Analytical Hierarchy Process

- A systematic method for comparing a list of objectives or alternatives
- When used in the systems engineering process, AHP can be a powerful tool for comparing alternative design concepts
- Reference: Ernest H. Forman, *Decision by Objectives*, http://mdm.gwu.edu/Forman/DBO.pdf

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- Assume that a set of objectives has been established (VSD, OH), and that we are trying to establish a normalized set of weights to be used when comparing alternatives using these objectives.
- For simplicity, we assume that there are 4 objectives: O_1, O_2, O_3, and O_4.

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- Form a pairwise comparison matrix *A*, where the number in the i_{th} row and j_{th} column gives the relative importance of O_i as compared with O_j
- Use a 1–9 scale, with
  - a_{ij} = 1 if the two objectives are equal in importance
  - a_{ij} = 3 if O_i is weakly more important than O_j
  - a_{ij} = 5 if O_i is strongly more important than O_j
  - a_{ij} = 7 if O_i is very strongly more important than O_j
  - a_{ij} = 9 if O_i is absolutely more important than O_j

  - a_{ij} = 1/3 if O_j is weakly more important than O_i

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- Thus we might arrive at the following matrix:

$$A = \begin{bmatrix} 1 & 1/5 & 1/3 & 1/7 \\ 5 & 1 & 3 & 5 \\ 3 & 1/3 & 1 & 3 \\ 7 & 1/5 & 1/3 & 1 \end{bmatrix} = \begin{bmatrix} 1.000 & 0.200 & 0.333 & 0.143 \\ 5.000 & 1.000 & 3.000 & 5.000 \\ 3.000 & 0.333 & 1.000 & 3.000 \\ 7.000 & 0.200 & 0.333 & 1.000 \end{bmatrix}$$

- To normalize the weights, compute the sum of each column and then divide each column by the corresponding sum
- Using an overbar to denote normalization, we get:

$$\overline{A} = \begin{bmatrix} 0.063 & 0.115 & 0.071 & 0.016 \\ 0.313 & 0.577 & 0.643 & 0.547 \\ 0.188 & 0.192 & 0.214 & 0.328 \\ 0.438 & 0.115 & 0.071 & 0.109 \end{bmatrix}$$

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- The numbers in the second row are generally larger than the rest of the numbers, except for the case of column 1
- This indicates some inconsistency in the comparisons used in the original matrix
- Ideally, the 4 normalized columns would all be identical if the pairwise comparisons were consistent
- In practice, one can compute a consistency measure using the eigenvalues of the normalized comparison matrix.

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- The next step is to compute the average values of each row and use these as the weights in the Objective Hierarchy
- For this example, the weights would be:

$$w = [0.066 \quad 0.520 \quad 0.231 \quad 0.183]^T$$

- Note that by construction, $\sum_{i=1}^{4} w_i = 1.$
- These weights would be used in summing the measures as required in the evaluation of the Objective Hierarchy.