AOE 3134
Stability and Control
Exam #1 Solutions

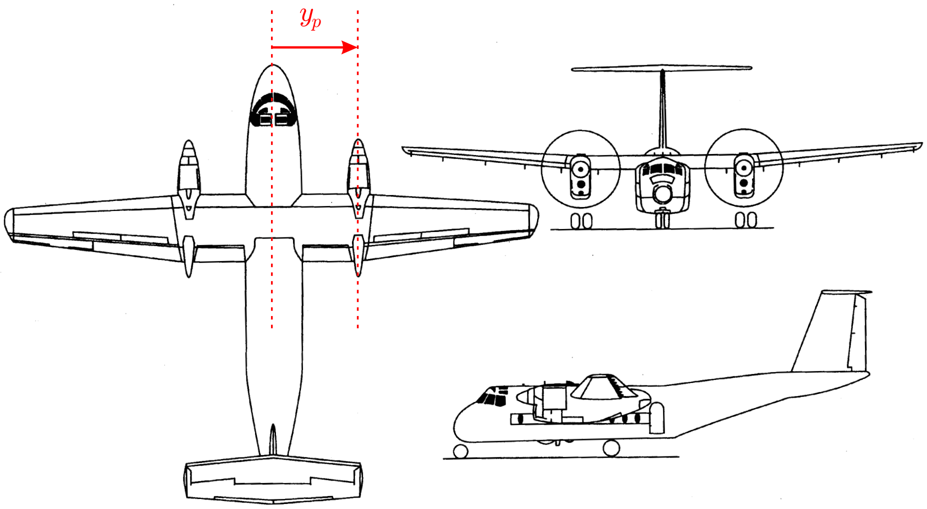

Figure 1: A STOL aircraft.

Figure 1 shows a twin-engine, short takeoff and landing aircraft. When flying at 10,000 ft[1] at Mach number $M = 0.370$, the two engines generate a combined 4,750 lbs of thrust. Mass and geometric parameters include:

$$W = 40{,}000 \text{ lbs} \qquad S = 945 \text{ ft}^2 \qquad \bar{c} = 10.1 \text{ ft} \qquad b = 96 \text{ ft} \qquad y_p = 15.6 \text{ ft}.$$

Stability and control derivatives are given in the following tables, with longitudinal quantities on the left and lateral-directional quantities on the right. Note: Derivatives with respect to angles are *per radian.*

	$C_{L_{(\cdot)}}$	$C_{m_{(\cdot)}}$
α	5.24	-0.780
δe	0.465	-2.12
q	7.83	-35.6

	$C_{Y_{(\cdot)}}$	$C_{l_{(\cdot)}}$	$C_{n_{(\cdot)}}$
β	-0.362	-0.125	0.101
δa	0.000	0.200	0.000
δr	0.233	0.0240	-0.107

Problem 1. (50 points) (a) Given that

$$\begin{aligned} C_L &= C_{L_\alpha}\alpha + C_{L_{\delta e}}\delta e \\ C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e \end{aligned}$$

with $C_{m_0} = 0.0120$, compute the equilibrium angle of attack and elevator angle.

(b) Compute the additional elevator increment $\Delta\delta e$ that would be required for a steady, wings level $2g$ pull-up (that is, a pull-up with load factor $n = 2$).

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[1] For the standard atmosphere at 10,000 ft altitude, density is $\rho = 1.7556 \times 10^{-3}$ slugs/ft^3 and the speed of sound is $a = 1077.4$ ft/s.

Solution. (a) The equilibrium flight conditions are

$$\begin{aligned} C_w &= C_{L_\alpha}\alpha + C_{L_{\delta e}}\delta e \\ 0 &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e \end{aligned}$$

where

$$C_w = \frac{W}{\left(\frac{1}{2}\rho V^2\right) S}.$$

Solving, we find that

$$\begin{aligned} \begin{pmatrix} \alpha \\ \delta e \end{pmatrix} &= \begin{pmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{m_\alpha} & C_{m_{\delta e}} \end{pmatrix}^{-1} \begin{pmatrix} C_w \\ -C_{m_0} \end{pmatrix} = \begin{pmatrix} 5.24 & 0.465 \\ -0.780 & -2.12 \end{pmatrix}^{-1} \begin{pmatrix} 0.303 \\ -0.0120 \end{pmatrix} \\ &= \begin{pmatrix} 0.0593 \\ -0.0162 \end{pmatrix} \text{rad} = \begin{pmatrix} 3.40^\circ \\ -0.927^\circ \end{pmatrix}. \end{aligned}$$

(b) The elevator increment required for a $2g$ pull-up is

$$\Delta\delta e = \frac{(n-1)C_w}{C_{L_\alpha}C_{m_{\delta e}} - C_{L_{\delta e}}C_{m_\alpha}}\left[-C_{m_\alpha} + \left(C_{m_\alpha}C_{L_q} - C_{L_\alpha}C_{m_q}\right)\left(\frac{\rho\bar{c}S}{4m}\right)\right] = -0.0392 \text{ rad} = -2.25^\circ.$$

Problem 2. (25 points) Suppose that the left engine fails and that the right engine is throttled up to produce the same total thrust $T = 4,750$ lbs. Compute the aileron and rudder deflections, δa and δr, necessary to maintain equilibrium flight, along with the resulting constant roll angle ϕ.

Solution. The nondimensional yaw moment due to the asymmetric thrust condition is

$$C_{n_T} = -\frac{Ty_p}{QSb} = -0.00586.$$

Requiring that the sideslip angle be zero, we obtain the following:

$$\begin{aligned} \begin{pmatrix} \delta a \\ \delta r \\ \phi \end{pmatrix} &= \begin{pmatrix} 0 & C_{Y_{\delta r}} & C_w \\ C_{l_{\delta a}} & C_{l_{\delta r}} & 0 \\ C_{n_{\delta a}} & C_{n_{\delta r}} & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ -C_{n_T} \end{pmatrix} = \begin{pmatrix} 0 & 0.233 & 0.303 \\ 0.200 & 0.0240 & 0 \\ 0 & -0.107 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0.00586 \end{pmatrix} \\ &= \begin{pmatrix} 0.00657 \\ -0.0547 \\ 0.0420 \end{pmatrix} \text{rad} = \begin{pmatrix} 0.376^\circ \\ -3.14^\circ \\ 2.41^\circ \end{pmatrix}. \end{aligned}$$

Problem 3. (25 points) For each of the following questions, circle the letter corresponding to the correct answer. Each question is worth five points. (Don't forget to turn these answers in with your solutions to Problems 1 and 2!)

1. Consider a dynamical system which is in a state of equilibrium. Suppose that, after a small perturbation is applied that slightly changes the state, the system initially tends to move back toward the equilibrium. One may reasonably infer that this system is

(a) statically stable (b) dynamically stable (c) statically unstable (d) dynamically unstable.

2. For a given conventional airplane, moving the center of gravity (CG) forward increases static longitudinal stability. Ignoring the stall phenomenon, moving the CG forward also

(a) increases the range of achievable equilibrium α.
(b) decreases the range of achievable equilibrium α.
(c) leaves the range of achievable equilibrium α unchanged.

3. For a stable conventional airplane with reversible controls, the stick-free static margin is generally

(a) smaller than the stick-fixed static margin.
(b) greater than the stick-fixed static margin.
(c) the same as the stick-fixed static margin.

4. For an airplane which is statically stable in pitch, yaw, and roll:

(a) $C_{m_\alpha} < 0$, $C_{n_\beta} < 0$, and $C_{l_\beta} < 0$.
(b) $C_{m_{\delta e}} < 0$, $C_{n_{\delta r}} < 0$, and $C_{l_{\delta a}} > 0$.
(c) $C_{m_\alpha} < 0$, $C_{n_\beta} > 0$, and $C_{l_\beta} < 0$.
(d) $C_{m_q} < C_{m_\alpha}$.

5. A common consequence of the coupling between roll and yaw is that $C_{n_{\delta a}} < 0$. This phenomenon is referred to as

(a) weathercock stability (b) free aileron effect (c) dihedral effect **(d) adverse yaw.**