AOE 3134
Stability and Control
Exam #1 Solutions

Problem 1. (30 points) An aircraft in equilibrium flight at a given airspeed and altitude drops its empty fuel tanks. The resulting change in weight coefficient ($C_{W_{\text{final}}} - C_{W_{\text{initial}}}$) is $\Delta C_W = -0.1$. Assuming that the aircraft re-establishes equilibrium flight at the same airspeed and altitude, compute the change in trim angle of attack $\Delta\alpha$ and elevator angle $\Delta\delta e$ given that

$$C_{L_\alpha} = 5.1 \text{ rad}^{-1}, \quad C_{L_{\delta e}} = 0.2 \text{ rad}^{-1}, \quad C_{m_\alpha} = -1.0 \text{ rad}^{-1}, \quad \text{and} \quad C_{m_{\delta e}} = -2.0 \text{ rad}^{-1}.$$

Solution.

$$\begin{pmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{m_\alpha} & C_{m_{\delta e}} \end{pmatrix} \begin{pmatrix} \Delta\alpha \\ \Delta\delta e \end{pmatrix} = \begin{pmatrix} \Delta C_W \\ 0 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \Delta\alpha \\ \Delta\delta e \end{pmatrix} = \begin{pmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{m_\alpha} & C_{m_{\delta e}} \end{pmatrix}^{-1} \begin{pmatrix} \Delta C_W \\ 0 \end{pmatrix}$$

or

$$\begin{pmatrix} \Delta\alpha \\ \Delta\delta e \end{pmatrix} = \left(\frac{1}{C_{L_\alpha} C_{m_{\delta e}} - C_{m_\alpha} C_{L_{\delta e}}} \right) \begin{pmatrix} C_{m_{\delta e}} & -C_{L_{\delta e}} \\ -C_{m_\alpha} & C_{L_\alpha} \end{pmatrix} \begin{pmatrix} \Delta C_W \\ 0 \end{pmatrix}.$$

Substituting values,

$$\begin{pmatrix} \Delta\alpha \\ \Delta\delta e \end{pmatrix} = -\frac{1}{10} \begin{pmatrix} -2 & -0.2 \\ 1 & 5.1 \end{pmatrix} \begin{pmatrix} -0.1 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.02 \\ 0.01 \end{pmatrix}.$$

Problem 2. (40 points) Figure 1 depicts a weathervane in a steady north wind. Assuming that $\psi \ll 1$, the motion of this device is well-described by the equation

$$I_z \ddot{\psi} = N_\psi \psi + N_r \dot{\psi} \tag{1}$$

where I_z is the moment of inertia about the pivot. The terms on the right represent the total aerodynamic moment. (Ignore the frictional moment about the pivot point.)

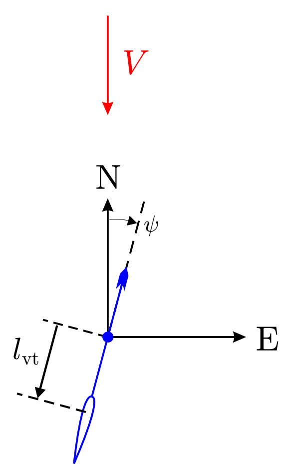

Figure 1: A weathervane in a steady north wind.

A. Derive expressions for N_ψ and N_r. Show your work. Include diagrams, as necessary.

B. Equation (1) describes a damped oscillator. Consider the following parameter values:

$$\rho = 1 \text{ kg/m}^3, \quad S_{\text{vt}} = 0.02 \text{ m}^2, \quad V = 10 \text{ m/s},$$

$$l_{\text{vt}} = 0.1 \text{ m}, \quad I_z = 0.005 \text{ kg m}^2 \quad \text{and} \quad C_{L_{\alpha_{\text{vt}}}} = 1 \text{ rad}^{-1}.$$

Assume that the oscillator is underdamped, so that (1) may be rewritten in the form

$$\ddot{\psi} + 2\zeta\omega_n\dot{\psi} + \omega_n^2\psi = 0$$

where $\omega_n > 0$ and $|\zeta| < 1$. Compute ω_n and ζ.

Solution. (A.) Using small angle approximations in the moment balance,

$$N_\psi = -\left(\left(\frac{1}{2}\rho V^2\right) S_{\text{vt}}\right) l_{\text{vt}} C_{L_{\alpha_{\text{vt}}}}.$$

For small angular rates $\dot{\psi}$, the increment in angle of attack due to $\dot{\psi}$ is

$$\Delta\alpha_{\text{vt}} = \frac{l_{\text{vt}}\dot{\psi}}{V},$$

so

$$N_r = -\left(\left(\frac{1}{2}\rho V^2\right) S_{\text{vt}}\right) \frac{l_{\text{vt}}^2}{V} C_{L_{\alpha_{\text{vt}}}}.$$

(B.) Writing out

$$I_z\ddot{\psi} - N_r\dot{\psi} - N_\psi\psi = 0$$

with the expressions from Part A gives

$$I_z\ddot{\psi} + \left[\left(\left(\frac{1}{2}\rho V^2\right) S_{\text{vt}}\right) \frac{l_{\text{vt}}^2}{V} C_{L_{\alpha_{\text{vt}}}}\right]\dot{\psi} + \left[\left(\left(\frac{1}{2}\rho V^2\right) S_{\text{vt}}\right) l_{\text{vt}} C_{L_{\alpha_{\text{vt}}}}\right]\psi = 0.$$

Substituting values and dividing by I_z gives

$$\ddot{\psi} + \underbrace{0.2}_{2\zeta\omega_n}\dot{\psi} + \underbrace{20}_{\omega_n^2}\psi = 0.$$

Thus, we find that

$$\omega_n = \sqrt{20} \text{ rad/s} \approx 4.5 \text{ rad/s} \qquad \text{and} \qquad \zeta = 0.022.$$

In the given wind conditions, the weathervane is extremely underdamped.

Problem 3. (30 points) For each of the following questions, circle the letter corresponding to the best answer. Each question is worth five points. (Don't forget to turn these answers in with your solutions to Problems 1 and 2!)

1. The sidewash angle σ

(a) does not affect the restoring moment generated by a top-side vertical tail.
(b) increases the restoring moment generated by a top-side vertical tail.
(c) is the negative of the sideslip angle β.
(d) is independent of the sideslip angle β.

2. If the CG is shifted aft in a statically stable aircraft,

(a) the aircraft becomes more stable in pitch.
(b) the aircraft becomes more stable in yaw.
(c) the aircraft becomes more stable in roll.
(d) the elevator becomes more effective.

3. For an aircraft in a wings level pull-up, the pitch rate q

(a) increases with increasing load factor.
(b) increases with increasing speed.
(c) induces a nose-up pitch moment.
(d) does none of the above.

4. The fundamental requirement for static longitudinal stability is that

(a) $C_{m_{0L}} < 0$ and $C_{m_\alpha} < 0$.
(b) $C_{m_{0L}} > 0$ and $C_{m_\alpha} < 0$.
(c) $C_{m_{0L}} > 0$ and $C_{m_\alpha} > 0$.
(d) $C_{m_{0L}} < 0$ and $C_{m_\alpha} > 0$.

5. For an aircraft in a steady, banked turn to the right,

(a) $\dot{\theta} > 0$, (b) $q = 0$, (c) $\dot{\psi} = 0$, **(d) none of the preceding.**

6. Which of the following steps would contribute a negative increment to C_{l_β}?

(a) Increasing the dihedral angle.
(b) Mounting the wing above the fuselage.
(c) Sweeping the wings aft.
(d) All of the above.