Local Sidereal Time Calculation

Local sidereal time (LST), denoted θ, is the angle between the local line of longitude and the $\hat{\mathbf{I}}$ axis (vernal equinox). (See Figs. 2.8.3 and 2.8.4 in BMW.) Since the earth is rotating relative to the IJK frame, θ varies for a fixed point on earth as

$$\begin{aligned}\dot{\theta} = \omega_{\oplus} &= 360^{\circ}/\text{sidereal day}\\ &= 1.0027379093 \times 360^{\circ}/\text{day}\\ &= 1.0027379093 \times 2\pi\text{rad/day}\\ &= 7.292115856 \times 10^{-5}\text{rad/sec}\end{aligned}$$

These numbers are just four different ways of expressing the angular velocity of the earth, $\omega_{\oplus}$.

Thus if $\theta_o = \theta(t_o)$ is known, $\theta(t)$ can be calculated by

$$\theta(t) = \theta_o + \omega_{\oplus}(t - t_o)$$

As pointed out in the text, one usually refers to a table to find the LST of Greenwich (0 longitude) at t_o, denoted θ_{go}. Then the LST of the point of interest is calculated by

$$\theta = \theta_g + \lambda_E = \theta_{go} + \omega_{\oplus}(t - t_o) + \lambda_E$$

where λ_E is the east longitude of the point of interest. This (λ_E) is clearly constant, and θ_{go} is simply a lookup. Thus the only "hard" calculation to make is $\omega_{\oplus}(t - t_o)$.

To do this, you simply have to express $\omega_{\oplus}$ and $t - t_o$ in consistent units, *e.g.*, rad/sec and sec, °/sec and sec, °/day and days, *etc.*

The two equations on p. 104 in BMW give the formula for θ_g in terms of °/day and days, and rad/day and days, respectively.

So, you only need to figure out the $\Delta t = t - t_o$, put it in the "right" units (don't convert it!), and multiply by $\omega_{\oplus}$, using the same units. What could be easier?

For example, for Homework 3Problem 3, $\theta_{go} = 99.990704^{\circ}$ (BMW, p. 104), and $t - t_o =$ 27 hrs, since 1 day plus 3 hours has passed since the t_o for which θ_{go} is given. And the longitude of Goose Bay is about 60° west, so $\lambda_E = -60^{\circ}$. Thus

$$\begin{aligned}\theta &= 99.990704^{\circ} + 1.0027379093 \times 360^{\circ}/\text{day} \times \frac{1}{24\text{hrs/day}} \times 27\text{hrs} - 60^{\circ}\\ &= 446.0996^{\circ} = 86.0996^{\circ}\end{aligned}$$

The 27/24 is the D used on p. 104 in BMW.

By the way, in the example on pp. 107–108 in BMW, they use $\omega_{\oplus} = 15^{\circ}/\text{hour}$, thereby adding to the confusion. Of course, they should've used $\omega_{\oplus} = 15.04106864^{\circ}/\text{hour}$, but then they wouldn't've got the answer in the book.