fixed reference frame defined relative to a particular wings-level equilibrium flight condition. The frame is defined such that, in nominal flight,

(a) $w = 0$ (b) $u = 0$ (c) $z = 0$ (d) $I_{\text{xz}} = 0$

3. Which of the following statements is *not* a standard assumption in formulating the equations of motion for an aircraft

(a) Y, L, and N do not depend on θ, u, w, or q.
(b) X, Z, and M do not depend on ϕ, ψ, v, p, or r.
(c) N does not depend on v.
(d) X does not depend on q or $\dot{w}$.

4. The terms $Z_{\dot{w}}$ and $M_{\dot{w}}$ are primarily due to

(a) the apparent mass of the wing and horizontal tail in the vertical direction.
(b) an aeroelastic effect involving vertical oscillations of the wing.
(c) second order effects of the propulsion system.
(d) a time delay between changes in wing lift and their effect on downwash at the tail.

5. For a constant thrust propulsion system,

(a) $\left.\frac{\partial T}{\partial u}\right|_0 = 0$, (b) $C_{T_u} = 0$, (c) $C_{X_u} = 0$, (d) $C_{L_\alpha} = 0$

6. For a conventional, dynamically stable airplane:

(a) $C_{m_q} < 0$, $C_{n_r} > 0$, and $C_{l_p} < 0$.
(b) $C_{m_q} < 0$, $C_{n_r} < 0$, and $C_{l_p} > 0$.
(c) $C_{m_q} > 0$, $C_{n_r} > 0$, and $C_{l_p} > 0$.
(d) $C_{m_q} < 0$, $C_{n_r} < 0$, and $C_{l_p} < 0$.