Dynamics and Control
Preliminary Examination Topics

1. Particle and Rigid Body Dynamics
    The topics of interest can be found in:
        Meirovitch, Leonard; *Methods of Analytical Dynamics*, McGraw-Hill, Inc New York, NY, 1970
            Chapters 1 - 5

2. Atmospheric Flight Mechanics
    The topics of interest can be found in:
        Etkin, Bernard, and Reid, Loyd Duff; *Dynamics of Flight, Stability and Control* Third Edition, John Wiley and Sons, Inc., New York, NY, 1996
            Chapters 4 - 7
        Anderson, John; *Introduction to Flight, 4th ed*, McGraw-Hill, Inc. New York, NY, 1999
            Chapter 6 (Performance chapter)

3. Space Flight Mechanics
    The topics of interest can be found in:
        Bate, Roger, Mueller, Donald, and White, Jerry; *Fundamentals of Astrodynamics,* Dover Publications, New York, NY, 1970
            Chapters 1(all), 2 (2.1- 2.9), 3(all), 4 (except 4.3-4.5), 7 (7.4)

4. Control
    A. Linear System Theory
        The topics of interest can be found in:
        Bay, John, *Fundamentals of Linear state Space Systems*, McGraw Hill, New York, NY, 1998
            Chapters 1 - 8

    B. Linear Optimal Control
        The topics of interest can be found in:
        Kwakernaak, Huibert and Sivan, Raphael; *Linear Optimal Control Systems,* Wiley Interscience, (John Wiley & Sons, New York, NY. 1972
            Chapters 3 - 5

    C. Optimal Control
        The topics of interest can be found in:
        Gill, Philip, Murray, Walter, and Wright, Margaret; *Practical Optimization,* Academic Press, New York, NY, 1981
            Chapters 1 - 3, 4(4.1-4.5), 5 (5.1-5.2), 6 (6.1 - 6.6)
        Betts, J. T.; *Practical Methods for Optimal Control Using Nonlinear Programming*, in SIAM book Advances in Design and Control 3, SIAM Press, 2001
        Leitman, George; *The Calculus of Variations and Optimal Control, Kluwer Acade;mic Publishers*, 1981
            Chapters 10, 11

Dynamics and Control - Preliminary Examination Format

In general, each question covers more than one of the topics outlined previously. However questions are grouped into roughly the four categories as indicated. There are two questions from each category. The student is required to answer a total of four questions, from at least three different categories

Dynamics and Control Sample Problem

You are to analyze the problem of minimum ground-roll during take-off for a jet-powered vehicle with thrut vectoring Treat the problem in point-mass approximation assuming the entire ground-roll takes place with the vehicle at a fixed incidence. The gross thrust can be continuously vectored; the gross-thrust angle to the horizontal (θ) can be varied between specified bounds ($\theta_{min} \leq \theta(t) \leq \theta_{max}$). The angle-of-attack during the ground-roll is constant. The problem is to determine the thrust-angle history $\theta^*(t)$ and the angle-of-attack (or lift coefficient) (C_L) to achieve a specified take-off speed in the shortest distance.

a) Completely define an appropriate dynamic model (states, controls, differential equations, bounds).

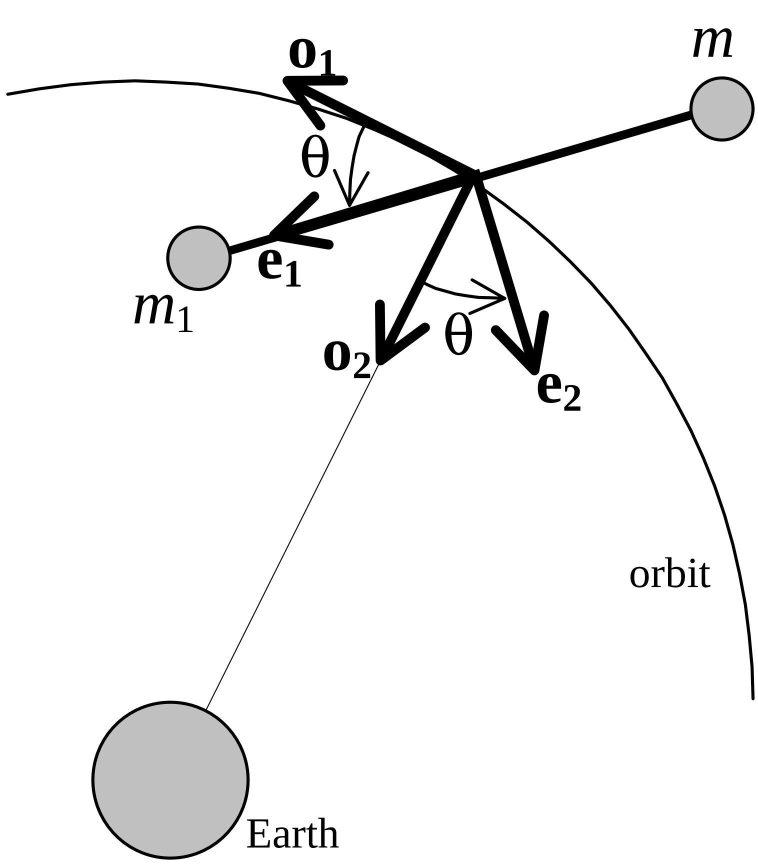
b) Completely specify an optimal control problem (initial conditions, end conditions, cost functional).

c) Apply the Minimum Principle and characterize extremal paths (so that someone knowledgeable about scientific programming, but not about control theory, could use your analysis to compute a solution).

Preliminary Exam Dynamics Problem

The figure shows a simplified model of a tethered satellite. The simplified model defines the two end bodies as equal point masses ($m_1 = m_2 = m$) and the tether as a rigid, massless rod of constant length $2d$. The center of mass orbits the Earth (gravitational parameter μ) in a circular orbit of constant radius R. The orbital frame is a rotating reference frame ($\mathbf{o}_1$, $\mathbf{o}_2$, $\mathbf{o}_3$ where $\mathbf{o}_3 = \mathbf{o}_1 \times \mathbf{o}_2$), with angular velocity $\omega_o = \omega_o \mathbf{o}_3$ relative to inertial space. The position and velocity vectors of the mass center are $\mathbf{R} = -R\mathbf{o}_2$ and $\mathbf{V} = v\mathbf{o}_1$, respectively. The body frame is a rotating reference frame ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ where $\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2$), with angular velocity $\omega_b = \dot{\theta}\ \mathbf{e}_3$ relative to the orbital frame.

a) Develop a 2nd order differential equation of motion for m_1 in terms of the angle θ. State all assumptions and show all steps in this derivation.
b) Simplify the equation by using the assumption that $R >> d$.
c) Determine the stability of all steady solutions. Use symmetry to reduce the number of steady solutions if necessary.
d) If there are any unstable steady solutions, choose one and develop a control strategy to stabilize the unstable steady motion.



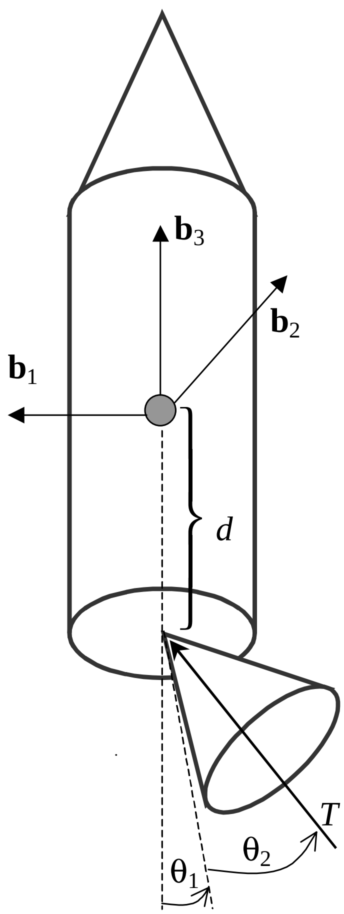
<u>Preliminary Exam Flight Dynamics and Control Problem</u>

A rocket uses a thrust-vector control system to swivel the rocket engine to maintain correct orientation of the vehicle. As shown in the figure, the engine gimbal angles are θ_1 and θ_2. Note that θ_1 is a positive rotation about the "1" axis, and θ_2 is a positive rotation about the "2" axis. The thrust T is applied at a point a distance d aft of the mass center. The moments of inertia of the rocket, about the principal axes shown are A, A, and C, respectively. Ignore the change in these moments of inertia due to swiveling the nozzle.

a) Develop equations of motion for the rocket.

b) Linearize these equations for small ω_1, ω_2, and ω_3, and small θ_1 and θ_2.

c) Develop a feedback control law such that the rotational motion is stable.



Flight Dynamics and Control Problem

The equations of motion for a body moving in a plane are given in plane polar coordinates by:

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \qquad (1)$$

a) For the case where $\dot{\theta} = \omega_{ref} = const$, find a value of r_{ref} in terms of μ and ω_{ref} , that is, the value of r_{ref} in this "steady state" case.

b) Linearize the system (1) for motion in the neighborhood of the reference flight condition established in (a). Select for state variables the states $x^T = [\Delta v_r, \Delta r, \Delta\omega]$, (note that θ is ignorable here and the θ equation can be dropped), where Δ(•) is the small perturbation away from the reference flight condition.

c) Find the eigenvalues and the associated eigenvectors for the third order linearized system found in part (b). Discuss the significance of the eigenvalues and their associated eigenvectors with regard to the concepts of stability and mode shapes.

d) Assume now that we have a control, a_r, which acts only in the radial direction. Modify the linear system found in part (b) to include this control. Furthermore, assume that we can only measure the angular rate. Discuss the controllability and observability of this modified system.

e) For this part of the problem assume that we can sense Δr and Δv_r only. Design a simple control system that will insure that any disturbance in the radial direction will return to the nominal radius in 0.1 TU. For this problem assume that $\mu = 1\ DU^3/TU^2$, $r_{ref} = 1.1$ DU. (TU = time unit, DU= distance unit). Note that to return to the nominal radius, we mean that it will eliminate 98% of the initial displacement.

Flight Dynamics and Control Problem

An approximation to the longitudinal Phugoid motion can be obtained by examining the equations:

$$\dot{V} = \frac{1}{m}[T - D - mg\sin\gamma]$$
$$\dot{\gamma} = \frac{1}{mV}[L - mg\cos\gamma] \qquad (1)$$

Here we will assume that Thrust (T) = a constant and that lift (L) and drag (D) are functions velocity (V) only. However in this approximation, we will hold the angle-of-attack constant so that the only variables that appear are velocity and flight-path-angle (γ). Assume a reference flight condition that is not level with a flight path angle (γ_{ref}) and velocity (V_{ref}).

a) Write a version of the equations-of-motion (1) that are valid for the reference flight condition.

b) Linearize the equations-of-motion about the reference flight condition, and put the results in the form, $\Delta\dot{x} = A\,\Delta x + B\,\Delta u$, where $\Delta x = [\Delta V \;\; \Delta\gamma]^T$, and $\Delta u = \Delta T$ (Note these results should be as simplified as possible).

c) Determine the characteristic equation for this system, and discuss stability issues. Py special attention to the slopes of L and D with V, and also the effects of flight path angle γ_{ref}.

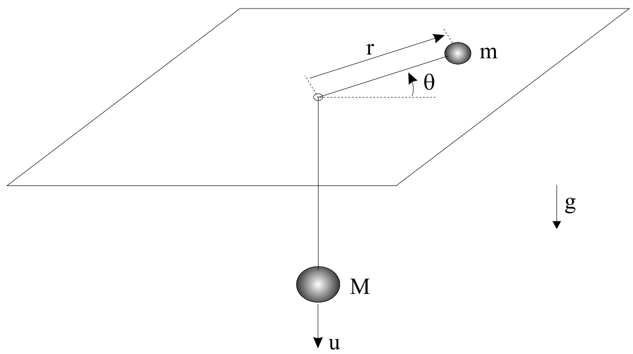
d) Assume a level flight reference flight condition ($\gamma_{ref} = 0$). Also assume the following reference flight values and parameters: Weight, W = 38,200 lbs, Wing area, S=542.5 ft^2, Air density, $\rho = 0.00238$ slugs/ft^3. Reference speed, V = 223.3 ft/sec. $C_{L\,ref} = 1.19$, $C_{D\,ref} = 0.095$.

1) For uncontrolled response to initial conditions, determine:
   - i) Frequency (rad/sec)
   - ii) Period (sec)
   - iii) Time to ½ or double amplitude
   - iv) damping ratio (ζ)

e) Determine the steady state response to a step input in ΔT

f) Outline and discuss how you would construct and altitude-hold controller for this vehicle.

Sample Problem for the AOE D&C Preliminary Exam



Consider two point masses m and M which are connected by an inextensible cord that passes through a small hole in a horizontal plate. The smaller mass m slides along the plate while the larger mass M hangs vertically. A control force u acts on M in the vertical direction as shown. Assume there is no friction in the system.

1. Derive the equations of motion using your favorite technique.
2. Compute the rate of change of energy and angular momentum.
3. Assume a constant (nonzero) value for the angular momentum and find conditions for a circular orbit with $u = 0$. Linearize the radial dynamics about the circular orbit that you found and compute the eigenvalues of the linear state matrix. Comment on the stability of this "relative equilibrium."
4. Assume that r and $\dot{r}$ are available for feedback and compute the control law u which minimizes

$$J = \frac{1}{2} \int_0^\infty R u(\tau)^2 d\tau$$

where $R > 0$ is a scalar constant. (*Hint:* You must solve a 2×2 algebraic Riccati equation.) Comment on stability of the desired circular orbit under this choice of feedback.

The various criteria for the determination of minimum carrier approach speed (V_{PA}) include this: "The slowest acceptable approach airspeed must provide adequate over the nose field of view." Evaluation of this criterion is specified as follows: With the airplane at an altitude of 600 ft AGL in level flight and with the pilot's eye at the design eye position, the waterline at the stern of the ship must be visible upon intersecting a 4 deg optical glide slope. The source of the optical glide slope is 500 ft forward of the ramp of the ship and 65 ft above the water. Figure 1 below represents this condition.

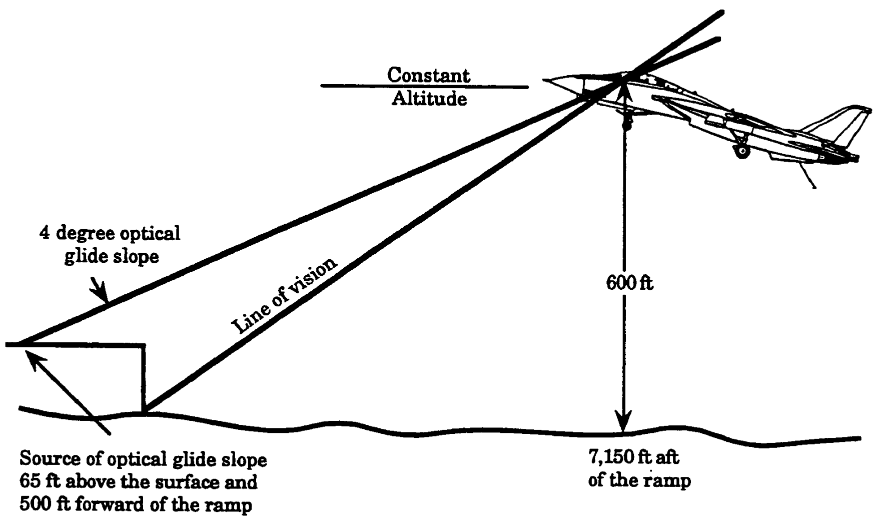


Figure 1: Over the Nose Field of View

Analyze the relationship of this criterion to the distance at which the pilot loses sight of the waterline during an approach. Variables that must be considered include the Wind-Over-Deck ($V_{Atmos/Ship}$); the airspeed of the aircraft, $V_{Acft/Atmos}$, which has magnitude V_{PA}; and the actual glideslope angle θ_{GS} which is either 3.5° or 4.0°. State all assumptions made.