AOE 3134
Stability and Control
Exam #1 Solutions

Problem 1. (40 points) To a good approximation, a particular airplane has the following static lift and pitch moment characteristics:

$$\begin{aligned} C_L &= 0.10\,\alpha \\ C_m &= 0.10 - 0.02\,\alpha - 0.05\,\delta e \end{aligned}$$

where all angles are measured in degrees. Other parameters include

$$b = 20 \text{ m}, \qquad S = 50 \text{ m}^2, \qquad \bar{c} = 2.5 \text{ m}, \qquad V_{\mathrm{H}} = 1, \qquad W = 100 \text{ kN}.$$

1. Determine the stick-fixed static margin.
2. Determine the equilibrium angle of attack and elevator deflection corresponding to steady, wings-level flight at speed $V = 100$ m/s in air of density $\rho = 1$ kg/m^3.

Solution. The stick-fixed static margin is

$$K_n = -\left(h - h_n\right) = -\frac{C_{m_\alpha}}{C_{L_\alpha}} = -\frac{-0.02}{0.10} = 0.2.$$

To determine the equilibrium flight conditions, we first compute

$$C_W = \frac{W}{\left(\frac{1}{2}\rho V^2\right) S} = \frac{100,000}{\left(\frac{1}{2}(1)(100)^2\right)(50)} = 0.4.$$

We must solve the system

$$\begin{pmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{m_\alpha} & C_{m_{\delta e}} \end{pmatrix} \begin{pmatrix} \alpha \\ \delta e \end{pmatrix} = \begin{pmatrix} C_W \\ -C_{m_0} \end{pmatrix}$$

or

$$\begin{pmatrix} 0.1 & 0 \\ -0.02 & -0.05 \end{pmatrix} \begin{pmatrix} \alpha \\ \delta e \end{pmatrix} = \begin{pmatrix} 0.4 \\ -0.1 \end{pmatrix}.$$

The solution is

$$\begin{aligned} \begin{pmatrix} \alpha \\ \delta e \end{pmatrix} &= \begin{pmatrix} 0.1 & 0 \\ -0.02 & -0.05 \end{pmatrix}^{-1} \begin{pmatrix} 0.4 \\ -0.1 \end{pmatrix} \\ &= \frac{1}{-0.005} \begin{pmatrix} -0.05 & 0 \\ 0.02 & 0.1 \end{pmatrix} \begin{pmatrix} 0.4 \\ -0.1 \end{pmatrix} \\ &= -200 \begin{pmatrix} -0.02 \\ -0.002 \end{pmatrix} = \begin{pmatrix} 4^\circ \\ 0.4^\circ \end{pmatrix}. \end{aligned}$$

Problem 2. (35 points) A sophisticated new airplane uses "smart materials" and "smart structures" to provide the necessary control moments. Although the control deflections are still denoted δa and δr, according to the standard convention, these new actuators completely decouple roll and yaw control the lateral aerodynamic force no longer depends on the control deflections. For this aircraft,

	$C_{Y_{(\cdot)}}$	$C_{l_{(\cdot)}}$	$C_{n_{(\cdot)}}$
β	-0.1	-0.1	0.2
δa	0	0.1	0
δr	0	0	-0.1

All coefficients in the table above are per radian. The airplane has the following characteristics:

$$W = 18,000 \text{ N}, \qquad S = 20 \text{ m}^2, \qquad b = 10 \text{ m}$$

Compute the control deflections δa and δr and the roll angle ϕ necessary for this airplane to land at sea level in a 6 m/s cross-wind coming from the right side. The total airspeed at landing (including the cross-wind) is $V = 60$ m/s. (Note that β is a small angle so that $\sin\beta \approx \beta$.) Assume that the local air density is $\rho = 1$ kg/m^3.

Solution. From the equilibrium requirements

$$\begin{aligned} C_{Y_\beta}\beta + C_{Y_{\delta a}}\delta a + C_{Y_{\delta r}}\delta r + C_W\phi &= 0 \\ C_{l_\beta}\beta + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r &= 0 \\ C_{n_\beta}\beta + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r &= 0, \end{aligned}$$

we find that we need

$$\begin{pmatrix} C_{Y_{\delta a}} & C_{Y_{\delta r}} & C_W \\ C_{l_{\delta a}} & C_{l_{\delta r}} & 0 \\ C_{n_{\delta a}} & C_{n_{\delta r}} & 0 \end{pmatrix} \begin{pmatrix} \delta a \\ \delta r \\ \phi \end{pmatrix} = - \begin{pmatrix} C_{Y_\beta} \\ C_{l_\beta} \\ C_{n_\beta} \end{pmatrix} \beta$$

where

$$\beta = \sin^{-1}\left(\frac{\text{Crosswind velocity}}{\text{Total airspeed}}\right).$$

For this problem,

$$\beta = \sin^{-1}\left(\frac{6}{60}\right) \approx 0.1 \text{ rad.}$$

Also,

$$C_W = \frac{W}{QS} = \frac{18,000}{\left(\frac{1}{2}(1)(60)^2\right)(20)} = 0.5$$

Substituting parameter values,

$$\begin{pmatrix} 0 & 0 & 0.5 \\ 0.1 & 0 & 0 \\ 0 & -0.1 & 0 \end{pmatrix} \begin{pmatrix} \delta a \\ \delta r \\ \phi \end{pmatrix} = - \begin{pmatrix} -0.1 \\ -0.1 \\ 0.2 \end{pmatrix} (0.1) = \begin{pmatrix} 0.01 \\ 0.01 \\ -0.02 \end{pmatrix}.$$

Note that these three equations are decoupled. Rearranging the order of variables on the left, we have

$$\begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.1 \end{pmatrix} \begin{pmatrix} \phi \\ \delta a \\ \delta r \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.01 \\ -0.02 \end{pmatrix}$$

which is easily solved:

$$\begin{aligned}
\begin{pmatrix} \phi \\ \delta a \\ \delta r \end{pmatrix} &= \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.1 \end{pmatrix}^{-1} \begin{pmatrix} 0.01 \\ 0.01 \\ -0.02 \end{pmatrix} \\
&= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & -10 \end{pmatrix} \begin{pmatrix} 0.01 \\ 0.01 \\ -0.02 \end{pmatrix} \\
&= \begin{pmatrix} 0.02 \\ 0.1 \\ 0.2 \end{pmatrix} \mathrm{rad} \approx \begin{pmatrix} 1.1^\circ \\ 5.7^\circ \\ 11.5^\circ \end{pmatrix}.
\end{aligned}$$

Problem 3. (25 points) For each of the following questions, circle the letter corresponding to the correct answer. Each question is worth five points. (Don't forget to turn these answers in with your solutions to Problems 1 and 2!)

1. The downwash angle ϵ

(a) increases with increasing sideslip angle.
(b) decreases with increasing sideslip angle.
(c) increases with increasing wing angle of attack.
(d) is unaffected by changes in wing angle of attack.

2. Viewed from aft, the tail of a "V-tail" airplane forms a "V" (hence the name). This alternative configuration combines the roles of the vertical and horizontal stabilizers. An "inverted V-tail" hangs below the fuselage, forming an upside-down "V." Compared with an upright V-tail, an inverted V-tail

(a) provides more roll stability.
(b) provides less roll stability.
(c) is unaffected by downwash.
(d) eliminates the need for control surfaces.

3. If the CG is shifted forward in a statically stable aircraft, the elevator's effectiveness at changing the trim angle of attack

(a) remains the same, (b) becomes greater, **(c) becomes smaller.**

4. For an aircraft in a steady turn or a wings level pull-up, the increment in pitch moment due to the pitch rate q

(a) opposes the pitching motion.
(b) tends to increase the pitching motion.
(c) causes the aircraft to roll.
(d) causes the aircraft to yaw.

5. The fundamental requirement for static longitudinal stability is that

(a) $C_{m_{0L}} > 0$ and $C_{m_\alpha} > 0$.
(b) $C_{m_{0L}} < 0$ and $C_{m_\alpha} > 0$.
(c) $C_{m_{0L}} < 0$ and $C_{m_\alpha} < 0$.
(d) $C_{m_{0L}} > 0$ **and** $C_{m_\alpha} < 0$**.**